1.1 Introduction

A power system mainly consists of generating stations, transmission lines and distribution systems. Generating stations and distribution systems are connected through transmission lines, which also connect one power system grid to another. A distribution system connects all loads in a particular area to the transmission lines.

A three phase power system is said to be symmetrical, when the system viewed from any phase is similar. This means that, in symmetrical systems, the self impedances of all the three phases are equal and the mutual impedances, if any, between the three phases are same. The three phase voltages (or currents) are said to be balanced if the three voltages (or currents) are equal in magnitude and have the same phase angle difference with respect to each other. In symmetrical, balanced three phase systems, the phase angle difference between the voltages (or currents) will be equal to 120° electrical.

For planning the operation, improvement and expansion of power system, it is required to make a thorough analysis of the system. This necessitates the modeling of the power system network. A complete model of a large interconnected power system representing all the three phases becomes too complicated, rendering it almost impossible for analysis. However, because of symmetry of the system and the balanced nature of the voltages, a three phase, symmetrical, balanced system can be reduced to a single phase system for the purpose of analysis. This results in considerable simplification of the three phase network.

1.2 Circuit Models of Power system components

Synchronous machines, transformers, transmission lines, static and dynamic loads are the major components of a power system. In this section, the single phase equivalent circuits of these components are discussed in brief.

a) Equivalent circuit of a synchronous machine (non-salient type)

The three-phase equivalent circuit of a synchronous generator depicting the voltage generated and the impedance of each phase is shown in fig. 1.1(a). Let us consider the generator to be balanced and perfectly symmetrical, then $E_a=E_b=E_c=E_g$ (say) and $Z_a=Z_b=Z_c=Z_s$ (say). Therefore, the three phase network can be replaced by a single phase network as shown in fig. 1.1(b).

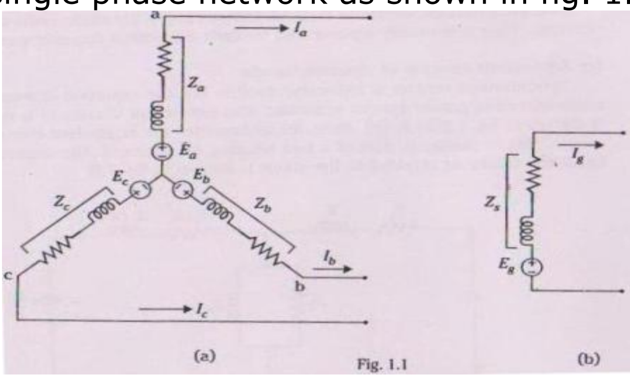
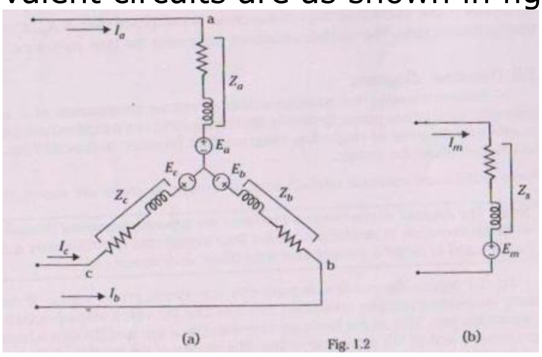


Fig. 1.1

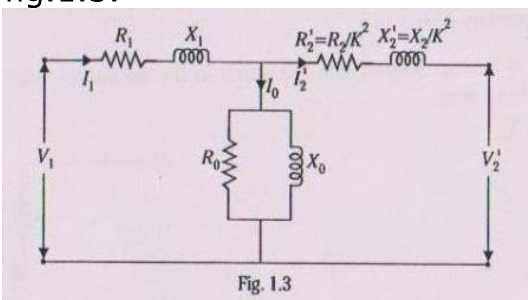
Note:

A synchronous motor receives electrical power and converts it into mechanical power. Therefore the direction of current in motor is opposite to that of generator. Hence its equivalent circuits are as shown in fig. 1.2(a) and (b)



b) Equivalent circuit of a two winding transformer

The well known equivalent circuit of a two winding transformer referred to its primary side as shown in fig.1.3.



Here, R_1 and X_1 are the resistance and reactance of the primary side, R_2 and X_2 are the resistance and reactance of the secondary side. $K=N_2/N_1=V_2/V_1$ is the voltage transform ratio. R_0 and X_0 constitute the exciting circuit of the transformer.

c) Equivalent circuit of a transmission line.

The transmission line is represented usually by its nominal p-circuit. This is shown in fig. 1.4.

Where,

Z= total series impedance of the line per phase.

Y= total shunt admittance per phase.

V_s and I_s = sending end voltage and current respectively.

V_R and I_R = receiving end voltage and current respectively.

d) Equivalent circuit of a three winding transformer.

Both the primary and secondary winding of a two winding transformer have the same kVA rating, but all three windings of a three winding transformer may have different kVA ratings.

The symbol of a three winding transformer is shown in fig. 1.5(a). The three windings are designed as primary, secondary and tertiary windings. The impedances of these windings are connected in star to represent the single-phase equivalent circuit (with magnetizing current neglected) as shown in fig. 1.5(b). The common point is fictious and unrelated to the neutral of the system.

Let

Z_{ps}= leakage impedance measured in the primary with secondary short circuited and tertiary open.

Z_{pt}= leakage impedance measured in the primary with tertiary short circuited and secondary open.

Z_{st}= leakage impedance measured in the secondary with tertiary short circuited and primary open.

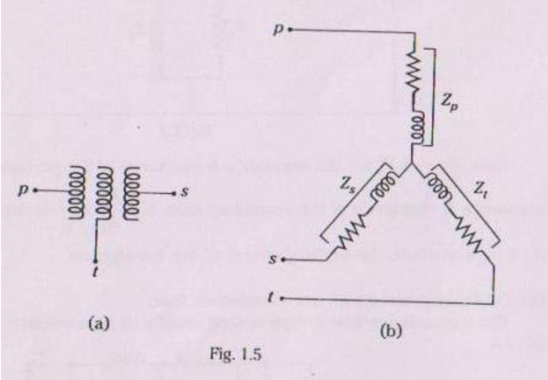


Fig. 1.5

Then, from transformer theory, it can be proved that,

$Z_{ps}=Z_p+Z_s$ …………1.1

$Z_{pt}=Z_p+Z_t$ …………1.2

$Z_{st}=Z_s+Z_t$ …………1.3

Equations 1.1 + 1.2 – 1.3 yield,

$Z_{ps}+ Z_{pt} - Z_{st} = 2Z_p$

Or

$Z_p = ½ (Z_{ps}+ Z_{pt} - Z_{st})$ …………………..1.4

Similarly,

$Z_s= ½ (Z_{st}+Z_{ps}-Z_{pt})$…………………1.5

And

$Z_t= ½ (Z_{st}+Z_{pt}-Z_{ps})$ ……………………1.6

The above formulae are used to compute the impedances of the three windings.

Applications of three winding transformers

i) They are used for interconnecting three transmission lines, each working at different voltage and power levels.

ii) Find extensive utilization in high voltage laboratories.

iii) Static capacitors or synchronous condensers may be connected to tertiary windings for reactive power injection into the system.

e) Equivalent circuits of static loads

Electric furnaces, induction heaters, lamps etc are the static loads on a power system network. They are usually represented by their equivalent impedances. (Example 1.9)

f) Equivalent circuits of dynamic loads

Synchronous motors and induction motors are the common dynamic loads that are encountered in power system networks. The equivalent circuit of a synchronous motor is shown in fig. 1.2(a) and (b). Now, let us consider the equivalent circuit of an induction motor. This is similar to that of a two winding transformer. The equivalent circuit of an induction motor as referred to the stator is shown in fig. 1.6.

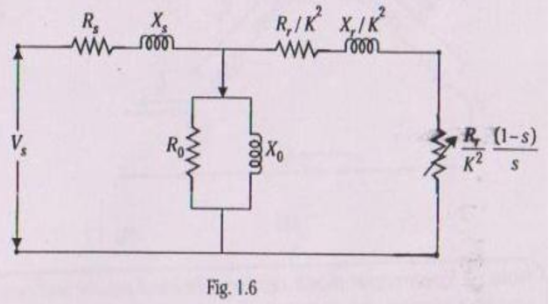


Fig. 1.6

Here,

R_s and X_s denotes stator resistance and reactance respectively. R_r and X_r are rotor resistance and reactance. The exciting or magnetizing circuit is composed of R_0 and X_0. 'K' is the voltage transformation ratio. The variable resistance represents the load on the motor.

1.3 One line diagram

A diagram showing the interconnection of various components of a symmetrical, balanced, three-phase power system by standard symbols on a single-phase basis is called as one-line diagram or single-line diagram. This provides, in concise form, significant information about the system.

Some of the most common symbols used in one-line diagrams are shown in table 1.1

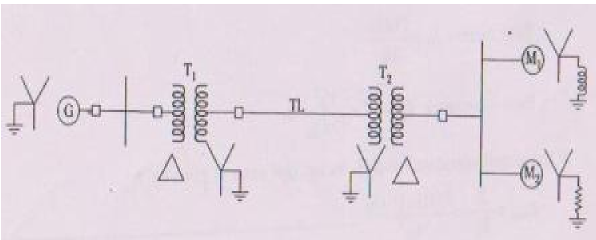
Note: The neutrals of synchronous machines are generally grounded through resistors or inductance coils to reduce the current flow through the neutral during a fault. The coil so used is called a ground fault neutralizer, or Petersen coil.

Fig. 1.7. depicts the one-line diagram of a very simple power system. It consists of a solidly grounded generator connected to a bus and through a step-up transformer to a transmission line. Two motor loads are connected to a bus and through a transformer to the opposite end of the transmission line. The ratings of the machines, their reactances and few relevant datas are also shown.

Sl. No.	COMPONENT	SYMBOL
1.	Line or cable or bus bar	
2.	Circuit breaker	or
3.	Rotating machine	
4.	Two - winding power transformer	or
5.	Three - winding power transformer	or
6.	Current transformer	
7.	Potential transformer	or
8.	Three - phase delta connection	
9.	Three - phase star connection, neutral ungrounded	
10.	Three - phase star connection, solidly grounded	
11.	Three - phase star, neutral grounded through a reactor	
12.	Three - phase star, neutral grounded through a resistor	

Table 1.1

G: 300MVA, 20kV, X”=1.2Ω
T_1: 350MVA, 230V Y/20kV Δ, X=15.2 Ω/ph
T_2: 300MVA, 230V Y/ 13.2kV Δ, X=16 Ω/ph
TL: l=64km, XTL= 0.5 Ω/km
M_1: 200MVA, 13.2kV, X”=1.6 Ω
M_2: 100MVA, 13.2kV, X”= 1.6 Ω
Fig 1.7



1.4 Impedance and reactance diagrams

The one line diagram provides a concise information about the system. The performance of the system load conditions or upon the occurrence of a short circuit cannot be directly calculated using the one-line diagram. It is necessary to obtain an equivalent circuit of the system for the purpose of analysis under the aforesaid conditions. The impedance and reactance diagram enter the screen at this juncture.

The impedance diagram is obtained by replacing each component of the power system by its single-phase equivalent circuit. The synchronous machine is represented by and emf source in series with an appropriate impedance. The transmission line replaced by its equivalent p-circuit, transformer y its equivalent circuit and loads by their equivalent impedances.

The simplified diagram got after omitting all resistances, the magnetizing circuit of the transformer and the capacitance of the transmission line in the impedance diagram is called as the reactance diagram. The simplified representation of some of the important components of power system in reactance diagram are shown in table 1.2

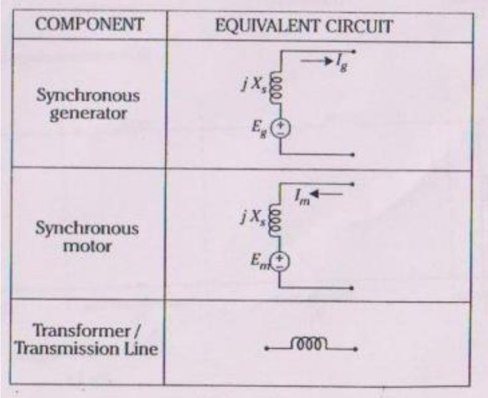


Table 1.2

Of course, omission of resistance introduces some error in the analysis, But, the results may be in almost all cases satisfactory, since the reactance of the system is much larger than its resistance.

Fig 1.8 shows the one line diagram, impedance diagram and reactance diagram of a sample power system.

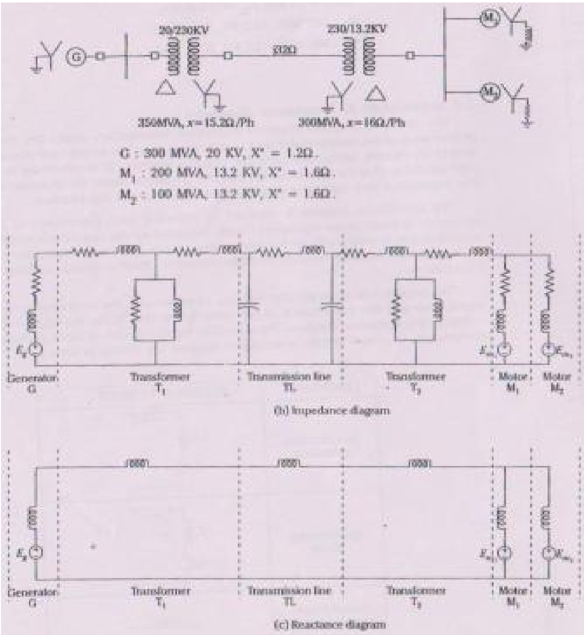


Fig. 1.8

It is re-emphasized here that only a balanced, symmetrical three phase system can be reduced to a single-phase system. In a balanced system, no current flows through the neutral and hence the current limiting impedances are not shown in the equivalent impedance and reactance diagrams.

The impedance and reactance diagram are sometimes called as positive-sequence diagram. This designation will become apparent in chap. 4.

It can be observed from the one line diagram that there are three voltage levels (13.2, 20 and 230 kV) present in the system. The analysis would proceed by transforming all voltages and impedances to ant selected voltage level, say that of transmission line (230 kV). The voltages of generators and motors are transformed in the ratio of transformation and all impedances by the square of the ratio of transformation. This is a very tedious procedure for a large interconnected network with several voltage levels. The per unit (P.U) method discussed in the following section is found quite convenient for power system analysis.

1.5 Per Unit (P.U) system

The per unit value of any quantity is defined as:

$$\frac{\text{the actual value of the quantity in any unit}}{\text{the base or reference value in the same unit}}$$

since the base value always has the same units as the actual value, the per unit value is dimensionless.

If we choose, 50A as the base current, then a current of 30 A is equal to 30/50 = 0.6 in per unit, a current of 80A is equal to 80/50= 1.6 in per unit. It is usual to express voltage, current, volt-amperes and impedance of an electrical system in per unit quantities.

1.5.1 Per Unit system applied to single phase circuits

Let,
Base voltamperes $=(VA)_B$
Base voltage $= V_B$
then
Base current $I_B= (VA)_B/V_B$
Base impedance $Z_B=V_B/I_B=V^2_B/(VA)_B \quad \Omega$
If the actual impedance is Z Ω, its per unit value is given by
$Z_{p.u} = Z/Z_B= Z(\Omega)\times(VA)_B/ V^2_B$ 1.7

For a power system, practical choice of base value are:
Base voltamperes $= (MVA)_B$
Base voltage $= (kV)_B$
Hence,
$Z_{p.u}= Z(\Omega)\times(MVA)_B/ (kV)^2_B$.......................................1.8

1.5.2 Per unit system extended to three phase circuits

Here, rather than obtaining the per unit values using per phase base quantities, the per unit values can be obtained directly by using three phase base quantities. Let

Three phase base megavoltamperes $=(MVA)_B$

Line to line base kilovolts $= (kV)_B$

Assuming star connection (or equivalent star can always be found),

Base current $I_B=((MVA)_B / 3) / ((kV)_B / \sqrt{3}) = ((MVA)_B\times10^6) / (((10^6 / 10^6) \times (\sqrt{3} \times \sqrt{3}) \times 10^3 (kV)_B) / \sqrt{3})$

$= (10^3 \times (MVA)_B) / (\sqrt{3} \times (kV)_B) = (1000 \times (MVA)_B) / (\sqrt{3} \times (kV)_B)$

Base impedance $Z_B= (10^3 \times (kV)_B) / (\sqrt{3}\, I_B) = (1000 \times (kV)_B) / (\sqrt{3} (1000 \times (MVA)_B) / (\sqrt{3} \times (kV)_B))$

$= (kV)^2_B / (MVA)_B \quad \Omega$

If the actual impedance is Z Ω, its per unit impedance is given by,

$Z_{p.u}= Z(\Omega) / Z_B = Z(\Omega) / (kV)^2_B / (MVA)_B$

or $Z_{p.u}= Z(\Omega)\times(MVA)_B / (kV)^2_B$ 1.9

Note:

Sometimes per cent values are used instead of per unit values.

Per cent value = per unit value × 100

per cent value is not convenient for use as the factor of 100 has to be carried in computations.

1.6 Change of Base Quantities

The impedance of a device or a component is usuall specified in per unit or per cent on the basis of its own rated MVA and rated kV. In a large interconnected power system, there will be various devices with different MVA and kV ratings. Hence , it will be convenient for analysis to have a common base for the entire power system. Since all impedance in any part of a system must be expressed on the common base, it is necessary to have a means of converting per unit impedances from one base to another.

Let $(kV)_{B,\ old}$ and $(MVA)_{B,\ old}$ represent old base values and $(kV)_{B,\ new}$ and $(MVA)_{B,\ new}$ represent new base values.

Then, by virtue of Eq. 1.9 , we can write

$Z_{p.u,\ old}$ = p.u impedance of a circuit element on old base

$= Z(\Omega)\times(MVA)_{B,\ old} / (kV)^2_{B,\ old}$ 1.10

$Z_{p.u,\ new}$ = p.u impedance of a circuit element on new base

$= Z(\Omega)\times(MVA)_{B,\ new} / (kV)^2_{B,\ new}$ 1.11

Dividing equation 1.11 by equation 1.10 and rearranging, we get

$Z_{p.u,\ new} = Z_{p.u,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$ 1.12

Eq. 1.12 can be used to convert the p.u impedance expressed on one base value (old) to another base (new).

Example 1.1: Calculate the per unit impedance of a synchronous motor rated 200kVA, 13.2kV and having a reactance of 50 Ω /ph

Solution:
Base values:
The ratings of the machine itself is considered as base values.
Therefore,
Base voltage $(kV)_B$ = 13.2kV
Base megavoltamperes $(MVA)_B$= 200 / 1000 = 0.2 MVA
Then, the reactance of the motor in p.u is given as
$X_{p.u} = X\ (\Omega)\ (MVA)_B\ /\ (kV)^2_B = 50 \times (0.2) / (13.2)^2 = 0.0574$ p.u

Example 1.2 : The primary and secondary sides of a single phase 1 MVA, 4kV / 2kV transformer have a leakage reactance of 2 Ω each. Find the p.u reactance of the transformer referred to the primary and secondary side.

Solution:
Base values:
$(MVA)_B$ = 1 MVA
primary base voltage $(kV_1)_B$ = 4kV
secondary base voltage $(kV_2)_B$ = 2kV
also, it is given that $X_1 = X_2 = 2\ \Omega$.
Primary side:
the total impedances as referred to the primary side $X_{01} = X_1 + X_2'$
where,
$X_2' = X_2\ (kV_1 / kV_2)^2 = 2\ (4 / 2)^2 = 8\ \Omega$
therefore $X_{01} = 2+8 = 10\ \Omega$
$(X_{01})_{p.u} = X_{01}(\Omega) \times (MVA)_B / (kV_1)^2_B = 10\ (1 / 4)^2 = 0.625$p.u ....a)

secondary side:
the total impedance as referred to the secondary side is $X_{02} = X_2 + X_1'$
$X_1' = X_1\ (kV_2 / kV_1)^2 = 2\ (2 / 4)^2 = 0.5\ \Omega$
therefore $X_{02} = 2+0.5 = 2.5\ \Omega$
$(X_{02})_{p.u} = X_{02}(\Omega) \times (MVA)_B / (kV_2)^2_B = 2.5\ (1 / 2)^2 = 0.625$p.u ....b)
from eq. a and b, it can be observed that p.u reactance of the transformer referred to primary side and secondary side is the same, though their ohmic values are different.

Example 1.3: show that the per unit impedance of a transformer is the same irrespective of the side on which it is calculated.

Solution:

Base values:
Let,
$(MVA)_B$= rated MVA of the transformer.
$(kV_1)_B$= base voltage in the primary side.
$(kV_2)_B$= base voltage in the secondary side.
Also, let Z_{01} be the impedance of the transformer reffered to primary side and Z_{02} the impedance as referred to the secondary.
We have,
$(Z_{01})_{p.u} = Z_{01}(\Omega) \times (MVA)_B / (kV_1)^2_B$ ....a)
and
$(Z_{02})_{p.u} = Z_{02}(\Omega) \times (MVA)_B / (kV_2)^2_B$ ....b)
where,
$Z_{02}(\Omega)=Z_{01}(\Omega) \times ((kV_2)^2_B / (kV_1)^2_B)$ ....c)
substituting eq. c) in eq. b), we get,
$(Z_{02})_{p.u}= Z_{01}(\Omega) \times ((kV_2)^2_B / (kV_1)^2_B) \times ((MVA)_B / (kV_2)^2_B) = Z_{01}(\Omega) \times (MVA)_B / (kV_1)^2_B$
$(Z_{02})_{p.u}=(Z_{01})_{p.u}$
Thus, it is proved that the per unit impedance of a transformer is the same whether computed from primary or secondary side.

Example 1.4: three winding transformer has rating as follows:
Primary : Y connected, 6.6kV, 15MVA
secondary: Y connected, 33kV, 10MVA
tertiary : Δ connected, 2.2kV, 7.5MVA
Leakage impedance measured from primary side as Z_{ps}=j0.232 Ω , Z_{pt} = j0.29 Ω and on the secondary side Z_{st}= j8.7 Ω Find the star connected equivalent on a base of 15MVA, 6.6kV in the primary circuit. Neglect resistances.

Solution:
Base values:
primary side: $(MVA)_B$= 15MVA, $(kV_p)_B$= 6.6kV
secondary side: $(MVA)_B$= 15MVA, $(kV_s)_B$= 6.6 x 33 / 6.6 =33kV
per unit leakage impedance.
$(Z_{ps})_{p.u}= Z_{ps}(\Omega) \times (MVA)_B/(kV_p)_B^2 = j0.232 \times 15 / 6.6^2 = j0.08$p.u
$(Z_{pt})_{p.u}= Z_{pt}(\Omega) \times (MVA)_B/(kV_p)_B^2 = j0.29 \times 15 / 6.6^2 = j0.1$p.u
$(Z_{st})_{p.u}= Z_{st}(\Omega) \times (MVA)_B/(kV_s)_B^2 = j8.7 \times 15 / 33^2 = j0.12$p.u

Therefore,
$Z_p = ½ (Z_{ps}+Z_{pt}- Z_{st}) = ½ (j0.08+j0.1-j0.12) = j0.03$p.u
$Z_s = ½ (Z_{ps}+Z_{st}- Z_{pt}) = ½ (j0.08+j0.12-j0.1) = j0.05$p.u
$Z_t = ½ (Z_{st} + Z_{pt} - Z_{ps}) = ½ (j0.12+j0.1-j0.08) = j0.07$p.u
hence, the star connected equivalent circuit is as shown in fig 1.9

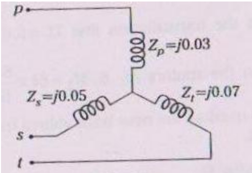


Fig. 1.9

Note:
since the resistances are neglected, the impedances are represented as pure reactances in the above equivalent circuit.

Example 1.5:
Three generators are rated as follows:
Generator 1 : 100 MVA, 33kV, reactance 10%.
Generator 2: 150MVA, 32kV, reactance 8%.
Generator 3: 110MVA, 30kV, reactance 12%.
Determine the reactance of the generator corresponding to base values of 200MVA, 35kV.

Solution:
Here, the reactances of the generators are specified on the basis of their own rated MVA and kV. We consider this as old values. Therefore,
$(X_{g1})_{p.u,\ old}$ = 10% = 0.1p.u on 100MVA, 33kV old bases
$(X_{g2})_{p.u,\ old}$ =8% = 0.08p.u on 150MVA, 32kV old bases
$(X_{g3})_{p.u,\ old}$ =12% = 0.12 p.u on 110MVA, 30kV old bases
the new base values are 200MVA, 35kV. (Given)
hence using the formula
$X_{p.u,\ new} = X_{p.u,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$
we get,
$(X_{g1})_{p.u,\ new} = 0.1 \times (200/100) \times (33^2/35^2) = 0.1777$ p.u
$(X_{g1})_{p.u,\ new} = 0.08 \times (200/150) \times (32^2/35^2) = 0.08916$ p.u
$(X_{g1})_{p.u,\ new} = 0.12 \times (200/110) \times (30^2/35^2) = 0.1603$ p.u

1.7 Advantages of per unit computations

1) Manufacturers usually specify the impedance of an apparatus in per unit or per cent value on the base of the name plate rating of the apparatus.
2) The per unit impedance of the same type of machines, may be of different ratings, lie within a narrow range. However, the ohmic values differ materially for machines of different ratings. Hence, if the per unit impedance of a generator is not known, say, then it can be chosen from a set of tabulated values.
3) The per unit impedance of transformer is the same referred to either side of it.
4) The method of connection of transformers (Y-Y, Y- Δ etc) do not effect the per unit impedance of the transformer.

5) The greatest advantage of using per unit values is that it makes the calculations relatively easier.

Note:
The per unit systems is not without its drawbacks which include:
1) Some equations that hold in the unscaled case are modified when scaled into per unit. Factors such as √3 and 3 are removed or added by the method.
2) Equivalent circuits of the components are modified, making them some what more abstract.

1.8 Per unit impedance and reactance diagrams

In these diagrams, the impedance or reactance of the components are not expressed in their ohmic values, but in per unit values expressed on a common base. This greatly simplifies the task involved in analysis.

Procedure to form per unit reactance diagram from one-line diagram.

From a one-line diagram of a power system we can directly draw the p.u reactance (or impedance) diagram by the following steps given below:

1) Choose a base kilovoltampere $(kVA)_B$ or megavoltampere $(MVA)_B$. This value remains the same for all sections of the power system and preferably is the rated kVA or MVA of one of the electrical apparatus of the system. In case of three phase power system, the $(kVA)_B$ or $(MVA)_B$ is the three phase power rating.

2) Select a base kilovolt $(kV)_B$ for one section of power system. In case of three phase power system, the $(kV)_B$ is a line value. This should preferably be the rated KV of the apparatus whose power rating has been taken as the base. The various sections of the power system works at different voltage levels and the voltage conversion is achieved by means of transformers. Hence the $(kV)_B$ of one section of the power system should be converted to a $(kV)_B$ corresponding to another section using the transformer voltage ratio. In case of three phase transformer (or a bank of three single phase transformers), line to line voltage ratio is used to transfer the $(kV)_B$ on one section to another.

$(kV)_B$ on primary winding of transformer OR $(kV)_B$ on primary side of transformer = $(kV)_B$ on secondary winding of a transformer OR $(kV)_B$ on secondary side of the transformer × (primary winding given voltage rating / secondary winding given voltage rating)

$(kV)_B$ on secondary winding of transformer OR $(kV)_B$ on secondary side of transformer = $(kV)_B$ on primary winding of a transformer OR $(kV)_B$ on primary side of the transformer × (secondary winding given voltage rating / primary winding given voltage rating)

3) In the given problem, the impedance of the components of power system are expressed either in ohms or in p.u which is calculated using the component rating as the base values. In reactance diagram, the resistance are neglected and reactances of all components are expressed on a common base. Hence, starting from one end of power system the reactances of each component should be converted to p.u reactances on the selected new base.

When the specified reactance of the component is in ohms then,

$X_{p.u} = X(\Omega) \times (MVA)_{B,new} / (kV)^2_{B,new}$

When the specified reactance of the component is in p.u on the component rating as base values, then consider the component rating as old base values and selected MVA base, calculated kV base values as new bases. Now the p.u reactance on new base can be calculated using the formula,

$$X_{p.u,\ new} = X_{p.u,\ old} \times \left((MVA)_{B,\ new} / (MVA)_{B,\ old} \right) \times \left((kV)^2_{B,\ old} / (kV)^2_{B,\ new} \right)$$

Example 1.6 : draw the reactance diagram of the system shown in fig. 1.10. the ratings of the components are

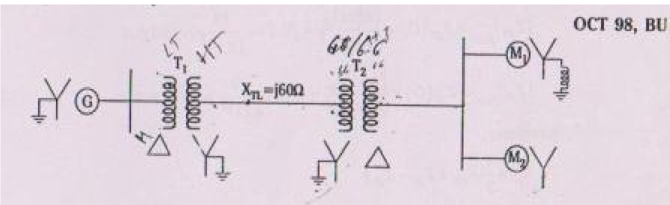


Fig. 1.10

G: 15MVA, 6.6kV, X" = 12%

T_1= 20 MVA, 6.6/66 kV, X=8%

T_2= 20 MVA, 66/6.6 kV, X=8%

M_1 & M_2: 5MVA, 6.6kV, X"=20%

solution:

Base values:

Let us consider(choose) the ratings of the generator as base values.

Therefore,

base megavoltamperes, $(MVA)_B$ = 15MVA (this is same for the entire system)

base kilovolt on the generator G, $(kV)_B$ = 6.6kV

The various sections of the power system works at different voltage levels. Hence, the base kilovolts are different in the different sections. The voltage conversion is achieved by means of transformer voltage ratio.

Base kilovolt on the secondary side of the transformer T_1 OR base kV on transmission line section =
base kilovolts on the primary side of the transformer T_1× (its secondary winding voltage rating / its primary winding voltage rating)= 6.6 × (66 / 6.6) = 66 kV

Base kilovolts on the secondary winding of the transformer T_2=OR base kV on the motors M_1 & M_2 = 66 × (6.6 / 66) = 6.6 kV

These values are used as the new values for calculation of p.u reactances of the different components.

Reactance of generator G:
$X_{g,\,new} = X_{g,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$
$= j0.12 \times (15 / 15) \times (6.6^2 / 6.6^2)$
$= j\,0.12$ p.u
This value is the same as given because, the new base values are selected on its rating.

Reactance of transformer T_1: (calculated primary side)
$X_{T1,\,new} = X_{T1,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$
$= j0.08 \times (15 / 20) \times (6.6^2 / 6.6^2)$
$= j\,0.06$ p.u

Reactance of transformer T_1: (calculated secondary side)
$X_{T1,\,new} = X_{T1,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$
$= j0.08 \times (15 / 20) \times (66^2 / 66^2)$
$= j\,0.06$ p.u

"It is important to know that, it is confirmed by above calculations, in calculating reactance of transformer T_1 that p.u reactance calculated on either side of the transformer is the same. Therefore we can calculate reactance of the transformer any side of it considering either primary side of the base kilovolts old and new values else secondary side base kilovolts old and new values"

Reactance of 60 Ω transmission line TL:
$X_{TL} = X(\Omega) \times (MVA)_B / (kV)^2_B = j60 \times 15 / 66^2 = j0.207$p.u

Reactance of transformer T_2: (calculated primary side)
$X_{T2,\,new} = X_{T2,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$
$= j0.08 \times (15 / 20) \times (66^2 / 66^2)$
$= j\,0.06$ p.u

Reactance of transformer T_2: (calculated secondary side)

$X_{T2,\ new} = X_{T2,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$

$= j0.08 \times (15 / 20) \times (6.6^2 / 6.6^2)$

$= j\ 0.06$ p.u

"calculate reactance of the transformer in p.u at any one side, as we know that p.u reactance calculated is same on either side of it"

Reactance of motor M_1:

$X_{M1,\ new} = X_{M1,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$

$= j0.2 \times (15 / 5) \times (6.6^2 / 6.6^2)$

$= j\ 0.6$ p.u

Reactance of motor M_2:

$X_{M2,\ new} = X_{M2,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$

$= j0.2 \times (15 / 5) \times (6.6^2 / 6.6^2)$

$= j\ 0.6$ p.u

as ratings for motor M_1 & M_2 are same,

$X_{M1,\ new} = X_{M2,\ new} = j\ 0.6$ p.u

The reactance diagram is as shown in fig 1.11

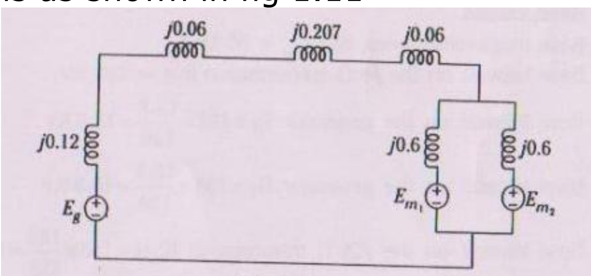


Fig. 1.11

Example 1.7:

Obtain the impedance diagram of the electrical power system shown in fig. 1.12. Mark all impedance values in per unit on a base of 50MVA, 138kV in the 40 ohm line. The machine ratings are:

G_1: 20MVA, 13.2kV, X”=15%

G_2: 20MVA, 13.2kV, X”=15%

M: 30MVA, 6.9kV, X”=20%

Three phase Y-Y transformers: 20MVA, 13.8/138kV, X=10%

Three phase Y- Δ transformers: 15MVA, 6.9/138kV, X=10%

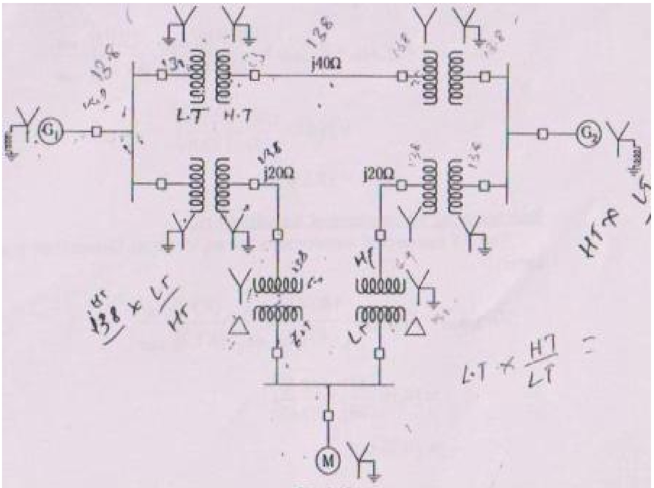


Fig. 1.12

Solution:
Given that to choose:
Base MVA= 50
Base kV on the j40 Ω transmission line = 138
We calculate,
Base kV on the generator section G1= 138 × 13.8 / 138 = 13.8
base kV on the generator section G2= 138 × 13.8 / 138 = 13.8
base kV on the j20 Ω transmission lines = 13.8 × 138 / 13.8 = 138
base kV on the motor section M = 138 × 6.9 / 138 = 6.9
Reactance of j40 ohm transmission line:
$X_{TL1} = X_{TL1}(\Omega) \times (MVA)_B / (kV)^2_B = j40 \times 50 / 138^2 = j0.105$ p.u

Reactance of generators G1 & G2:
the generators G_1 & G_2 are identical. Hence their p.u reactances are the same.
$X_{G1, new} = X_{G2, new} = X_{G1, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$
$= j0.15 \times (50 / 20) \times (13.2^2 / 13.8^2)$
$= j\, 0.343$ p.u

Reactance of Y-Y connected transformers:(calculated considering primary winding old and new base kV values)
The Y-Y connected transformers are all identical. Hence their p.u reactances are the same.
$X_{TR1, new} = X_{TR1, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$
$= j0.1 \times (50 / 20) \times (13.8^2 / 13.8^2)$
$= j\, 0.25$ p.u

Reactance of j20 Ω transmission lines :
It is observed that both the sections of the j20 ohm transmission lines have the same values of reactances and same base values. Hence their p.u reactances will be the same.
$X_{TL2} = X_{TL2}(\Omega) \times (MVA)_B / (kV)^2_B = j20 \times 50 / 138^2 = j0.053$ p.u

Reactance of Y- Δ connected transformers:(calculated considering primary winding old and new base kV values)

Since the Y- Δ connected transformers are all identical. Hence their p.u reactances are the same.

$X_{TR2,\ new} = X_{TR2,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$

$= j0.1 \times (50 / 15) \times (138^2 / 138^2)$

$= j\ 0.33$ p.u

Reactance of motor M:

This motor is connected on to the secondary windings of the Y- Δ transformers (i.e Low Voltage side or Low tension side)

$X_{M1,\ new} = X_{M1,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$

$= j0.2 \times (50 / 30) \times (6.9^2 / 6.9^2)$

$= j\ 0.33$ p.u

Thus, the impedance (reactance) diagram of the system is as in fig 1.13

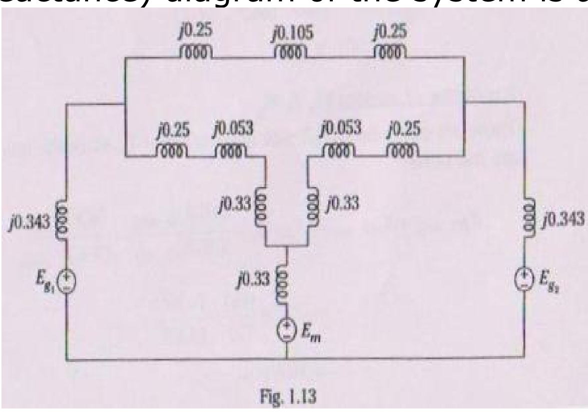


Fig. 1.13

Example 1.8:

The one-line diagram of an unloaded generator is shown in the fig 1.14 Draw the p.u impedance diagram. Choose a base of 50MVA, 13.8kV in the circuit of generator G1.

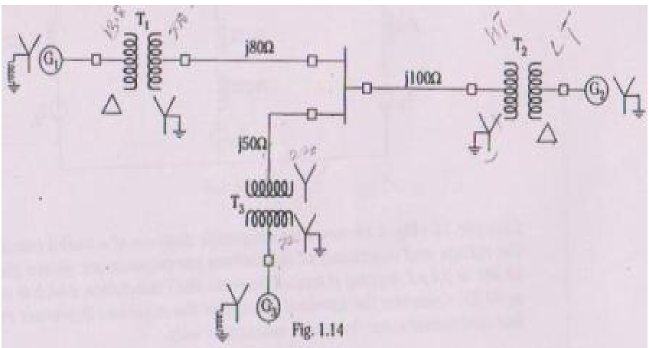


Fig. 1.14

The generators and transformers are rated as follows:

G_1: 20MVA, 13.8kV, X”=0.2 p.u

G_2: 30MVA, 18kV, X”=0.2 p.u

G_3: 30MVA, 20kV, X”=0.2 p.u

T_1: 25MVA, Y220 kV/13.8 kV Δ , X=10%

T_2: Three single phase units each rated 10MVA, 127/18kV, X=10%

T_3=35MVA, 220kV Y/22kV Y, X=10%

solution:
Base values:
given that to choose,
base MVA=50
base kV on the generator G_1=13.8
we calculate,
base kV on the j80 ohm transmission line= 13.8 × 220 / 13.8 = 220
base kV on the j50 ohm transmission line=220
base kV on the j100 ohm transmission line=220
(as all the transmission lines are connected to the same bus, so base kV on them is the same)
base kV on the generator G3= 220 × 22 / 220 = 22

The transformer T_2 is a three phase bank formed using three single phase transformers with a voltage rating of 127/18kV. In this, the HT side is star connected and LT side is delta connected.

Voltage ratio of line voltage of 3-phase transformer bank T_2= √3 × 127 / 18 = 220kV/18kV
(as primary winding is star connected $V_{line}=\sqrt{3}\,V_{ph}$, and secondary is Δ, $V_{line}=V_{ph}$)

base kV on the generator G_2= 220 × 18 / 220 = 18

Reactance of generator G_1:

$$X_{G1,\,new} = X_{G1,\,old} \times \left((MVA)_{B,\,new} / (MVA)_{B,\,old} \right) \times \left((kV)^2_{B,\,old} / (kV)^2_{B,\,new} \right)$$
$$= j0.2 \times (50 / 20) \times (13.8^2 / 13.8^2)$$
$$= j\,0.5 \text{ p.u}$$

Reactance of transformer T_1: (calculated primary side)

$$X_{T1,\,new} = X_{T1,\,old} \times \left((MVA)_{B,\,new} / (MVA)_{B,\,old} \right) \times \left((kV)^2_{B,\,old} / (kV)^2_{B,\,new} \right)$$
$$= j0.1 \times (50 / 25) \times (13.8^2 / 13.8^2)$$
$$= j\,0.2 \text{ p.u}$$

Reactance of transmission lines:
j80 ohm line,
$X_{TL1}= X_{TL1}(\,\Omega\,) \times (MVA)_B / (kV)^2_B = j80 \times 50 / 220^2 = j0.083$p.u
j100 ohm line,
$X_{TL2}= X_{TL2}(\,\Omega\,) \times (MVA)_B / (kV)^2_B = j100 \times 50 / 220^2 = j0.1033$p.u
j50 ohm line,
$X_{TL3}= X_{TL3}(\,\Omega\,) \times (MVA)_B / (kV)^2_B = j50 \times 50 / 220^2 = j0.0516$p.u

Reactance of transformer T_2: (calculated secondary side)
as this is a bank of three single phase transformers, hence,
base MVA old = 10 ×3 = 30

$$X_{T2,\ new} = X_{T2,\ old} \times \left((MVA)_{B,\ new} / (MVA)_{B,\ old} \right) \times \left((kV)^2_{B,\ old} / (kV)^2_{B,\ new} \right)$$
$$= j0.1 \times (50 / 30) \times (220^2 / 220^2)$$
$$= j\,0.1667 \text{ p.u}$$

Reactance of generator G_2:
this is connected to the LT side of T2,

$$X_{G2,\ new} = X_{G2,\ old} \times \left((MVA)_{B,\ new} / (MVA)_{B,\ old} \right) \times \left((kV)^2_{B,\ old} / (kV)^2_{B,\ new} \right)$$
$$= j0.2 \times (50 / 20) \times (18^2 / 18^2)$$
$$= j\,0.333 \text{ p.u}$$

Reactance of transformer T_3: (calculated secondary side of it)

$$X_{T3,\ new} = X_{T3,\ old} \times \left((MVA)_{B,\ new} / (MVA)_{B,\ old} \right) \times \left((kV)^2_{B,\ old} / (kV)^2_{B,\ new} \right)$$
$$= j0.1 \times (50 / 30) \times (22^2 / 22^2)$$
$$= j\,0.143 \text{ p.u}$$

Reactance of generator G_3:

$$X_{G3,\ new} = X_{G3,\ old} \times \left((MVA)_{B,\ new} / (MVA)_{B,\ old} \right) \times \left((kV)^2_{B,\ old} / (kV)^2_{B,\ new} \right)$$
$$= j0.2 \times (50 / 30) \times (20^2 / 22^2)$$
$$= j\,0.275 \text{ p.u}$$

Using the above values, the reactance diagram is as constructed in fig.1.15

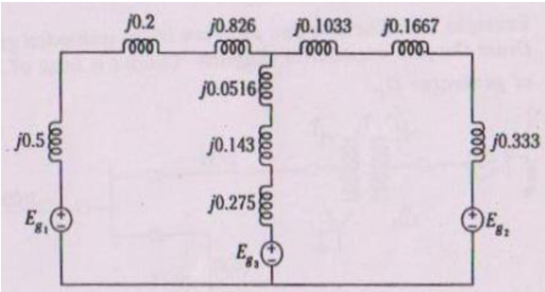


Fig. 1.15

Example 1.9:

Fig 1.16 shows the schematic diagram of a radial transmission system. The ratings and reactances of the various components are shown therin. A load of 60MW at 0.9p.f lagging is tapped from the 66kV substation which is to be maintained at 60kV. Calculated the terminal voltage of the machine. Represent the transmission line and transformer by series reactances only.

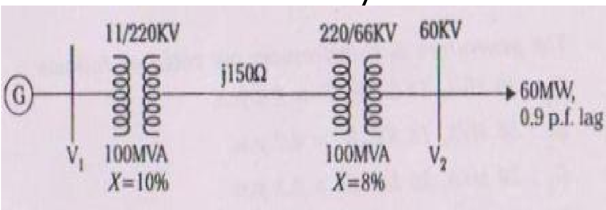


Fig. 1.16

let us choose the base MVA throughout the system be 100
choose the base kV in the transmission line j150 ohm = 220
we calculate,
base kV on the load= 220 × 66 / 220 = 66
base kV on the generator side = 220 × 11 / 220 = 11

Reactance on 11/220kV transformers:(calculated secondary side of it)
$X_{T1,\,new} = X_{T1,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$
$= j0.1 \times (100 / 100) \times (220^2 / 220^2)$
$= j\,0.1$ p.u

Reactance of j150 ohm transmission line:
$X_{TL} = X_{TL}(\Omega) \times (MVA)_B / (kV)^2_B = j150 \times 100 / 220^2 = j0.31$p.u

Reactance on 220/66kV transformers:(calculated primary side of it)
$X_{T2,\,new} = X_{T2,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$
$= j0.08 \times (100 / 100) \times (220^2 / 220^2)$
$= j\,0.08$ p.u

Impedance of load:
The exact value of the impedance of the load is not required in this problem. However, if inevitably to calculate, then it can be determined using the formula,
$Z = |V_L|^2 / (P - jQ)$
where,
V_L=voltage at the terminals of the laod
P=active component of the power at the load
Q=reactive component of the power at the laod.
Hence,
the reactance diagram of the system is as shown in fig1.17

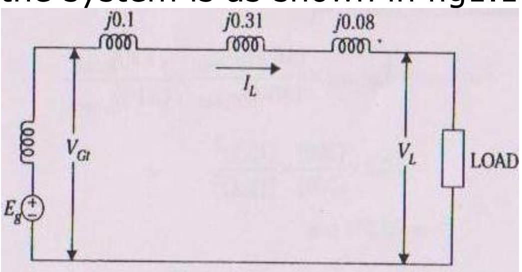


Fig. 1.17

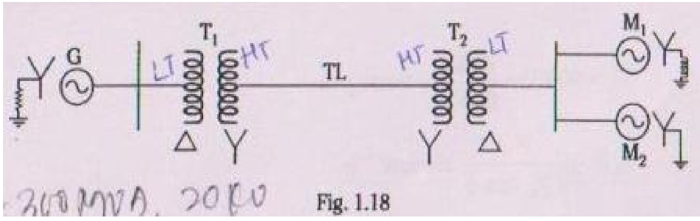
Terminal voltage of the machine:
Let V_{gt}=terminal voltage of the machine
V_L=load voltage
I_L=load current
V_L=60kV= 60kV/66kV = 0.909p.u
$I_L = (P / (\sqrt{3} \times V_L \cos\varphi)) \angle -\cos^{-1}\varphi$
$I_L = ((60\times10^6) / (\sqrt{3} \times 60\times10^3 \times 0.9)) \angle -\cos^{-1}0.9$
$I_L = 641.5 \angle -25.48°$ A
for computation, the value of I_L should be in p.u. Hence, first we determine the base current I_B

I_B=(1000 × $(MVA)_B$) / (√3 × $(kV)_B$)= (1000 × 100) / (√3 ×66)= 874.77 A
therefore, $I_L$in p.u =I_L/I_B=641.5 ∠-25.48° / 874.77 = 0.733∠-25.48° p.u
from fig 1.17 it can be observed that,

$(V_{Gt})_{p.u}$ = $(V_L)_{p.u}$ + I_L (X_{T1}+X_{TL}+X_{T2})
=0.909+0.733∠-25.48°(j0.1+j0.31+j0.08)
=0.909+0.359∠64.16°
=0.909+0.156+j0.323
=1.065+j0.323
=1.112∠16.87° p.u

therefore,
V_{Gt} in kilovolts=$(V_{Gt})_{p.u}$ × base kV on the generator=1.112×11=13.232kV
$|V_{Gt}|$=12.232kV
this is the desired answer.

Example 1.10:
A 300MVA, 20kV, 3φ generator has a reactance of 20%. The generator supplies two motors M1 and M2 over a transmission line of 64km as shown in one line diagram.



The ratings of the components are as follows:
T_1: 350 MVA, 230KV-Y/20KV-Δ, X=10%
T_L: L=64Km, X_{TL}= j0.5Ω/Km.
T_2: Composed of three single phase transformers each rated 127/13.2 KV, 100 MVA with leakage reactance of 10%.
M_1: 200 MVA, 13.2 KV, X"= 20%
M_2: 100 MVA, 13.2 KV, X"= 20%
Select the generator ratings as the base & draw the reactance diagram with all reactance's marked in p.u. If the motors M_1 & M_2 have inputs of 120 MW & 60 MW at 13.2KV and operate at upf, find the voltage at the terminals of the generator.

Solution:
base values:
given that to choose base MVA=300
base kV on the generator =20
base kV on the transmission line = 20 × 230 / 20 = 230
base kV on the motors= 230 × 13.2 / (127√3) =13.8

Reactance of generator G:
$X_{G, new} = X_{G, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$

$= j0.2 \times (300 / 300) \times (20^2 / 20^2)$
$= j\,0.2$ p.u

Reactance of transformer T_1: (calculated considering secondary side of it)

$X_{T1,\ new} = X_{T1,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$
$= j0.1 \times (300 / 350) \times (230^2 / 230^2)$
$= j\,0.085$ p.u

Reactance of transmission line TL:

$X_{TL,new} = X_{TL}(\Omega) \times (MVA)_B / (kV)^2_B = (j0.5 \times 64)\ 300 / 220^2 = j0.181$p.u

Reactance of transformer T_2: (calculated considering primary side)

$X_{T2,\ new} = X_{T2,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$
$= j0.1 \times (300 / 300) \times ((127\sqrt{3})^2 / 230^2)$
$= j\,0.09$ p.u

Reactance of generator M_1:

$X_{M1,\ new} = X_{M1,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$
$= j0.2 \times (300 / 200) \times (13.2^2 / 13.8^2)$
$= j\,0.274$ p.u

Reactance of generator M_2:

$X_{M2,\ new} = X_{M2,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$
$= j0.2 \times (300 / 100) \times (13.2^2 / 13.8^2)$
$= j\,0.549$ p.u

The reactance diagram is as shown in fig 1.19

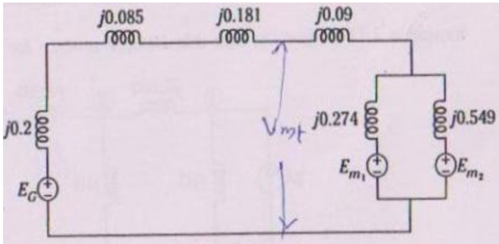


Fig. 1.19

Let,

V_{Mt}=terminal voltage at the motor ends.
V_{Gt}=terminal voltage of the generator.
The total electrical power that flows into the motors is,
P=120+60=180MW at 13.2kV, upf.
Therefore,
the current drawn by the motors,
$I_m = (180 \times 10^6) / (\sqrt{3} \times 13.2 \times 10^3 \times 1) = 7873$ A
it is required to express the current in p.u. Hence, the base current I_B=
$(1000 \times 300) / (\sqrt{3} \times 13.8) = 12551$A
therefore,
I_m in p.u$=I_m/I_B = 7873/12551 = 0.627\angle 0°$p.u

also,
V_{mt} in p.u= 13.2∠0°/13.8=0.96∠0° p.u
from fig 1.19
it can be observed that,
$(V_{Gt})_{p.u} = (V_{Mt})_{p.u} + I_m (X_{T1}+X_{TL}+X_{T2})$
=0.96∠0°+0.627(j0.085+j0.181+j0.09)
=0.96∠0°+j0.223
=0.986∠13.07°
therefore,
$|V_{Gt}|=|V_{Gt}$ in p.u$|\times$20kV=0.986×20=19.72kV.

1.9 Node Equations and Bus Admittance Matrix

The junctions formed when two or more elements (R, L, C etc) are connected are called Nodes. In a power system network, the buses can be treated as nodes and the voltages of all buses (nodes) can be solved by using the conventional nodal analysis.

Let us consider an example of equation formulation by nodal analysis method. The circuit of fig.1.20 contains four independent nodes(buses) as shown by the circled 1,2,3 and 4. These nodes are called major or principal nodes. The node O, with respect to which all voltages are measured is called as the reference node.

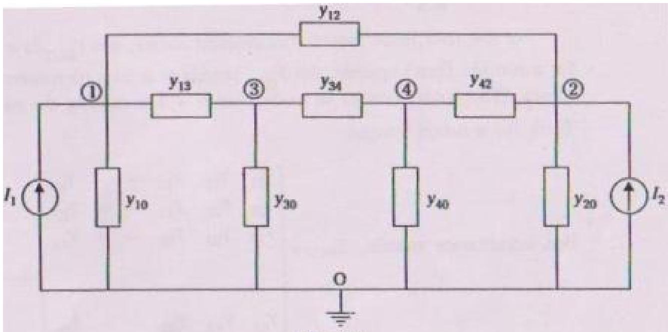


Fig. 1.20

Let V_1, V_2, V_3 & V_4 be the voltages at the respective nodes(buses). The admittances are marked as shown.

At node-1,

$I_1=(V_1-V_3)y_{13}+(V_1-V_2)y_{12}+V_1y_{10}$
$=(y_{10}+y_{12}+y_{13})V_1-y_{12}V_2-y_{13}V_3$
$=Y_{11}V_1+Y_{12}V_2+Y_{13}V_3+Y_{14}V_4$ 1.13

where,

$Y_{11}=y_{10}+y_{12}+y_{13}$
$Y_{12}=-y_{12}$
$Y_{13}=-y_{13}$
$Y_{14}=0$ 1.14

(Here y_{10} is the shunt charging admittance at node-1). Similarly, we can formulate nodal current equations at other nodes as,

$I_2=Y_{21}V_1+Y_{22}V_2+Y_{23}V_3+Y_{24}V_4$ 1.15
$I_3=Y_{31}V_1+Y_{32}V_2+Y_{33}V_3+Y_{34}V_4$ 1.16
$I_4=Y_{41}V_1+Y_{42}V_2+Y_{43}V_3+Y_{44}V_4$ 1.17
these equations can be written in a matrix form as follows:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

..................1.18

or in a more compact form, the above equations can be written as
I=Y V...................1.19

The Y matrix above is designed as Y_{BUS}, and is called as bus admittance matrix. The node voltages are called bus voltages in power system analysis.

Thus, the performance equations 1.19 can be written as,
$I=Y_{BUS}$ V 1.20

For the four node system considered above, the Y_{BUS} is (4×4) matrix. In general, for a n-node(bus)system, the Y_{BUS} is a (n×n) matrix where n is the number of buses, for a n-bus system
Bus admittance matrix,

$$Y_{BUS} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & - & - & Y_{1n} \\ Y_{21} & Y_{22} & Y_{23} & - & - & Y_{2n} \\ Y_{31} & Y_{32} & Y_{33} & - & - & Y_{3n} \\ . & . & . & & & . \\ . & . & . & & & . \\ Y_{n1} & Y_{n2} & Y_{n3} & - & - & Y_{nn} \end{bmatrix}$$

..................1.21

Each admittance Y_{ii} (i+1,2,3,..................n) is called the self admittance (or driving point admittance) of node 1 and equals the algebraic sum of all the admittances terminating on the node .(Refer eq. 1.14). Each off-diagonal term Y_{ik} (i, k=1,2,3.....n) is the mutual admittance(or the transfer admittance) between nodes i & k and equals the negative of the sum of all admittances connected directly between these nodes. Further, $Y_{ik}=Y_{ki}$ (Refer eq. 1.14)

Thus, as seen above, Y_{BUS} is a symmetric matrix (except when phase shifting transformers are involved, this case is not considered here). Furthermore, Y_{ik} =0 if the buses i & k are not connected (eg. Y_{14}=0). In a large power network, each bus (node) is connected only to a few other buses (usually to three or four buses), thus, the Y_{BUS} of a large network is very sparse i.e it has a large number of zero elements. (this may not be evident in a small system like the sample system of fig 1.20). In a large system consisting of 100 nodes, the non-zero elements may be as small as 2% of the total elements. This greatly reduces the numerical computations required for analysis. Bus admittance matrix is often used in solving load flow problems. It has gained widespread application owing to its simplicity of data preparation & handling.

1.10 Formation of Y_{BUS} by inspection

There are different techniques of assembling the Y_{BUS}, namely the method of singular transformation (MST), the K. Nagappan style and others. Here we confine ourselves to the inspection method only. Accordingly, the admittance matrix may be assembled as

i)The diagonal element of each node is the sum of the admittance connected to it.

ii)The off-diagonal element is the negated admittance between the nodes.

Example 1.11:

Find the bus admittance matrix for the circuit shown in fig.1.21

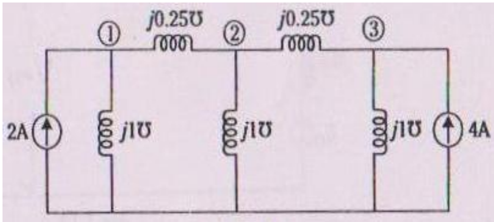


Fig. 1.21

solution:

There are three major nodes in the circuit shown. Using the inspection method, we can write,

Y_{11}=j(1+0.25)=j1.25 mho

Y_{22}=j(0.25+1+0.25)=j1.50 mho

Y_{33}=j(1+0.25)=j1.25 mho

$Y_{12}=Y_{21}$=-j0.25 mho

$Y_{23}=Y_{32}$=-j0.25 mho

$Y_{13}=Y_{31}$=0 (As they are not directly connected.

Thus, the bus admittance matrix of the system is,

$$Y_{BUS}=\begin{bmatrix} j1.25 & -j0.25 & 0 \\ -j0.25 & j1.50 & -j0.25 \\ 0 & -j0.25 & j1.25 \end{bmatrix}$$

Example 1.12:

Determine the Y_{BUS} for a four bus transmission line system the bus diagram of which is as shown in fig 1.22. The impedances and line charging admittances are as tabulated.

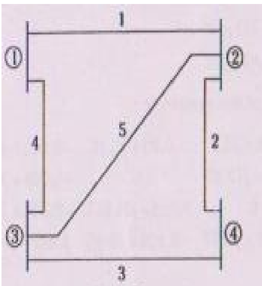


Fig. 1.22

LINE NO.	BUS-CODE p-r	IMPEDANCE Z_{pr}	LINE CHARGING ADMITTANCE FOR EACH LINE, $y_{pr}/2$
1.	1 - 2	j0.20	j0.04
2.	2 - 4	j0.25	j0.05
3.	3 - 4	j0.15	j0.03
4.	3 - 1	j0.10	j0.02
5.	2 - 3	j0.30	j0.06

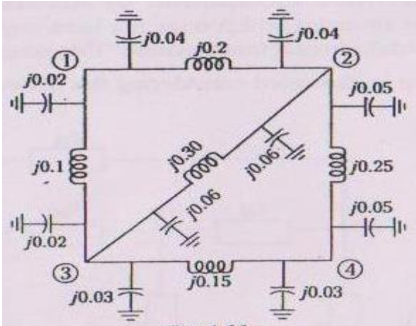


Fig. 1.23

Solution:

including the line charging admittances, the equivalent circuit is as shown in the fig.1.23

The series components are marked with their impedance values, whereas the shunt values are marked with their admittance values.

Hence,

Y_{11}= (1/j0.2)+(1/j0.1)+j0.04+j0.02= -j14.94

Y_{22}=(1/j0.2)+(1/j0.3)+(1/j0.25)+j0.04+j0.06+j0.05= -j12.18

Y_{33}=(1/j0.1)+(1/j0.3)+(1/j0.15)+j0.02+j0.03+j0.06= -j19.89

Y_{44}=(1/j0.25)+(1/j0.15)+j0.03+j0.05= -j10.58

Y_{12}=Y_{21}= -(1/j0.2)=j5

Y_{13}=Y_{31}= -(1/j0.1)= j10

Y_{14}=Y_{41}=0

Y_{23}=Y_{32}= -(1/j0.3)=j3.3

Y_{24}=Y_{42}= -(1/j0.25)=j4

Y_{34}=Y_{43}= -(1/j0.15)= j6.67

Thus,

the Y_{BUS} of the transmission system is:

$$Y_{BUS} = \begin{bmatrix} -j14.94 & j5 & j10 & 0 \\ j5 & -j12.18 & j3.3 & j4 \\ j10 & j3.3 & -j19.89 & j6.67 \\ 0 & j4 & j6.67 & -j10.58 \end{bmatrix}$$

Example 1.13:

Find the bus admittance matrix of the system shown in fig. 1.24. Given that all the lines are characterized by a series impedance of (0.1+j0.7) ohm/km and a shunt admittance of $j0.35\times10^{-5}$ mho/km. Using base values of 220kV and 100MVA, express all impedances & admittances in p.u.

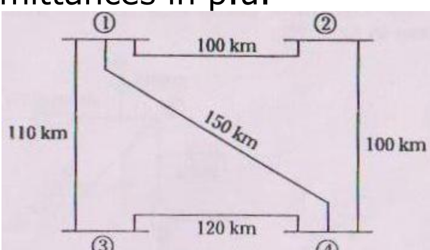


Fig. 1.24

Solution:

First, we find the total series impedance of the line. Then express it in p.u using the formula,

$Z_{p.u} = Z(\Omega)\times(MVA)_B / (kV)^2_B$.

Here,

it is given that $(MVA)_B = 100$

and $(kV)^2_B = 220kV$

then,

find the series admittance of the line.

Next,

the total shunt admittance of of the line is estimated.

Then,

it is expressed in p.u using the formula,

$Y_{p.u} = Y(mho)\times(kV)^2_B/(MVA)_B$

Carefully note that if y_{pr} is the shunt admittance(line charging admittance) of the line,

then it should be divided as $y_{pr}/2$ at both the ends of the line as shown in fig. 1.25

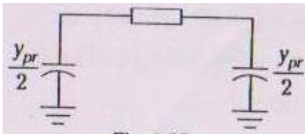


Fig. 1.25

These are tabulated as follows:

BUS-CODE	TOTAL SERIES IMPEDANCE IN Ω	TOTAL SERIES IMPEDANCE IN p.u.	TOTAL SERIES ADMITTANCE IN p.u.	TOTAL SHUNT ADMITTANCE IN ℧	TOTAL SHUNT ADMITTANCE IN p.u.
1-2	10+j70	0.02066+j0.1446	0.968-j6.77	$j350\times10^{-6}$	j0.1694
1-3	11+j77	0.023+j0.159	0.89-j6.16	$j385\times10^{-6}$	j0.1863
1-4	15+j105	0.031+j0.217	0.645-j4.52	$j525\times10^{-6}$	j0.2541
2-4	10+j70	0.02066+j0.1446	0.968-j6.77	$j350\times10^{-6}$	j0.1694
3-4	12+j84	0.0248+j0.1736	0.806-j5.65	$j420\times10^{-6}$	j0.2033

The equivalent circuit depicting the shunt admittances and series admittance is as shown in fig. 1.26

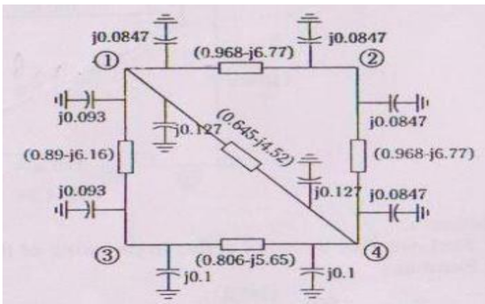


Fig. 1.26

Hence,

$Y_{11}=(0.968-j6.77)+(0.89-j6.16)+(0.645-j4.52)+j0.0847+j0.127+j0.093=(2.5=j17.145)p.u$

$Y_{22}=(0.968-j6.77)+(0.968-j6.77)+j0.0847+j0.0847=(1.936-j13.37)p.u$

Y_{33}=(0.89-j6.16)+(0.806-j5.65)+j0.093+j0.1=(1.696-j11.617)p.u

Y_{44}=(0.645-j4.52)+(0.968-j6.77)+(0.806-j5.65)+j0.0847+j0.127+j0.1=(2.419-j16.63)p.u

$Y_{12}=Y_{21}= -y_{12}$= (-0.968+j6.77)p.u

$Y_{13}=Y_{31}= -y_{13}$= (-0.89+j6.16)p.u

$Y_{14}=Y_{41}= -y_{14}$= (-0.645+j4.52)p.u

$Y_{23}=Y_{32}= -y_{23}$= 0

$Y_{24}=Y_{42}= -y_{24}$= (-0.968+j6.77)p.u

$Y_{34}=Y_{43}= -y_{34}$= (-0.806+j5.65)p.u

Thus, the bus admittance matrix of the system is:

$$Y_{BUS} = \begin{bmatrix} 2.5-j17.45 & -0.968+j6.77 & -0.89+j6.16 & -0.645+j4.52 \\ -0.968+j6.77 & 1.936-j13.37 & 0 & -0.968+j6.77 \\ -0.89+j6.16 & 0 & 1.696-j11.617 & -0.806+j5.65 \\ -0.645+j4.52 & -0.968+j6.77 & -0.806+j5.65 & 2.419-j16.63 \end{bmatrix}$$

Additional Examples:

Example 1.18:

A d.c series motor rated at 220kV, 100A has an armature resistance of 0.15 ohm and field resistance of 0.2 ohm. The friction and windage loss is 1650W. Calculated the efficiency of the machine. Use per unit system.

Solution:

Base values:

base kV=0.22

base current= 100A

hence,

the base power=220×100=0.022MVA

base MVA=0.022

The total resistance of the machine=R=0.15+0.2=0.35 ohm

$R_{p.u}= R(\Omega)\times(MVA)_B / (kV)^2_B$.

$= 0.35\times0.022 / 0.22^2=0.159$p.u

At rated load, the copper loss of the machine in p.u is

$I_{p.u}^2R_{p.u} =(100/100)^2\times0.159=0.159$p.u

Friction and windage loss in p.u is =1650/ (220×100) = 0.075p.u

Hence,

the total loss= 0.159+0.075=0.234p.u

Let the rated output be 1p.u

therefore,

the efficiency=1/(1+0.234)=0.81p.u

Example 1.19:

A 100 MVA, 33kV, 3φ generator has a sub-transient reactance of 155. The generator is connected to the motors through a transmission line and transformers as shown in fig 1.48. The motors have rated outputs of 30MVA, 20MVA and 50MVA at 30kV with 20% subtransient reactance each. The three-phase transformers are rated 100MVA, 32kV Δ/100kV Y with leakage reactance of 8%. The line has a reactance of 50 ohm. Selecting the generator rating as the base in the generator circuit, draw the p.u reactance diagram.

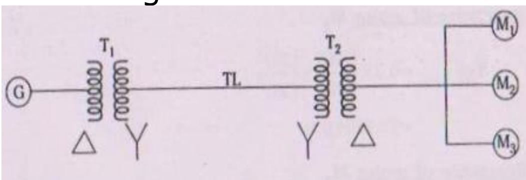


Fig. 1.48

Solution:
base values:
given that to choose,
base MVA= 100
base kV on the generator side= 33kV
we calculate,
base kV on the transmission line=33×100/32=103.125
base kV on the motor side=103.125×32/100=33
Reactance of generator G:
X_G=15%=j0.15p.u

Reactance of transformers T_1& T_2: (calculated considering primary side of T_2)
since the two transformers are one and same, their p.u reactaces are also the same.

$X_{T1, new} = X_{T2, new} = X_{T1, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$
$= j0.08 \times (100 / 100) \times (100^2 / 103.125^2)$
$= j\ 0.075$ p.u

Reactance of transmission line :
$X_{TL,p.u} = X_{TL}(\Omega) \times (MVA)_B / (kV)^2_B = j0.50 \times 100 / 103.125^2 = j0.47$p.u

Reactance of generator M_1:
$X_{M1, new} = X_{M1, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$
$= j0.2 \times (100 / 30) \times (30^2 / 33^2)$
$= j\ 0.5509$ p.u

Reactance of generator M_2:
$X_{M2, new} = X_{M2, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$
$= j0.2 \times (100 / 20) \times (30^2 / 33^2)$
$= j\ 0.8264$ p.u

Reactance of generator M_3:
$X_{M3, new} = X_{M3, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$
$= j0.2 \times (100 / 50) \times (30^2 / 33^2)$
$= j\ 0.3305$ p.u

Using the calculated per unit values of reactances, the p.u reactance diagram is drawn as shown in fig.1.49

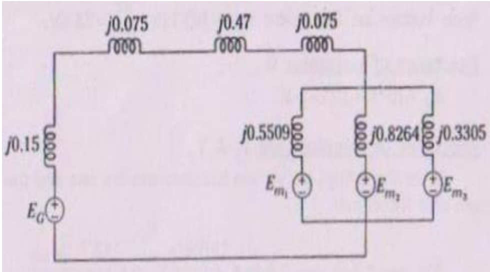


Fig. 1.49

Example 1.20:
The three parts of a single phase electric system are designated A,B,C and are connected to each other through transformers. The transformers are rated as follows:
A-B 10MVA, 13.8-138kV, leakage reactance 10%
B-C 10MVA, 69-138kV, leakage reactance 8%
if the base in circuit B is chosen as 10MVA, 138kV, find the p.u impedance of the 300 ohm resistive lead in circuit C referred to circuit C, B and A. Draw the impedance diagram of the system. Determine the voltage regulation if the voltage at the load is 66kV with the assumption that the voltage input to circuit A remains constant.

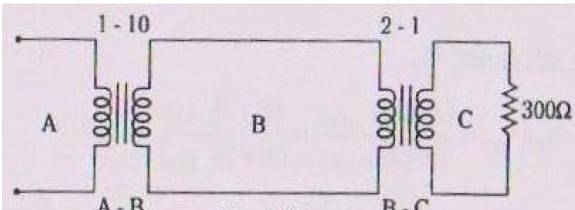


Fig. 1.50

Solution:
base values:
it is given to chose,
base MVA=10
base kV on circuit B=138
we calculate,
base kV on circuit A=138×13.8/138=13.8
base kV on circuit C=138×69/138=69
p.u reactance of the load connected in C=300×$(MVA)_B$/ $(kV)^2_B$=300×10/69^2=0.63p.u

load impedance as referred to circuit B=300×$138^2/69^2$=1200 ohm
load impedance in p.u as referred to B=1200×10/138^2=0.63p.u
similarly,
load impedance as referred to circuit C=1200×$13.8^2/138^2$=12 ohm
load impedance in p.u as referred to C=12×10/13.8^2=0.63p.u
it can be observed that the p.u impedance of the load referred to any part of the system is the same. The impedance diagram of the system is as shown in fig 1.51

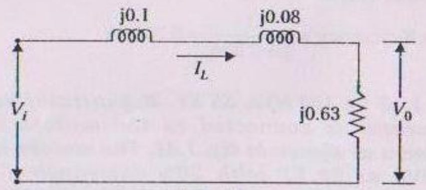


Fig. 1.51

voltage at the load, V_0=66kV/69kV=0.957 p.u
load current, I_L=0.957/0.63=1.52p.u
Voltage at input,
$V_i=I_L(j0.1+j0.08)+V0$
$=(1.52\times j0.18)+0.957=j0.2736+0.957=0.957+j0.2736=0.995\angle 15.95°$
therefore,
percentage regulation=$((|V_i|-|V_0|)/ |V_0|) \times 100=((0.995-0.957)/0.957)\times 100=3.97\%$

Example 1.21:
Fig 1.52 shows the one-line diagram of a simple four bus system. Table below gives the line impedances identified by the buses on which these terminate. The shunt admittance at all buses are negligible.
i)Find Y_{BUS} assuming that the line shown in dotted is not connected
ii)Find Y_{BUS} if the line shown in dotted is connected.

LINE	R_1 p.u.	X_1 p.u.
1 - 2	0.05	0.15
1 - 3	0.10	0.30
2 - 3	0.15	0.45
2 - 4	0.10	0.30
3 - 4	0.05	0.15

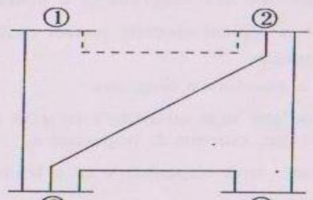


Fig. 1.52

Solution:
we need the admittance values to compute the Y_{BUS}. Hence we construct the following admittance table of the system (taking reciprocals of the impedances).

LINE	G_1 p.u.	B_1 p.u.
1-2	2.0	- 6.0
1-3	1.0	- 3.0
2-3	0.666	- 2.0
2-4	1.0	- 3.0
3-4	2.0	- 6.0

i) with the dotted line unconnected, the equivalent circuit is as shown in fig 1.53

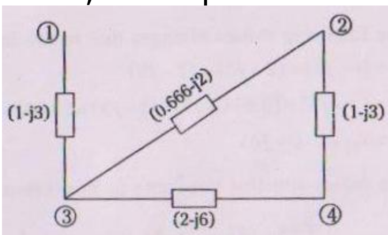


Fig. 1.53

$Y_{11}=(1-j3)$
$Y_{22}=(0.666-j2)+(1-j3)=(1.666-j5)$
$Y_{33}=(1-j3)+(0.666-j2)+(2-j6)=(3.66-j11)$
$Y_{44}=(2-j6)+(1-j3)=(3-j9)$
$Y_{12}=Y_{21}=0$
$Y_{13}=Y_{31}= -(1-j3)= -1+j3$
$Y_{14}=Y_{41}=0$
$Y_{23}=Y_{32}= -(0.666-j2)= -0.666+j2$
$Y_{24}=Y_{42}= -(1-j3)= -1+j3$
$Y_{34}=Y_{43}= -(2-j6) = -2+j6$
hence,
the Y_{BUS} of the system is,

$$Y_{BUS}=\begin{bmatrix}(1-j3) & 0 & (-1+j3) & 0\\ 0 & (1.66-j5) & (-0.666+j2) & (-1+j3)\\ (-1+j3) & (-0.666+j2) & (3.66-j11) & (-2+j6)\\ 0 & (-1+j3) & (-2+j6) & (3-j9)\end{bmatrix}$$

ii) with the dotted line connected, the equivalent circuit becomes as in fig 1.54

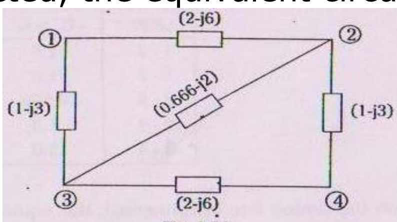


Fig. 1.54

only the following changes due to the inclusion of the line.
$Y_{11}=(1-j3)+(2-j6)=(3-j9)$
$Y_{22}=(2-j6)+(0.666-j2)+(1-j3)=(3.666-j11)$
$Y_{12}=Y_{21}= -(2-j6) = -2+j6$
all other values remains the same as in previous case. Hence the Y_{BUS} is,

$$Y_{BUS}=\begin{bmatrix}(3-j9) & (-2+j6) & (-1-j3) & 0\\ (-2+j6) & (3.666-j11) & (-0.666+j2) & (-1+j3)\\ (-1+j3) & (-0.666+j2) & (3.66-j11) & (-2+j6)\\ 0 & (-1+j3) & (-2+j6) & (3-j9)\end{bmatrix}$$

-----------END-----------