



MODULE-3

TRANSIENT AND STEADY STATE RESPONSE ANALYSIS

LESSON STRUCTURE:

Introduction**Time Response****Steady State Response****Routh's-Hurwitz Criterion**

OBJECTIVES:

- To demonstrate the time response and steady-state error of the system.
- To educate static and transient behavior of a system.
- To demonstrate stability of the various control systems by applying Routh's stability criterion.

Introduction:

Time is used as an independent variable in most of the control systems. It is important to analyse the response given by the system for the applied excitation, which is function of time. Analysis of response means to see the variation of output with respect to time. The output behavior with respect to time should be within these specified limits to have satisfactory performance of the systems. The stability analysis lies in the time response analysis that is when the system is stable output is finite

The system stability, system accuracy and complete evaluation is based on the time response analysis on corresponding results.

Time Response:

The response given by the system which is function of the time, to the applied excitation is called time response of a control system.

Practically, output of the system takes some finite time to reach to its final value. This time varies from system to system and is dependent on different factors. The factors like friction mass or inertia of moving elements some nonlinearities present etc. Example: Measuring instruments like Voltmeter, Ammeter.

Classification:

The time response of a control system is divided into two parts.

1 Transient response $c_t(t)$

2 Steady state response $c_{SS}(t)$

$$\therefore c(t) = c_t(t) + c_{SS}(t)$$

Where $c(t)$ = Time Response

Total Response = Zero State Response + Zero Input Response.



Steady State Response:

It is defined the part of the response which remains after complete transient response vanishes from the system output.

$$\text{i.e, } \lim_{t \rightarrow \infty} c_t(t) = c_{ss}(t)$$

The time domain analysis essentially involves the evaluation of the transient and Steady state response of the control system.

For the analysis point of view, the signals, which are most commonly used as reference inputs, are defined as **standard test inputs**.

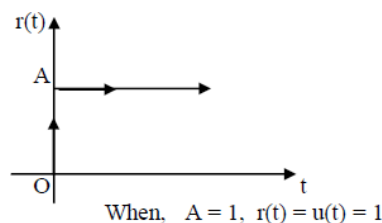
- The performance of a system can be evaluated with respect to these test signals.
- Based on the information obtained the design of control system is carried out. The
- commonly used test signals are
 1. Step Input signals.
 2. Ramp Input Signals.
 3. Parabolic Input Signals.
 4. Impulse input signal.

1. Step input signal (position function)

It is the sudden application of the input at a specified time as usual in the figure or instant any us change in the reference input

Example :-

- a. If the input is an angular position of a mechanical shaft a step input represent the sudden rotation of a shaft.
- b. Switching on a constant voltage in an electrical circuit.
- c. Sudden opening or closing a valve.



The step is a signal who's value changes from 1 value (usually 0) to another level A in Zero time.

In the Laplace Transform form $R(s) = A / S$

Mathematically $r(t) = u(t)$

$$= 1 \text{ for } t \geq 0$$

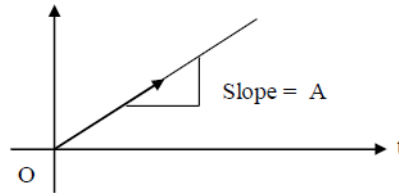
$$= 0 \text{ for } t < 0$$

**2. Ramp Input Signal (Velocity Functions):**

It is constant rate of change in input that is gradual application of input as shown in fig (2 b).

$r(t)$

Ex:- Altitude Control of a Missile



The ramp is a signal, which starts at a value of zero and increases linearly with time.

Mathematically $r(t) = At$ for $t \geq 0$

$$= 0 \text{ for } t \leq 0.$$

In LT form $R(S) = \frac{A}{S^2}$

$$S^2$$

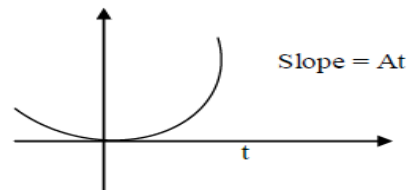
If $A=1$, it is called Unit Ramp Input

Parabolic Input Signal (Acceleration function):

- The input which is one degree faster than a ramp type of input as shown in fig (2 c) or it is an integral of a ramp .
- Mathematically a parabolic signal of magnitude

$$A \text{ is given by } r(t) = \frac{A t^2}{2} u(t)$$

$$r(t) = \begin{cases} \frac{A t^2}{2} & \text{for } t \geq 0 \\ 0 & \text{for } t \leq 0 \end{cases}$$



$$\text{In LT form } R(S) = \frac{A}{S^3}$$

- If $A = 1$, a unit parabolic function is defined as $r(t) = \frac{t^2}{2} u(t)$

$$\text{ie., } r(t) \quad \left\{ \begin{array}{l} \frac{t^2}{2} \text{ for } t \geq 0 \\ 0 \text{ for } t \leq 0 \end{array} \right.$$

$$\text{In LT for } R(S) = \frac{1}{S^3} =$$

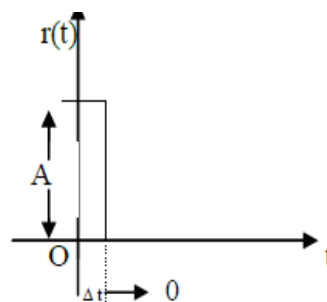
Impulse Input Signal :

It is the input applied instantaneously (for short duration of time) of very high amplitude as shown in fig 2(d)

Eg: Sudden shocks i e, HV due lightening or short circuit.

It is the pulse whose magnitude is infinite while its width tends to zero.

ie., $t \rightarrow 0$ (zero) applied momentarily





Area of impulse = Its magnitude

If area is unity, it is called **Unit Impulse Input** denoted as $\delta(t)$

Mathematically it can be expressed as

$$r(t) = A \text{ for } t = 0 \\ = 0 \text{ for } t \neq 0$$

In LT form $R(S) = 1$ if $A = 1$

Routh's-Hurwitz Criterion

E.J. Routh (1877) developed a method for determining whether or not an equation has roots with +ve real parts without actually solving for the roots.

A necessary condition for the system to be **STABLE** is that the real parts of the roots of the characteristic equation have -ve real parts. This insures that the impulse response will decay exponentially with time.

If the system has some roots with real parts equal to zero, but none with +ve real parts the system is said to be **MARGINALLY STABLE**.

It determines the poles of a characteristic equation with respect to the left and the right half of the S-plane without solving the equation.

The roots of this characteristic equation represent the closed loop poles. The stability of the system depends on these poles. The necessary, but not sufficient conditions for the system having no roots in the right half S-Plane are listed below.

- i. All the co-efficients of the polynomial must have the same sign.
- ii. All powers of S, must present in descending order.
- iii. The above conditions are not sufficient.

In a vast majority of practical systems. The following statements on stability are quite useful.

- i. If all the roots of the characteristic equation have -ve real parts the system is **STABLE**.
- ii. If any root of the characteristic equation has a +ve real part or if there is a repeated root on the j -axis, the system is **unstable**.
- iii. If condition (i) is satisfied except for the presence of one or more non repeated roots on the j -axis the system is limitedly **STABLE**

In this instance the impulse response does not decay to zero although it is bounded. Additionally certain inputs will produce outputs. Therefore **marginally stable** systems are **UNSTABLE**.

The Routh Stability criterion is a method for determining system stability that can be applied to an nth order characteristic equation of the form

$$s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + a_{n-3} s^{n-3} + \dots + a_1 s^1 + a_0 = 0$$

The criterion is applied through the use of a Routh Array (Routh table) Defined as follows:



s^n	a_n	a_{n-2}	a_{n-4}
s^{n-1}	a_{n-1}	a_{n-3}	a_{n-5}
s^{n-2}	b_1	b_2	b_3
s^{n-3}	c_1	c_2	c_3
s^{n-4}	d_1	d_2	
.			
.			
s^2	e_1	a_0	
s^1	f_1		
s^0	a_0		

The ROUTH STABILITY CRITERION is stated as follows,

All the terms in the first column of Routh's Array should have same sign, and there should not be any change of sign.

This is a necessary and sufficient condition for the system to be stable. On the other hand any change of sign in the first column of Routh's Array indicates,

- The System is Unstable, and
- The Number of changes of sign gives the number of roots lying in the right half of S-Plane

Example : find the stability of the system using Routh's criteria. For the equation

$$3S^4 + 10S^3 + 5S^2 + 5S + 2 = 0$$

S^4	3	5	2
S^3	10	5	0
S^2	(2)	(1)	0
S^1	7/2	2	0
S^0	-1/7	0	
S^0	2		

Here two roots are +ve (2 changes of sign) and hence the system is **unstable**.

OUTCOMES:

At the end of the unit, the students are able to:

- Obtain the time response and steady-state error of the system.
- Knowledge about improvement of static and transient behavior of a system.
- Determine stability of the various control systems by applying Routh's stability criterion.



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Notes

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SELF-TEST QUESTIONS:

1. Obtain an expression for time response of the first order system subject to step input.
2. Define
 - 1) Time response.
 - 2) Transient response.
 - 3) Steady state response.
 - 4) Steady state error.
3. Determine the stability of the system whose characteristic equation is given by $S^4 + 6S^3 + 23S^2 + 40S + 50 = 0$, Using Routh's criterion.