MODULE-2

BLOCK DIAGRAMS AND SIGNAL FLOW GRAPHS

LESSON STRUCTURE:

Transfer Functions definition
Block Diagram:
Signal Flow Graphs
Mason's Gain Formula

OBJECTIVES:

- To teach students the concepts of block diagrams and transfer functions.
- To teach students the concepts of Signal flow graph.

Transfer Functions definition

The transfer function of a control system is defined as the ration of the Laplace transform of the output variable to Laplace transform of the input variable assuming all initial conditions to be zero.

$$G(s) = \frac{C(s)}{R(s)} \Rightarrow R(s).G(s) = C(s)$$

function, block representation of systems elements, reduction of block diagrams, Signal flow graphs: Mason's gain formula.

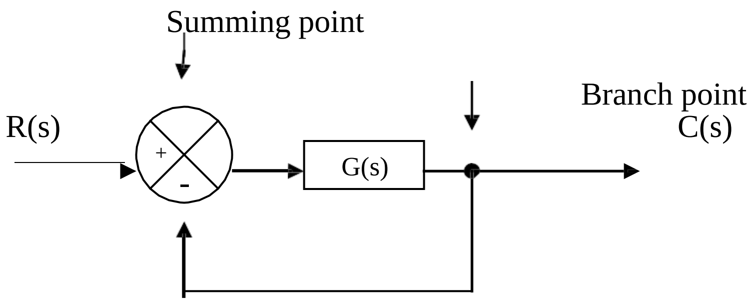
Block Diagram:

A control system may consist of a number of components. In order to show the functions performed by each component in control engineering, we commonly use a diagram called the **Block Diagram**.

A block diagram of a system is a pictorial representation of the function performed by each component and of the flow of signals. Such a diagram depicts the inter-relationships which exists between the various components. A block diagram has the advantage of indicating more realistically the signal flows of the actual system.

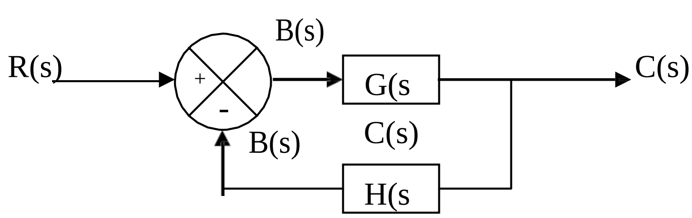
In a block diagram all system variables are linked to each other through functional blocks. The ―Functional Block or simply ―Block is a symbol for the mathematical operation on the input signal to the block which produces the output. The transfer functions of the components are usually entered in the corresponding blocks, which are connected by arrows to indicate the direction of flow of signals. Note that signal can pass only in the direction of arrows. Thus a block diagram of a control system explicitly shows a unilateral property.

BLOCK DIAGRAM OF A CLOSED LOOP SYSTEM .



The output C(s) is fed back to the summing point, where it is compared with reference input R(s). The closed loop nature is indicated in fig1.3. Any linear system may be represented by a block diagram consisting of blocks, summing points and branch points. A branch is the point from which the output signal from a block diagram goes concurrently to other blocks or summing points.

When the output is fed back to the summing point for comparison with the input, it is necessary to convert the form of output signal to that of he input signal. This conversion is followed by the feed back element whose transfer function is H(s) as shown in fig 1.4. Another important role of the feed back element is to modify the output before it is compared with the input.



The ratio of the feed back signal B(s) to the actuating error signal E(s) is called the open loop transfer function.

open loop transfer function = B(s)/E(s) = G(s)H(s)

The ratio of the output C(s) to the actuating error signal E(s) is called the feed forward transfer function .

Feed forward transfer function = C(s)/E(s) = G(s)

If the feed back transfer function is unity, then the open loop and feed forward transfer function are the same. For the system shown in Fig1.4, the output C(s) and input R(s) are related as follows.

$$C(s) = G(s)\,E(s)$$

$$E(s) = R(s) - B(s)$$

$$= R(s) - H(s)\,C(s) \qquad \text{but } B(s) = H(s)C(s)$$

Eliminating E(s) from these equations

$$C(s) = G(s)\,[R(s) - H(s)\,C(s)]$$

$$C(s) + G(s)\,[H(s)\,C(s)] = G(s)\,R(s)$$

$$C(s)[1 + G(s)H(s)] = G(s)R(s)$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\,H(s)}$$

C(s)/R(s) is called the closed loop transfer function.

The output of the closed loop system clearly depends on both the closed loop transfer function and the nature of the input. If the feed back signal is positive, then

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)\,H(s)}$$

SIGNAL FLOW GRAPHS

An alternate to block diagram is the signal flow graph due to S. J. Mason. A signal flow graph is a diagram that represents a set of simultaneous linear algebraic equations. Each signal flow graph consists of a network in which nodes are connected by directed branches. Each node represents a system variable, and each branch acts as a signal multiplier. The signal flows in the direction indicated by the arrow.

Definitions:
Node: A node is a point representing a variable or signal.
Branch: A branch is a directed line segment joining two nodes.
Transmittance: It is the gain between two nodes.
Input node: A node that has only outgoing branche(s). It is also, called as source and corresponds to independent variable.
Output node: A node that has only incoming branches. This is also called as sink and corresponds to dependent variable.
Mixed node: A node that has incoming and out going branches.
Path: A path is a traversal of connected branches in the direction of branch arrow.
Loop: A loop is a closed path.
Self loop: It is a feedback loop consisting of single branch.
Loop gain: The loop gain is the product of branch transmittances of the loop.
Non-touching loops: Loops that do not posses a common node.
Forward path: A path from source to sink without traversing an node more than once.
Feedback path: A path which originates and terminates at the same node.
Forward path gain: Product of branch transmittances of a forward path.

Properties of Signal Flow Graphs:

1. Signal flow applies only to linear systems.
2. The equations based on which a signal flow graph is drawn must be algebraic equations in the form of effects as a function of causes. Nodes are used to represent variables. Normally the nodes are arranged left to right, following a succession of causes and effects through the system.
3. Signals travel along the branches only in the direction described by the arrows of the branches.
4. The branch directing from node X_k to X_j represents dependence of the variable Xj on X_k but not the reverse.

5. The signal traveling along the branch X_k and X_j is multiplied by branch gain a_{kj} and signal $a_{kj}X_k$ is delivered at node X_j.

Guidelines to Construct the Signal Flow Graphs:

The signal flow graph of a system is constructed from its describing equations, or by direct reference to block diagram of the system. Each variable of the block diagram becomes a node and each block becomes a branch. The general procedure is

1. Arrange the input to output nodes from left to right.
2. Connect the nodes by appropriate branches.
3. If the desired output node has outgoing branches, add a dummy node and a unity gain branch.
4. Rearrange the nodes and/or loops in the graph to achieve pictorial clarity.

Mason's Gain Formula:

The relationship between an input variable and an output variable of a signal flow graph is given by the net gain between input and output nodes and is known as overall gain of the system. Masons gain formula is used to obtain the overall gain (transfer function) of signal flow graphs.

Gain P is given by

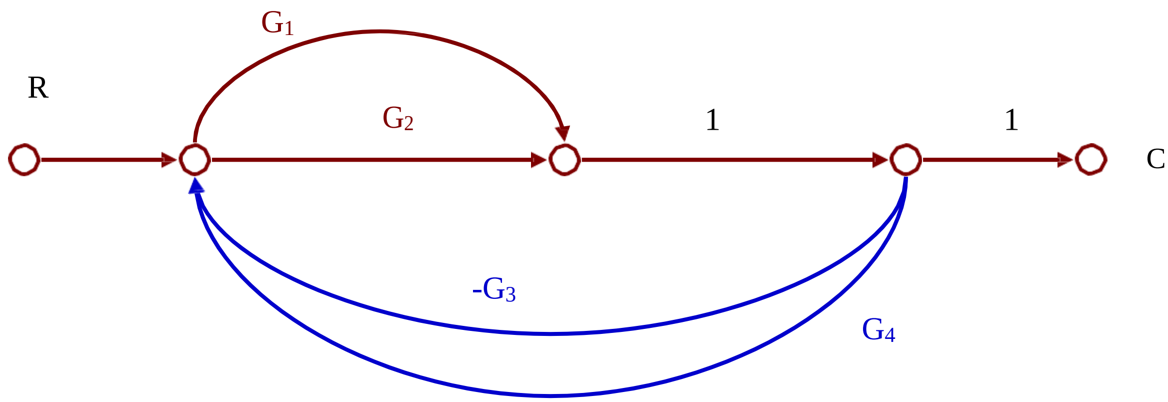
$$P = \frac{1}{\Delta} \sum_k P_k \Delta_k$$

Where, P_k is gain of k^{th} forward path,
Δ is determinant of graph

Δ=1-(sum of all individual loop gains) + (sum of gain products of all possible combinations of two nontouching loops – sum of gain products of all possible combination of three nontouching loops) + ···

Δ_k is cofactor of k^{th} forward path determinant of graph with loops touching k^{th} forward path. It is obtained from Δ by removing the loops touching the path P_k.

Example 1
Obtain the transfer function of C/R of the system whose signal flow graph is shown in Figure



There are two forward paths: Gain of path 1 :

$P_1=G_1$

Gain of path 2 : $P_2=G_2$

There are four loops with loop gains:

$L_1=-G_1G_3$, $L_2=G_1G_4$, $L_3= -G_2G_3$, $L_4= G_2G_4$

There are no non-touching loops.

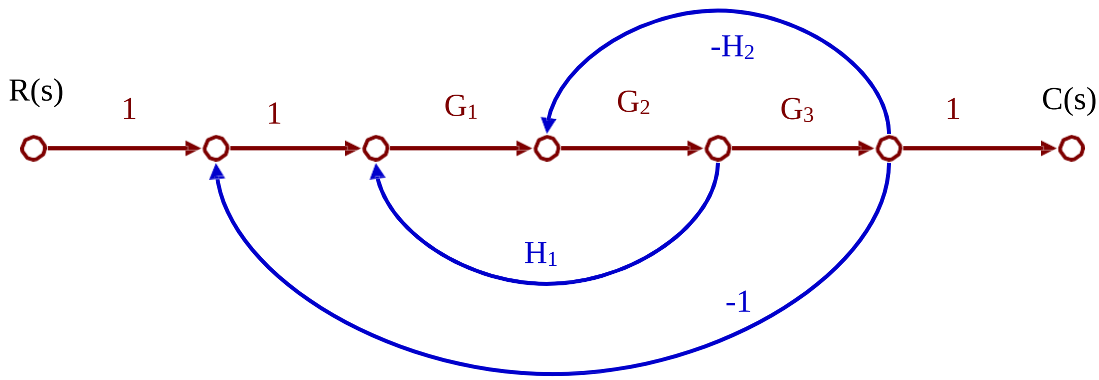
$\Delta = 1+G_1G_3-G_1G_4+G_2G_3-G_2G_4$

Forward paths 1 and 2 touch all the loops. Therefore, $\Delta_1= 1$, $\Delta_2= 1$

The transfer function T = $\dfrac{C(s)}{R(s)} = \dfrac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \dfrac{G_1 + G_2}{1+G_1G_3 - G_1G_4 + G_2G_3 - G_2G_4}$

Example 2

Obtain the transfer function of C(s)/R(s) of the system whose signal flow graph is shown in Figure



There is one forward path, whose gain is: $P_1=G_1G_2G_3$

There are three loops with loop gains:

$L_1=-G_1G_2H_1$, $L2=G_2G_3H_2$, $L3= -G_1G_2G_3$

There are no non-touching loops.

$\Delta = 1-G_1G_2H_1+G_2G_3H_2+G_1G_2G_3$

Forward path 1 touches all the loops. Therefore, $\Delta_1= 1$.

The transfer function $T =\frac{C(s)}{R(s)} = \frac{P_1\Delta_1}{\Delta} = \frac{G_1G_2G_3}{1-G\,G\,H\;+G\,G\,H\;+G\,G\,G}$

OUTCOMES:

At the end of the unit, the students are able to:

1. Represent the systems consisting of number of components in the form of block diagrams and signal flow graphs.
2. Develop mathematical models using reduction technique of these block diagrams and signal flow graphs.

SELF-TEST QUESTIONS:

1. Differentiate between Block diagram and Signal flow graph techniques.
2. Explain the rules for constructing Signal flow graph.
3. Reduce the block diagram shown in Figure 1, to its simplest possible form and find its closed loop transfer function.

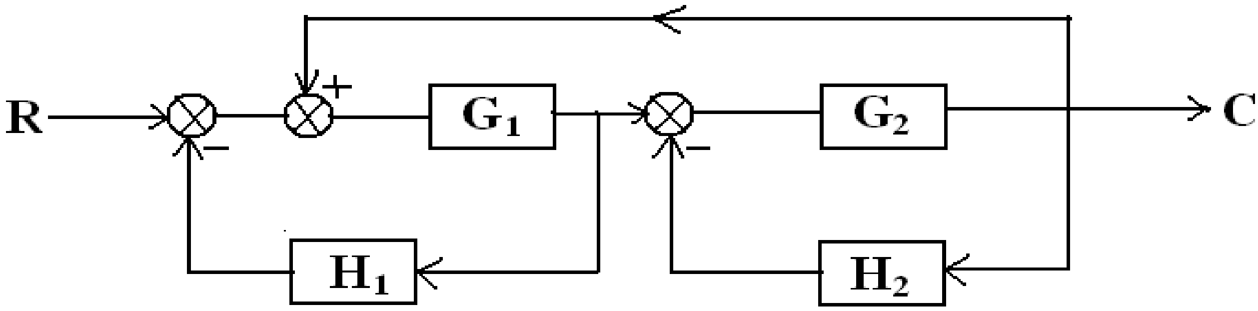


Figure 1

4. Find **C(S)/R(S)** for the following system using Mason's gain rule shown in figure 2.

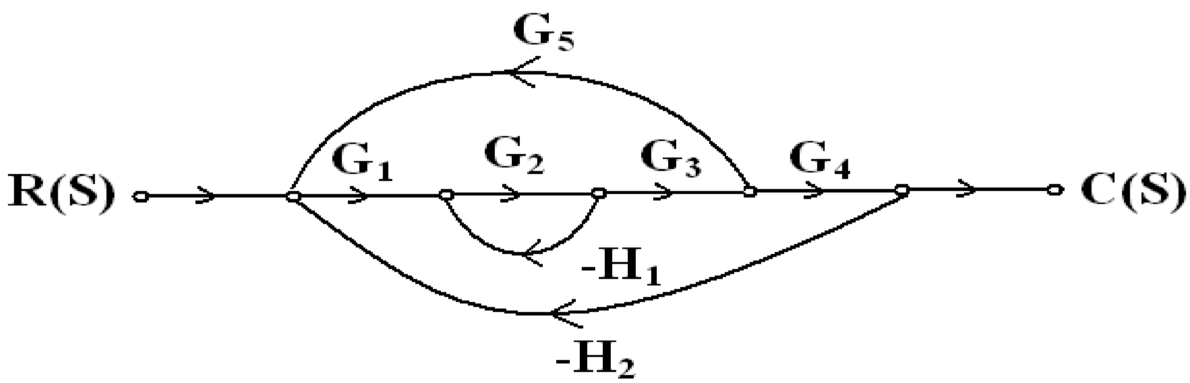


Figure 2