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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

CONTROL SYSTEMS NOTES (15EC43)
(As per Choice based Credit System (CBCS) Scheme)
IVTH SEMESTER

MODULE-2

Syllabus: Time Response of feedback control systems: Standard test signals, Unit step response of First and Second order Systems. Time response specifications, Time response specifications of second order systems, steady state errors and error constants. Introduction to PI, PD and PID Controllers (excluding design).

Study Material Referred:

- ✓ Modern Control engineering-K Ogata.
- ✓ Control Systems Engineering- J.Nagarath and M.Gopal.
- ✓ Automatic Control Systems-Benjamin C. Kuo.
- ✓ Linear Control Systems –B S Manke.
- ✓ Control Systems- Anand Kumar.
- ✓ **VTU Previous year Question papers (2010-2016)**.
- ✓ Problems and solution of controls systems -AK Jairath.

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Note: 1) Assignment Carries 05 Marks (To be submitted before 2ND Module test).

2) Quiz will be conducted after each Module test for 05 Marks.

3) Average of both will be taken as final marks for 05 Marks.

Module 2 : Time Response Of Feedback Control Systems

* → Time response is the variation of the Controlled output of the system with respect to time, when the system is subjected to test signal.

or

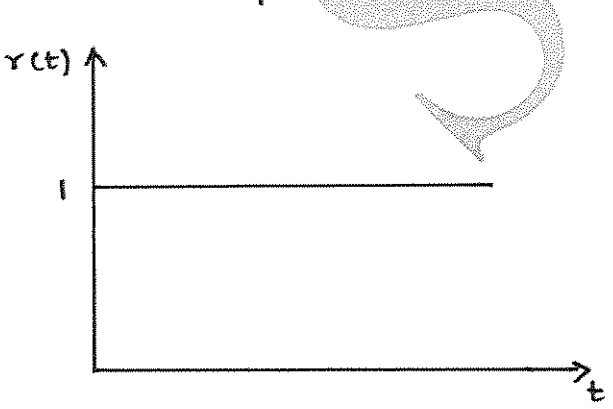
* → Time response of control system is defined as, To how a system behaves in accordance with time when a specified input test signal is applied.

* → Thus the time response of a control system is divided in to two parts, they are;

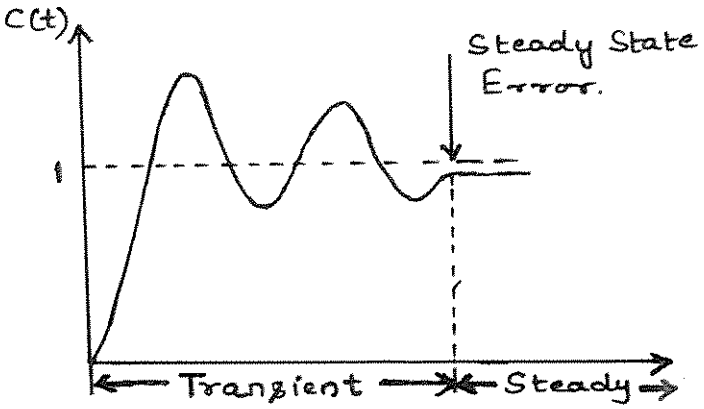
a) Transient Response.

b) Steady State Response.

* → The typical response of a Controlled System for a specified input test signal is as shown below.



(a) Input test Signal.



(b) Output Response.

* → Transient response of the system is defined as the part of time response that goes to zero as time becomes very large. Thus $C_t(t)$ has property that

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i.e, $\lim_{t \to \infty} C_{tr}(t) = 0$

* → The transient part of the time response reveals the nature of the response and gives an indication of speed

* → The steady state response is the part of total response that remains after the transient has died out.

* → The steady state part reveals the accuracy of Control System.

* → Steady state Error is observed if the actual o/p doesnot match with in the output.

* → From the fig, the o/p C(t) is given by.

$$C(t) = C_{tr}(t) + C_{ss}(t)$$

* → The different input Signals that are used to test a Control System

1) Step input.

2) Ramp input.

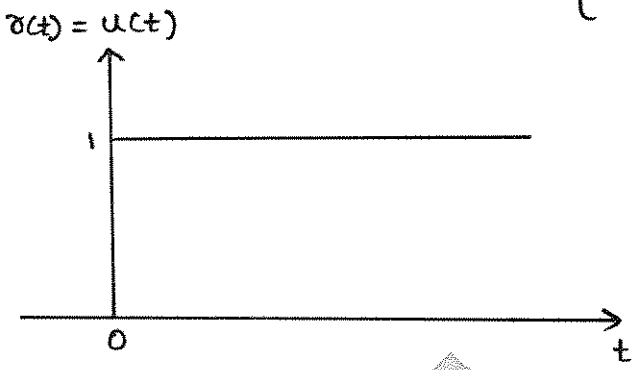
3) Parabolic input.

4) Impulse input.

5) Sinusoidal input.

1) Step Input:

* → It is defined as $r(t) = u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$



By taking Laplace transform of step function.

$$L\{u(t)\} = \frac{1}{s}$$

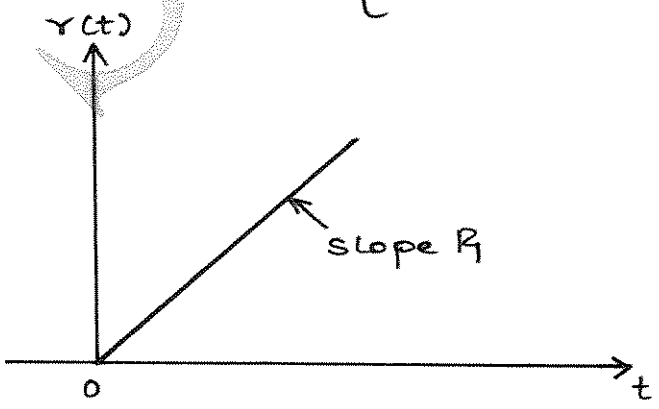
if $r(t) = A.u(t)$

Where A is the Amplitude of the step signal.

$$L[r(t)] = \frac{A}{s}$$

2) Ramp input:

* → It is defined as $r(t) = \begin{cases} R_1 t, & t \geq 0 \\ 0, & t < 0 \end{cases}$



* → For $R_1 = 1$, then $r(t) = t$ and the ramp function is called unit ramp function.

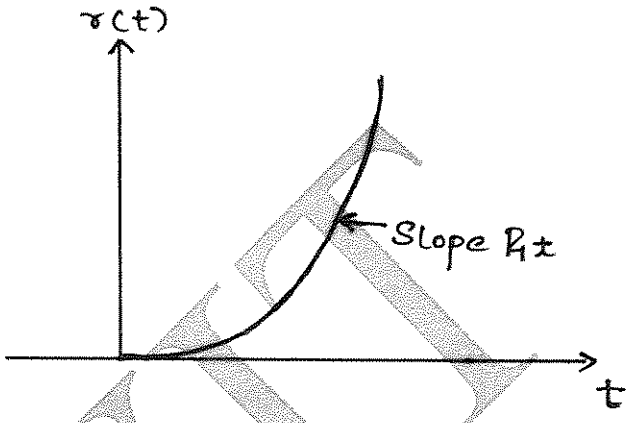
* → By taking Laplace transform

$$L[r(t)] = L[t.u(t)] = \frac{1}{s^2}$$

for $r(t) = A \cdot t\,u(t)$ then $L[r(t)] = \frac{A}{s^2}$

3) Parabolic Input:-

It is defined as $r(t) = \begin{cases} Rt^2/2, & \text{for } t \geq 0 \\ 0, & \text{for } t < 0 \end{cases}$



* → For $R=1$, then $r(t) = \frac{t^2}{2}$ and the parabolic function is called unit parabolic function.

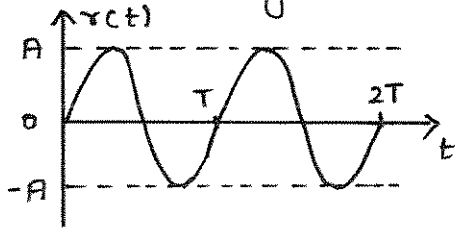
* → By taking laplace transform.

$$L[r(t)] = L\left[\frac{t^2}{2}\right] = \frac{1}{s^3}$$

4) Sinusoidal Input:-

It is defined as $r(t) = \begin{cases} A\sin\omega t, & t \geq 0 \\ 0, & t < 0 \end{cases}$

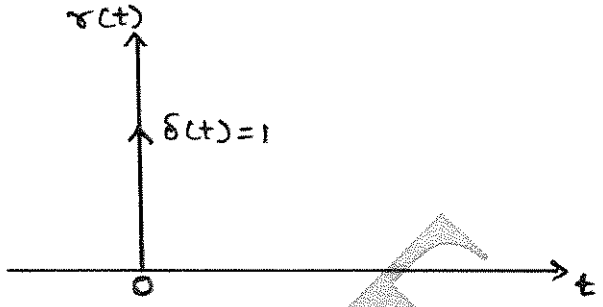
* → Where A is the Amplitude of the sinusoidal signal and ω is the Angular frequency.



$$L\left[A\sin(\omega t)\right] = \frac{A\cdot\omega}{s^2+\omega^2}$$

5) Impulse Input:

* → It is defined as $r(t) = \begin{cases} \delta(t) = 1, & t = 0; \\ 0, & t \neq 0; \end{cases}$



* → By taking Laplace transform

$$L\left[r(t)\right] = L\left[\delta(t)\right] = 1$$

Note :-

1) $L\left[A\cdot u(t)\right] = \frac{A}{s}$

2) $L\left[t^n\cdot u(t)\right] = \frac{n!}{s^{n+1}}$

3) $L\left[e^{\mp at}\right] = \frac{1}{s\mp a}$

4) $L\left[\sin\omega t\right] = \frac{\omega}{s^2+\omega^2}$

5) $L\left[\cos\omega t\right] = \frac{s}{s^2+\omega^2}$

6) $L\left[e^{\mp at}\sin\omega t\right] = \frac{\omega}{(s\mp a)^2+\omega^2}$

7) $L\left[e^{\mp at}\cos\omega t\right] = \frac{(s\mp a)}{(s\mp a)^2+\omega^2}$

8) $L\left[\delta(t)\right] = 1$

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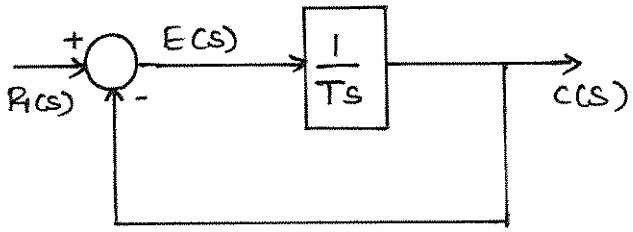
Time Response Of First Order Control System.

* → Consider the block diagram of a general first order Control System as shown below.



* → The Overall transfer function is given by.

$$\frac{C(S)}{R(S)} = \frac{1/Ts}{1 + \frac{1}{Ts} \cdot 1} = \frac{1}{Ts + 1}$$

$$\boxed{\frac{C(S)}{R(S)} = \frac{1}{sT + 1}}$$

* → 'T' is the time constant of the System.

* → The Highest power of 's' in the denominator of the Overall transfer function represents the Order of the Control System.

* → The Output of the System is Expressed as.

$$C(S) = R(S) \cdot \frac{1}{sT + 1}$$

W.K.T that $R(S) = \frac{1}{S}$. $\because R(t) = 1 \_ L[R(t)] = \frac{1}{S}$

$$\therefore C(S) = \frac{1}{S} \cdot \frac{1}{sT + 1}$$

Breaking R.H.S into to Partial fractions

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$$C(s) = \frac{1}{s} \cdot \frac{1}{sT+1} = \frac{A}{s} + \frac{B}{sT+1} \quad \rightarrow (1)$$

$$A = \lim_{s \to 0} \left[s \cdot C(s) \right] = \lim_{s \to 0} \left\{ \frac{1}{sT+1} \right\} = 1$$

$$B = \lim_{s \to -1/T} \left[(sT+1)\, C(s) \right] = \lim_{s \to -1/T} \left\{ \frac{1}{s} \right\} = -T$$

By substituting the value A and B in Equation (1)

$$C(s) = \frac{1}{s} - \frac{T}{sT+1}$$

$$C(s) = \frac{1}{s} - \frac{1}{s + \frac{1}{T}}$$

By taking Inverse laplace transform

$$\boxed{C(t) = 1 - e^{-t/T}} \quad \text{for } t \geq 0. \quad \rightarrow (2)$$

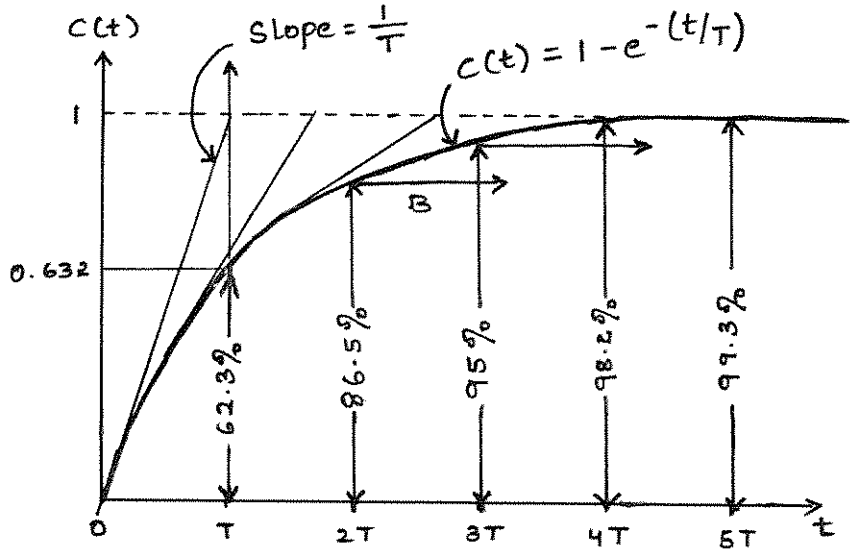
* → Equation 2 states that initially the Output C(t) is zero and finally it becomes unity.

* → At 't' = 'T', the value C(t) = 0.632, or the response C(t) has reached 63.2 % of its total change.

$$\text{i.e. } C(T) = 1 - e^{-T/T} = 1 - e^{-1} = 0.632$$

$$\boxed{C(T) = 0.632}$$

* → Note that the smaller the time constan T, the faster the system response. The response is as shown below.



*→The slope of the tangent line at t=0 is $1/T$ Since.

$$\left.\frac{dc}{dt}\right|_{t=0} = \frac{1}{T} e^{-t/T}\Big|_{t=0} = \frac{1}{T} \quad -(3)$$

* → The output would reach the final value at t=T, if it maintained its initial speed of response. The slope of the response curve c(t) monotonically from $1/T$ at t=0 to Zero at t=∞

* → In one time constant, the Exponential response curve has gone from 0 to 63.2% of the final value. (At t=T)

* → In two time constant, the response reaches 86% of the final value. (At t=2T)

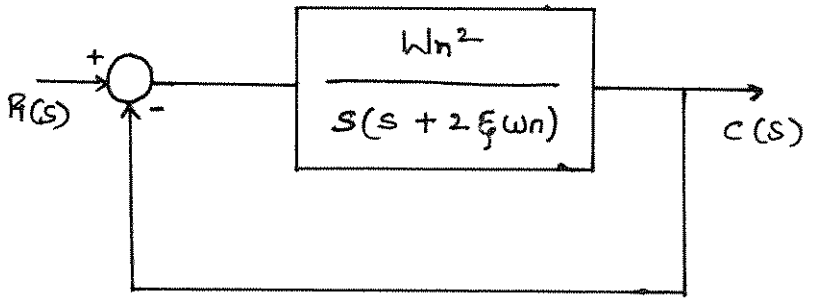
* → At t=3T, 4T and 5T, the response reaches 95%, 98.2% and 99.3% respectively of the final value.

* → Thus For t ≥ 4T, the response remains with in 2% of the final value

* → As seen from Equation (2) the steady state is reached mathematically only after an infinite time.

Time Response Of Second Order Control System

* → The block diagram of a general second order control system is as shown below.



* → The overall transfer function is given by.

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)} = \frac{\frac{\omega_n^2}{S(S+2\xi\omega_n)}}{1+\frac{\omega_n^2}{S(S+2\xi\omega_n)}} \cdot 1$$

$$\boxed{\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2+2\xi\omega_n S+\omega_n^2}}$$

* → ω_n is known as the natural frequency.

* → ξ is the damping ratio of the system.

* → If $\xi = 0$, the system is undamped.

* → If $0 < \xi < 1$, the system is under damped.

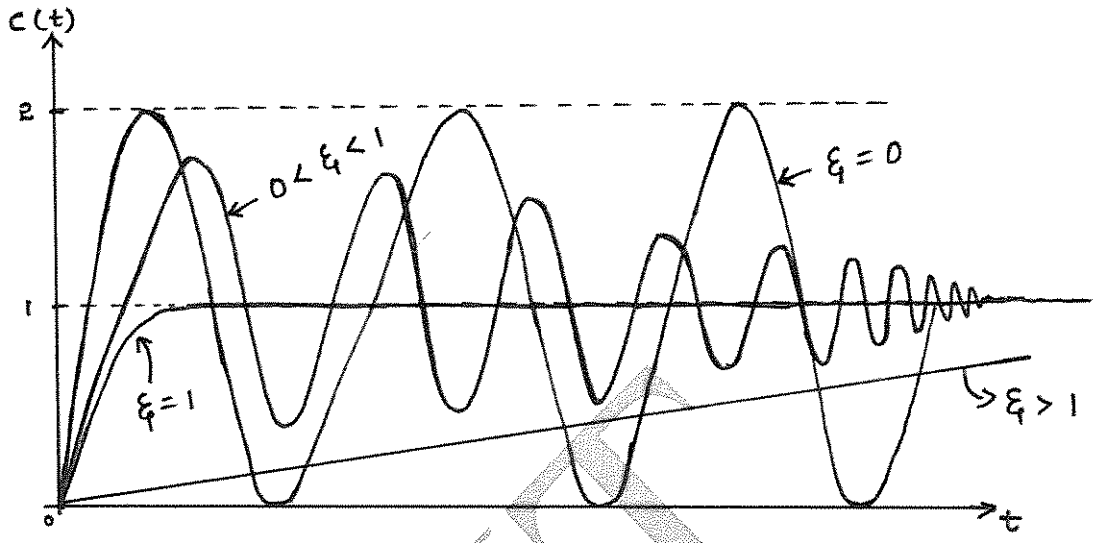
* → If $\xi = 1$, the system is Critically damped.

* → If $\xi > 1$, the system is over damped.

* → Denominator roots of $\frac{C(S)}{R(S)}$ are known as the poles of the closed loop transfer function and,

* → Numerator roots of $\frac{C(S)}{R(S)}$ are known as the zero's of the closed loop transfer function.

* → The time response of a second Order Control System subjected to a unit step input for different values of damping ratio is as shown below.



* → The time response of any system is characterized by the roots of the denominator polynomial, which in fact are the poles of the transfer function.

* → The denominator polynomial is therefore called the characteristic polynomial and it is called the characteristic Equation.

* → i.e $1+G(s)H(s)$ is known as characteristic Equation.

$$1+G(s)H(s) = 0$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$s = \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2} \qquad \because x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= -\xi\omega_n \pm \sqrt{\xi^2\omega_n^2 - \omega_n^2}$$

$$s = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$

* → When $\xi=0$ Poles are present on imaginary axis of plane.

* → When $\xi = 1$ Poles of the system are Real and Equal.
* → When $\xi > 1$ Poles of the system are Real and Un Equal.
* → When ξ lies between 0 & 1 Poles of the system are Complex.
* → for $0 < \xi < 1$ $\quad s = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$

$$\boxed{s = -\xi\omega_n \pm j\omega_d} \qquad \text{Note :-} \boxed{\omega_d = \omega_n\sqrt{1-\xi^2}}$$

Where $\xi\omega_n$ is known as the damping factor of the System. This terms decides the rate of delay of the transcient response.

ω_d is known as damped frequency of Osillation It is the Angular frequency of the transcient response.

⇒ Response of an Under damped System : $(0 < \xi < 1)$

* → Response of a second Order Control System Subjected to a unit step input. The Overall transfer function of a Second Order Control system is given by.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$C(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \cdot R(s) \quad \rightarrow (1)$$

W.K.T $R(s)$ = Unit step Input.

i.e $R(s) = \frac{1}{s}$

By substituting the value of $R(s)$ in (1)

$$C(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \cdot \frac{1}{s}$$

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

Breaking R.H.S into to partial fractions.

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)} = \frac{K_1}{s} + \frac{K_2 s + K_3}{s^2 + 2\xi\omega_n s + \omega_n^2} \rightarrow (2)$$

$$\omega_n^2 = K_1(s^2 + 2\xi\omega_n s + \omega_n^2) + (K_2 s + K_3)s$$

Put $s = 0$; $\omega_n^2 = K_1\omega_n^2$; $K_1 = \frac{\omega_n^2}{\omega_n^2}$ $\boxed{K_1 = 1}$

By Comparing the Co-efficients of 's^2'

s^2; $0 = K_1 + K_2$ or $K_2 = -K_1$

$$\boxed{K_2 = -1}$$

By Comparing the Co-efficients of 's'

s: $0 = 2\xi\omega_n K_1 + K_3$

$K_3 = -2\xi\omega_n K_1$ $\because \boxed{K_1 = 1}$

$$\boxed{K_3 = -2\xi\omega_n}$$

By substituting the value of K_1, K_2 and K_3 in (2)

$$C(s) = \frac{1}{s} + \frac{-s - 2\xi\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$= \frac{1}{s} - \frac{s + 2\xi\omega_n}{\underbrace{s^2 + 2\xi\omega_n \cdot s + (\xi\omega_n)^2}_{(a+b)^2} - (\xi\omega_n)^2 + \omega_n^2}$$

$$C(s) = \frac{1}{s} - \frac{(s + \xi\omega_n) + \xi\omega_n}{(s + \xi\omega_n)^2 + \omega_n^2 - \xi^2\omega_n^2}$$

$$C(s) = \frac{1}{s} - \frac{(s+\xi\omega_n) + \xi\omega_n}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)}$$

$$= \frac{1}{s} - \frac{(s+\xi\omega_n) + \xi\omega_n}{(s+\xi\omega_n)^2 + \omega_d^2}$$

We know that $\omega_d = \omega_n\sqrt{1-\xi^2}$

$$\therefore \omega_d^2 = \omega_n^2(1-\xi^2)$$

$$C(s) = \frac{1}{s} - \left[\frac{(s+\xi\omega_n)}{(s+\xi\omega_n)^2 + \omega_d^2} + \frac{\xi\omega_n}{(s+\xi\omega_n)^2 + \omega_d^2}\right]$$

By multiplying ω_d for both numerator and denominator of 3rd term

$$C(s) = \frac{1}{s} - \frac{(s+\xi\omega_n)}{(s+\xi\omega_n)^2 + \omega_d^2} - \frac{\xi\omega_n \cdot \omega_d}{\omega_d(s+\xi\omega_n)^2 + \omega_d^2}$$

$$C(s) = \frac{1}{s} - \frac{(s+\xi\omega_n)}{(s+\xi\omega_n)^2 + \omega_d^2} - \left(\frac{\xi\omega_n}{\omega_d}\right)\left(\frac{\omega_d}{(s+\xi\omega_n)^2 + \omega_d^2}\right)$$

By taking inverse laplace transforms.

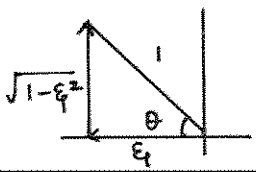
$$c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi\omega_n}{\omega_n\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin\omega_d t$$

$$c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin\omega_d t$$

By taking $\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}$ common

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\cos\omega_d t\sqrt{1-\xi^2} + \xi\sin\omega_d t\right]$$

Put $\sin\theta = \sqrt{1-\xi^2}$, $\therefore \cos\theta = \xi$; Note:



By Substituting We get

$$C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \left[\sin\theta \cos\omega_d t + \cos\theta \sin\omega_d t \right]$$

$$\boxed{C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \left[\sin(\omega_d t + \theta) \right]} \quad \text{for } t \geq 0 \rightarrow (3)$$

* → If the System to a step Input of Strength 'A' units, then the time response is given by.

$$\boxed{C(t) = A \left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \left[\sin(\omega_d t + \theta) \right] \right]}$$

* → Since $\omega_d = \omega_n \sqrt{1-\xi^2}$ and $\theta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$

* → By Substituting in Equation (3), Equation is rewritten as

$$\boxed{C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left[\omega_n \sqrt{1-\xi^2}\, t + \tan^{-1}\left[\frac{\sqrt{1-\xi^2}}{\xi}\right] \right]}$$

* → The Error Signal for the System is the difference between the Input and the Output and it is given by

$$e(t) = r(t) - C(t) \qquad \text{W.K.T } r(t) = 1$$

$$e(t) = 1 - \left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left[\omega_n \sqrt{1-\xi^2}\, t + \tan^{-1}\left[\frac{\sqrt{1-\xi^2}}{\xi}\right] \right] \right]$$

or

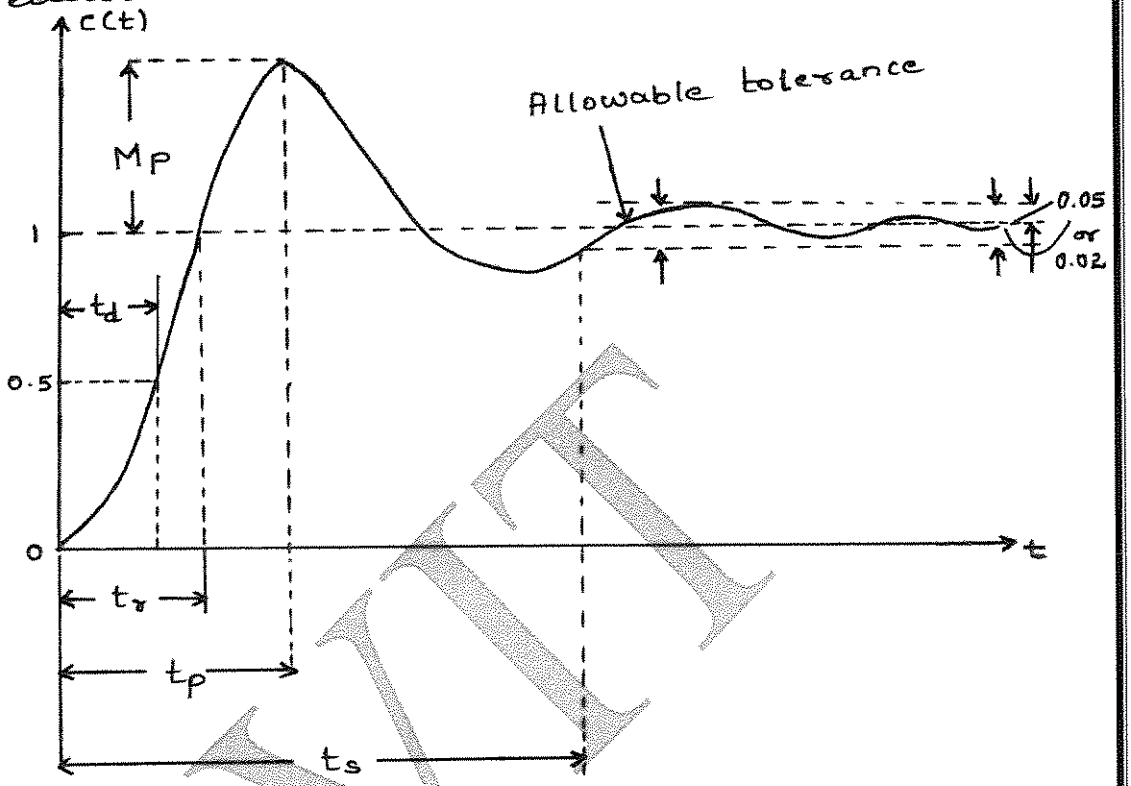
$$e(t) = \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \sin\left[\omega_n \sqrt{1-\xi^2}\, t + \tan^{-1}\left[\frac{\sqrt{1-\xi^2}}{\xi}\right] \right]$$

The steady state Error :-

$$\boxed{e_{ss} = \lim_{t \to \infty} \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left[\omega_n \sqrt{1-\xi^2}\, t + \tan^{-1}\left[\frac{\sqrt{1-\xi^2}}{\xi}\right] \right]}$$

Time Response Specifications

* → The time response of a second order Underdamped Control system subjected to a Unit step input, is as shown below.



* → The time response specifications are

i) Rise time, (tr)

ii) Delay time, (td)

iii) Peak time, (tp)

iv) Maximum Peak Overshoot, (Mp)

v) Setting time, (ts)

i) Rise time : tr

* Rise time is the time required for the response to rise from 0% to 100% of its final value at the very first instant.

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2) Delay time : t_d

* The delay time is the time required for the response to reach 50% of the final value at the very first time.

3) Peak time : t_p

* The peak time is the time required for the response to reach the first peak of the overshoot.

4) Maximum Overshoot : Mp

* The maximum peak overshoot is the maximum peak value of the response curve measured from unity.
* If the final steady-state value of the response differs from unity, then it is common to use to. maximum percent overshoot. It is defined by

Peak over shoot : $\boxed{Mp = c(t_p) - c(\infty)}$

Maximum percent Overshoot : $\boxed{\%Mp = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100}$

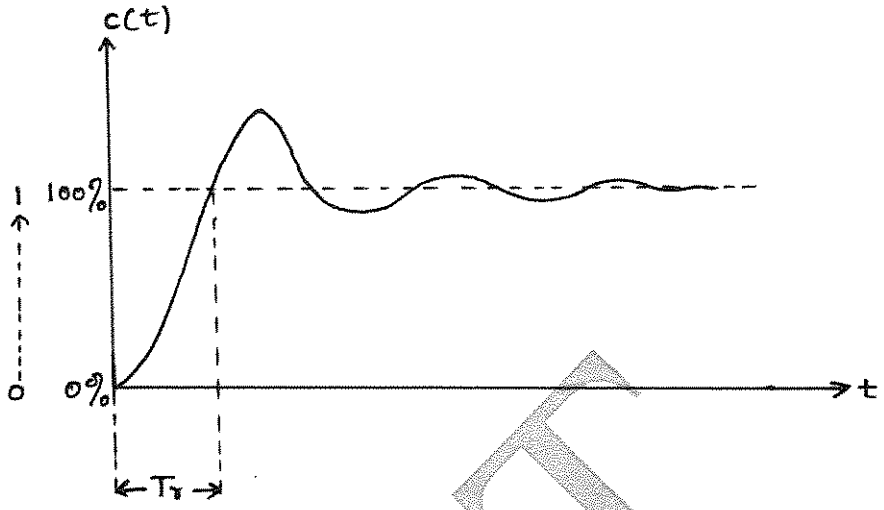
* The amount of the maximum (percent) overshoot directly indicates the relative stability of the system.

5) Settling time : t_s

* The Settling time is the time required for the response curve to reach and stay within specified range of its final value (within tolerance band, usually 2% or 5%).

Expression for Rise Time:

We know that, the Rise time is the time required to reach from 0% to 100% of it's final value.



* → The response of a Second Order Control System Subjected to a unit step input is given by

$$C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta) \quad \rightarrow ①$$

* → From the figure above At the risetime t_r.

$$C(t) = C(t_r) = 1$$

* → By substituting in Equation ① we get

$$1 = 1 - \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta) \quad \rightarrow ② \quad \text{W.K.T } t = t_r$$

* → Equation ② becomes

$$\frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta) = 0 \quad \rightarrow ③$$

* → Since in Equation ③

$$\frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \neq 0 \quad \because \text{the time } 't_r' \text{ is finite.}$$

* → $\sin(\omega_d t_r + \theta)$ must be equal to zero.

$$\therefore \sin(\omega_d t_r + \theta) = 0$$

$$\omega_d t_r + \theta = n\pi \; ; \; n = 1, 2, 3 \ldots$$

$$\omega_d t_r = n\pi - \theta$$

For the Ist time $n = 1$

$$\omega_d t_r = \pi - \theta$$

Therefore, the rise time "t_r" is given by

$$\boxed{t_r = \frac{\pi - \theta}{\omega_d}} \rightarrow (4)$$

W.K.T $\theta = \tan^{-1}\dfrac{\sqrt{1-\xi^2}}{\xi}$

$\omega_d = \omega_n\sqrt{1-\xi^2}$

* → By substituting in equation (4) we get.

$$\boxed{t_r = \frac{\pi - \tan^{-1}\dfrac{\sqrt{1-\xi^2}}{\xi}}{\omega_n\sqrt{1-\xi^2}}}$$

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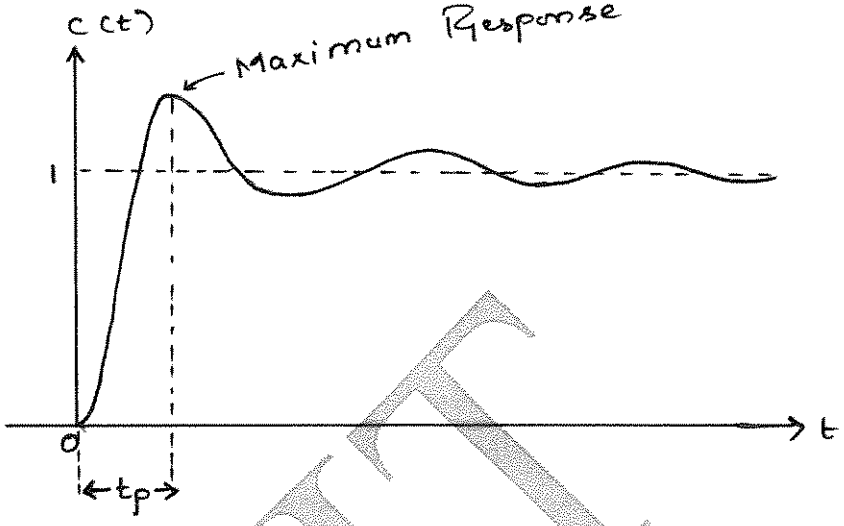
Expression For Peak Time:-

* → We know that, Peak Time is the time required for the response to reach the first peak of the Overshoot.



* → The output response of a second order control system subjected to a unit step input is given by.

$$C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta) \longrightarrow (1)$$

* → From the above figure, at the time $t = t_p$, the slope of $c(t)$ must be zero. Therefore. $\frac{dc(t)}{dt} = 0$

$$\left.\frac{dc(t)}{dt}\right|_{t=t_p} = 0 - \left[\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}(-\xi\omega_n)\sin(\omega_d t_p + \theta) + \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\cos(\omega_d t_p + \theta)\,\omega_d\right] = 0$$

$$\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\left[(\xi\omega_n)\sin(\omega_d t_p + \theta) + (\omega_d)\cos(\omega_d t_p + \theta)\right] = 0$$

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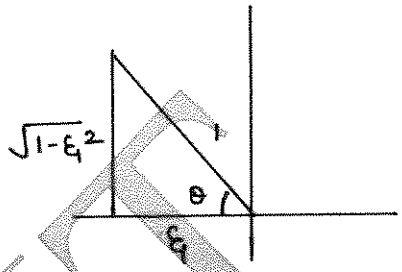
W.K.T $\quad W_d = W_n\sqrt{1-\xi^2}$

$$\frac{e^{-\xi W_n t_p}}{\sqrt{1-\xi^2}}\left[(\xi W_n)\sin(W_d t_p+\theta) - W_n\sqrt{1-\xi^2}\cos(W_d t_p+\theta)\right] = 0$$

$$\frac{e^{-\xi W_n t_p}}{\sqrt{1-\xi^2}} \cdot W_n\left[\xi \sin(W_d t_p+\theta) - \sqrt{1-\xi^2}\cos(W_d t_p+\theta)\right] = 0$$

But $\cos\theta = \xi$

$\sin\theta = \sqrt{1-\xi^2}$



$\therefore$ By substituting we get

$$\frac{e^{-\xi W_n t_p}}{\sqrt{1-\xi^2}} \cdot W_n\left[\cos\theta \sin(W_d t_p+\theta) - \sin\theta \cos(W_d t_p+\theta)\right] = 0$$

($\cos\theta$ → $\cos B$, $\sin(W_d t_p+\theta)$ → $\sin A$, $\sin\theta$ → $\sin B$, $\cos(W_d t_p+\theta)$ → $\cos A$)

$$\sin(A-B)$$

$$\frac{e^{-\xi W_n t_p}}{\sqrt{1-\xi^2}} \cdot W_n \sin(W_d t_p + \theta - \theta) = 0$$

$$\therefore \frac{e^{-\xi W_n t_p}}{\sqrt{1-\xi^2}} \cdot W_n \sin(W_d t_p) = 0$$

$$e^{-\xi W_n t_p} \cdot W_n \sin(W_d t_p) = 0$$

W.K.T $\quad e^{-\xi W_n t_p} \neq 0$ $\because$ 'tp' is finite

$\therefore \sin(W_d t_p) = 0$

$W_d t_p = n\pi$; $n = 1, 2, 3 \ldots\ldots$

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For the first peak over shoot $n = 1$

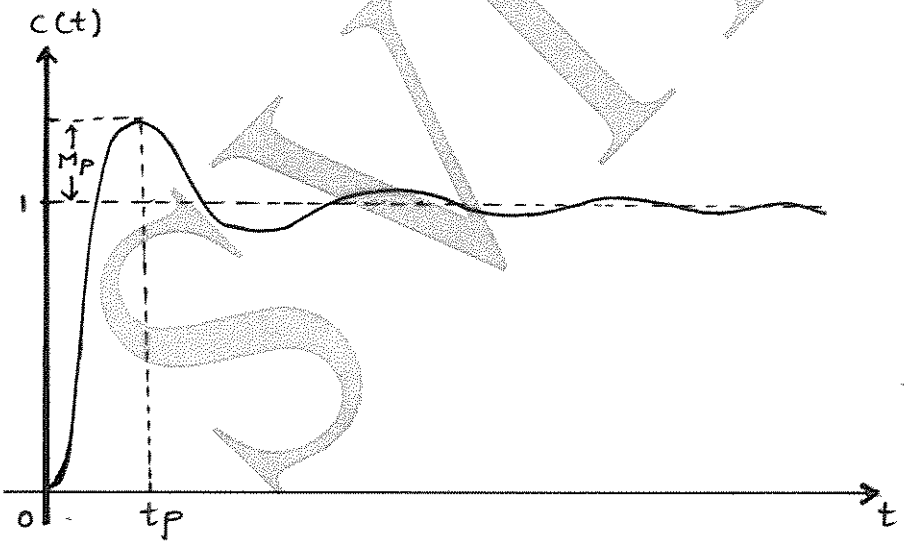
$$\therefore \omega_d t_p = \pi$$

$$\boxed{t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n \sqrt{1-\xi^2}}}$$

* To find time corresponding to first under shoot, Substitute $n = 2$ i.e $t_p = \frac{2\pi}{\omega_d}$.

* To find time corresponding to second over shoot, Substitute $n = 3$ i.e $t_p = \frac{3\pi}{\omega_d}$.

* Expression for Maximum Peak Overshoot (MP):



*→ We know that, the peak overshoot is the difference between the peak value and the final value.

*→ The output response of a second order control system subjected to a unit step input is given by,

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta) \longrightarrow (1)$$

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*→ By substituting $t = t_p$ in Equation (1), we get the peak response or maximum response.

$\therefore$ Maximum response $c(t_p) = 1 - \dfrac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \sin(\omega_d t_p + \theta)$

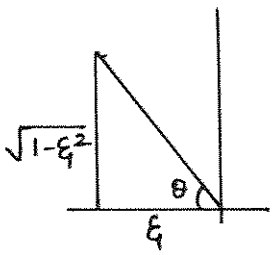
$$c(t_p) = 1 - \frac{e^{-\xi\omega_n \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \sin\left(\omega_d \cdot \frac{\pi}{\omega_d} + \theta\right) \quad \because t_p = \frac{\pi}{\omega_d} \text{ & } \omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\therefore \quad c(t_p) = 1 - \frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}} \sin(\pi+\theta)$$

$$\sin(\pi+\theta) = -\sin\theta$$

$$\therefore \quad c(t_p) = 1 - \frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}} \cdot (-\sin\theta) \quad \because \sin\theta = \sqrt{1-\xi^2}$$

i.e $$\boxed{c(t_p) = 1 + e^{-\xi\pi/\sqrt{1-\xi^2}}}$$



*→ We know that

Peak Over shoot is given by

$$M_p = c(t_p) - c(\infty)$$

$$M_p = 1 + e^{-\xi\pi/\sqrt{1-\xi^2}} - 1 \quad \because c(\infty) = 1 \text{ (final value)}$$

$$\boxed{M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}}$$

* → % peak overshoot is given by

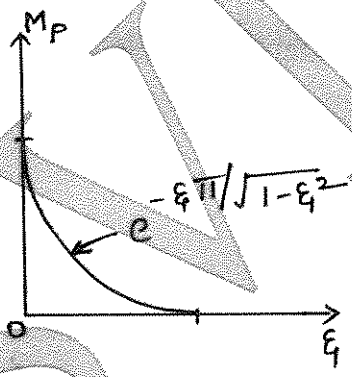
$$\%M_p = \frac{C(t_p) - C(\infty)}{C(\infty)} \times 100$$

As, in this case the $C(\infty) = 1$ % peak overshoot is given by

$$\%M_p = C(t_p) * 100$$

$$\therefore \boxed{\%M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} * 100}$$

* The variation of peak overshoot w.r.t damping ratio is as shown below.



* Note:- If the system is subjected to a step input of strength 'A' unit then the peak response is given by $c(t_p) = A\left(1 + e^{-\xi\pi/\sqrt{1-\xi^2}}\right)$

* Peak overshoot $M_p = C(t_p) - C(\infty)$

$$M_p = A\left(1 + e^{-\xi\pi/\sqrt{1-\xi^2}}\right) - A$$

$$M_p = A + Ae^{-\xi\pi/\sqrt{1-\xi^2}} - A$$

$$\boxed{M_p = Ae^{-\xi\pi/\sqrt{1-\xi^2}}}$$

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