MODULE-3

Design of Sections for Flexure: Analysis of members at ultimate strength - Preliminary Design - Final Design for Type 1members

Analysis of Members under Flexure Introduction Similar to members under axial load, the analysis of members under flexure refers to the evaluation of the following.

1) Permissible prestress based on allowable stresses at transfer.

2) Stresses under service loads. These are compared with allowable stresses underservice conditions.

3) Ultimate strength. This is compared with the demand under factored loads.

4) The entire load versus deformation behaviour.

The analyses at transfer and under service loads are presented in this section. The evaluation of the load versus deformation behaviour is required in special type of analysis. Assumptions The analysis of members under flexure considers the following.

1. Concrete is a homogeneous elastic material.

2. Within the range of working stress, both concrete & steel behave elastically, notwithstanding the small amount of creep, which occurs in both the materials under the sustained loading.

3. A plane section before bending is assumed to remain plane even after bending, which implies a linear strain distribution across the depth of the member.

4. Prestress Concrete is one in which there have been introduced internal stresses of such magnitude and distribution that stresses resulting from given external loading is counter balanced to a desired degree.

5. Plane sections remain plane till failure (known as Bernoulli's hypothesis).

6. Perfect bond between concrete and prestressing steel for bonded tendons.

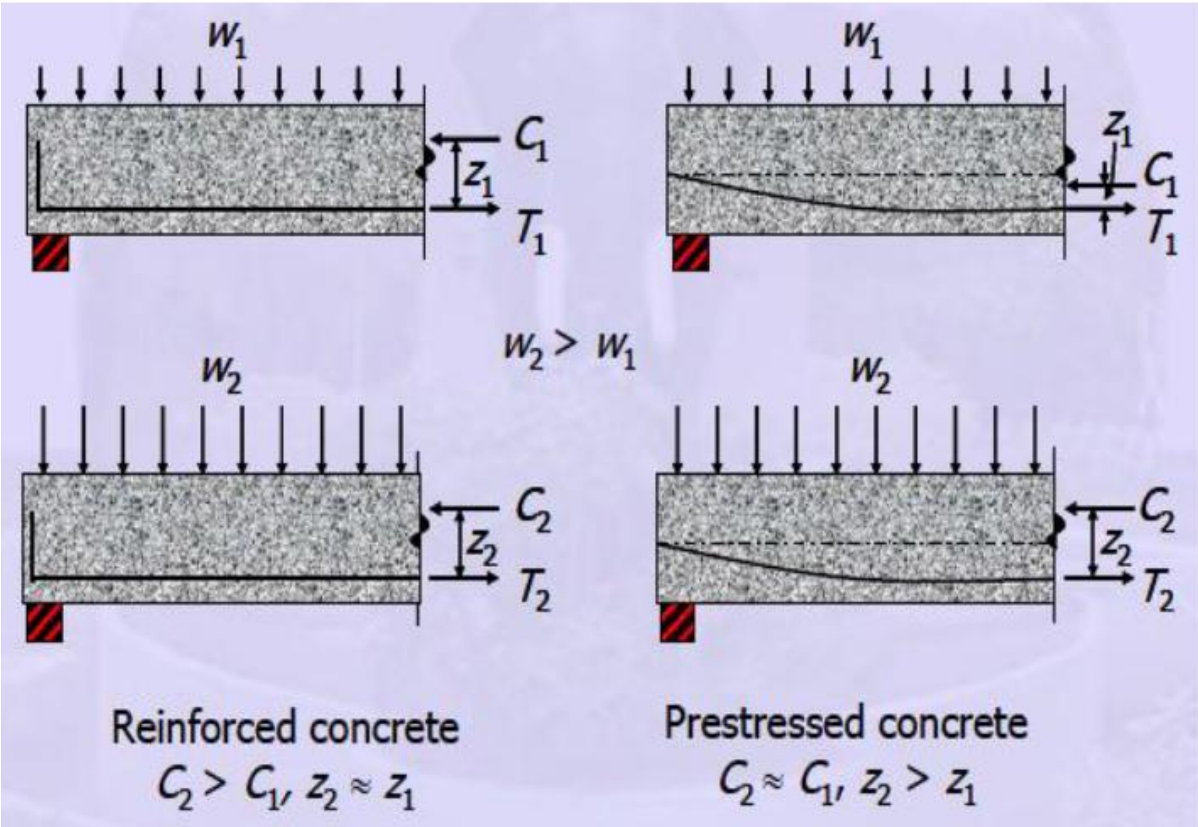
Principles of Mechanics The analysis involve three principles of mechanics.

1) Equilibrium of internal forces with the external loads. The compression in concrete (C) is equal to the tension in the tendon (T). The couple of C and T are equal to the moment due to external loads.

2) Compatibility of the strains in concrete and in steel for bonded tendons. The formulation also involves the first assumption of plane section remaining plane after bending. For unbonded tendons, the compatibility is in terms of deformation.

3) Constitutive relationships relating the stresses and the strains in the materials. Variation of Internal Forces In reinforced concrete members under flexure, the values of compression in concrete (C) and tension in the steel (T) increase with increasing external load. The change in the lever arm (z) is not large

In prestressed concrete members under flexure, t transfer of prestress C is located close to T. The couple of C and T balance only the self-weight. At service loads, C shifts up and the lever arm (z) gets large. The variation of C or T is not appreciable. The following figure explains this difference schematically for a simply supported beam under uniform load.



*C*1, *T*1 = compression and tension at transfer due to self-weight
*C*2, *T*2 = compression and tension under service loads
*w*1 = self-weight
*w*2 = service loads
*z*1 = lever arm at transfer
*z*2 = lever arm under service loads.

Analysis at Transfer and at Service

The analysis at transfer and under service loads is similar. Hence, they are presented together. A prestressed member usually remains uncracked under service loads. The concrete and steel are treated as elastic materials. The principle of superposition is applied. The increase in stress in the prestressing steel due to bending is neglected.

There are three approaches to analyse a prestressed member at transfer and under service loads. These approaches are based on the following concepts.

a) Based on stress concept.

b) Based on force concept.

c) Based on load balancing concept.

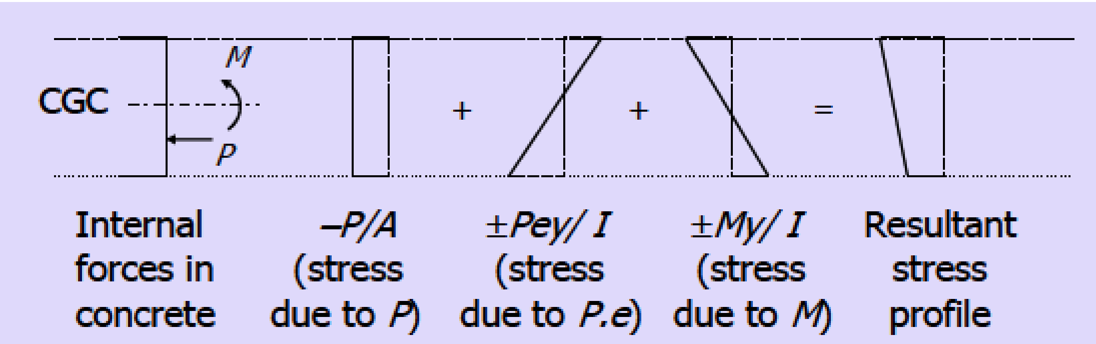
The following material explains the three concepts.

Based on Stress Concept

In the approach based on stress concept, the stresses at the edges of the section under the internal forces in concrete are calculated. The stress concept is used to compare the calculated stresses with the allowable stresses.

The following figure shows a simply supported beam under a uniformly distributed load (UDL) and prestressed with constant eccentricity (*e*) along its length.

The first stress profile is due to the compression *P*. The second profile is due to the eccentricity of the compression. The third profile is due to the moment. At transfer, the moment is due to self-weight. At service the moment is due to service loads.



Stress profiles at a section due to internal forces

The resultant stress at a distance *y* from the CGC is given by the principle of superposition as follows.

EXAMPLES

1. A posttensioned prestressed concrete beam having a rectangular section 150mm x 350mm has an effective cover of 50mm. if fck = 40 N/mm2, fp = 1600N/mm2 and the area of prestressing steel is 461 mm2, estimate the flexural strength of th section using (IS 1343) Indian standard code provisions.

Solution:

Given data

fck = 40 N/mm2

fp = 1600 N/mm2

AP = 461 N/mm2

D = d+d’ = 300mm

The effective ratio is calculated as (refer IS 1343-1980,TABLE-1)

(fpxAp)/(fckxbxd) = (1600x461)/(40x150x300) = 0.40

From table 11, the corresponding value of (pre-tensioning)

(fpu/0.87fy)= 0.9

Xu/d = 0.783

(Apxfp)/(bxdxfck) =0.40

Fpu = 0.87x0.9xfp

Xu = 0.783d

fpu = (0.87x0.9x1600)

=1252.8 N/mm2

Xu = (0.783x300)

=234.9mm

Now refer IS 1343

M =Mu = fpuxAp (d-0.42Xu)

= (1252.8x461)x(300-0.42x234.9)

=116.28x10^6 N-mm

=116.28KN-m

2. A bonded prestressed concrete beam of rectangular cross section (200x500)mm . the tendon consists of 500mm2 area and it is stress to 1500N/mm2. The tendons are located at 100mm from the soffit of the beam. The concrete characteristics strength is 40 N/mm2 and Ec = 35KN/mm2 and Es = 200KN/mm2. Use IS 1343 code to calculate the ultimate strength.

Solution:

Given data

fck = 40 N/mm2

fp = 1500 N/mm2

AP = 500 N/mm2

D = d+d' = 500mm

The effective ratio is calculated as (refer IS 1343-1980,TABLE-1)

(fpxAp)/(fckxbxd) = (1500x500)/(200x400x40) = 0.23

From table 11, the corresponding value of (pre-tensioning)

(fpu/0.87fy)= 1.0

Xu/d = 0.5

(Apxfp)/(bxdxfck) =0.40

Fpu = 0.87x0.9xfp

Xu = 0.5d

fpu = (0.87x0.9x1500)

=1305 N/mm2

Xu = (0.5x400)

=200

Now refer IS 1343

M =Mu = fpuxAp (d-0.42Xu)

= (1305x500)x(400-0.42x250)

=192.48x10^6 N-mm

=192.48 KN-m

3. A pre tensioned beam of rectangular section 400mmx 1000mm overall depth is prestressed by 800 mm2of high tensile steel wires at an eccentricity of 300mm. if fck = 40N/mm2, fp = 1600N/mm2, estimate the ultimate flexural strength of the section. As per IS 1343 codal provisions.

Solution:

Given data

fck = 40 N/mm2

fp = 1600 N/mm2

AP = 461 N/mm2

D = d+d' = 1000 mm

The effective ratio is calculated as (refer IS 1343-1980,TABLE-1)

(fpxAp)/(fckxbxd) = (1600x800)/(40x400x800) = 0.1

From table 11, the corresponding value of (pre-tensioning)

(fpu/0.87fp)= 1.0

Xu/d = 0.217

(Apxfp)/(bxdxfck) =0.1

Fpu = 0.87x0.9xfp

Xu = 0.217d

fpu = (0.87x1x1600)

=1392 N/mm2

Xu = (0.217x800)

=173.6 mm

Now refer IS 1343

M =Mu = fpuxAp (d-0.42Xu)

= (1392x800)x(800-0.42x173.6)

=809.68x10^6 N-mm

=809.68 KN-m

4. Find the ultimate moment of resistance of the prestressed beam of the posttensioned beam section of width 300mm and effective depth is 600mm for the following data.

Fck = N/mm2, fp = 1500 N/mm2, Ap= 500mm2, fpu = 0.87x fy

Solution:

Given data

fck = 40 N/mm2

fp = 1500 N/mm2

AP = 500 N/mm2

D = d+d' = 600mm

The effective ratio is calculated as (refer IS 1343-1980,TABLE-1)

(fpxAp)/(fckxbxd) = (1500x500)/(40x600x300) = 0.104

Xu/d = 0.217

(Apxfp)/(bxdxfck) =0.104

Fpu = 0.87xfp

Xu = 0.217d

fpu = (0.87x1500)

=1305 N/mm2

Xu = (0.217x600)

=130.2mm

Now refer IS 1343

M =Mu = fpuxAp (d-0.42Xu)

= (1305x500)x(600-0.42x130.2)

=355.82x10^6 N-mm

=355.82KN-m

5. The cross section of a composite beam is of T-section having a pretensioned rib, 80 mm wide and 240 mm deep, and in situ cast slab, 350 mm wide and 80 mm thick. The pretensioned beam is reinforced with eight wires of 5 mm diameter with an ultimate tensile strength of 1600 N/mm2, located 60 mm from the soffit of the beam. The compressive strength of concrete in the in situ cast and precast elements is 20 and 40N/mm2, respectively. If the adequate reinforcements are provided to prevent shear failure at the intereface, estimate the flexural strength of the composite section.

Solution:

Given data

fck = 20 N/mm2

fp = 1600 N/mm2

Ap = 20 x 8 = 160 N/mm2

d = 240 mm

b = 350 mm

The effective reinforcement ratio is given by (IS 1343-1980,Table-1)

(fpxAp)/(fckxbxd) = (1600x160)/(20x350x240) = 0.152

Xu/d = 0.326

(fpu)/(0.87fp) =1

Fpu = 0.87xfp

Xu = 0.326d

fpu = (0.87x1600)

=1392 N/mm2

Xu = (0.326x240)

=78mm

Now refer IS 1343

M =Mu = fpuxAp (d-0.42Xu)

= (1392x160)x(240-0.42x78)

=46.15 x10^6 N-mm

=46.15KN-m

6. Find the ultimate moment of resistance of unbounded beam section of width 300 mm and effective depth 600 mm for the following data.
Fck = 40 n/mm2, fp=1500N/mm2, Ap=500mm2, length = 10m.

Solution:

Given data

fck = 40 N/mm2, fp = 1500 N/mm2, Ap = 500 N/mm2, d = 600 mm

b = 300 mm, l=10m

The effective reinforcement ratio is given by (IS 1343-1980,Table-1)

(fpxAp)/(fckxbxd)= (1500x500)/(40x300x600) = 0.1

l/d = 10x10^3/600 = 16.67

By iteration

l/d=20	fpu/fp=1.26
l/d=16.67	fpu/fp=?
l/d=10	fpu/fp=1.45

Fpu=1.32 x 1500 =1980N/mm2

Similarly

l/d=20 → Xu/d=0.32

l/d=16.67 →Xu/d =?

l/d=10 →Xu/d =0.36

Xu/d=0.33

Xu=0.33 x 600

=198mm

Moment of resistance,Mu = fpuxAp (d-0.42Xu)

= (1980x500)x(600-0.42x198)

=511.67 x10^6 N-mm

=511.67KN-m

7. A post tensioned beam with unbounded tendons is of rectangular section 400 mm wide with an effective depth of 800 mm. The cross sectional area of the prestressing steel is 2840 mm2. The effective prestressing steel after all the losses is 900N/mm2. The effective span of the beam is 16m. If fck=40N/mm2, estimate the ultimate moment of resistance of the section using IS1343.

Solution:

Given data

fck = 40 N/mm2

fp = 900 N/mm2

Ap = 2840 N/mm2

d = 8600 mm

b = 400 mm

l=16m

The effective reinforcement ratio is given by (IS 1343-1980,Table-1)

(fpxAp)/(fckxbxd) = (2840x900)/(40x400x800) = 0.199=0.2

l/d = 16x10^3/800 = 20

Fpu/fp=1.16

Fpu=1.16 x 900

=1044N/mm2

Xu/d=0.58

Xu=0.58 x 800

=464mm

Moment of resistance,Mu = fpuxAp (d-0.42Xu)

= (1044x2840)x(800-0.42x464)

=1794.15 x10^6 N-mm

=11794.15KN-m