MODULE -2

Losses in Prestress, Loss of Prestress due to Elastic shortening, Friction, Anchorage slip, Creep of concrete, Shrinkage of concrete and Relaxation of steel - Total Loss. Deflection and Crack Width Calculations of Deflection due to gravity load. Deflection due to prestressing force -Total deflection - Limits of deflection - Limits of span-to-effective depth ratio - Calculation of Crack Width - Limits of crack width **10 hours**

Losses in Prestress

This section covers the following topics.

1. Introduction
2. Elastic Shortening

The relevant notations are explained first.

Notations

Geometric Properties

The commonly used geometric properties of a prestressed member are defined as follows.

A_c = Area of concrete section

= Net cross-sectional area of concrete excluding the area of prestressing steel.

A_p = Area of prestressing steel

= Total cross-sectional area of the tendons.

A = Area of prestressed member

= Gross cross-sectional area of prestressed member.

$= A_c + A_p$

A_t = Transformed area of prestressed member

= Area of the member when steel is substituted by an equivalent area of concrete.

$$= A_c + mA_p$$

$$= A + (m - 1)A_p$$

Here,

m = the modular ratio = E_p/E_c

E_c = short-term elastic modulus of elasticity.

The following figure shows the commonly used areas of the prestressed members.

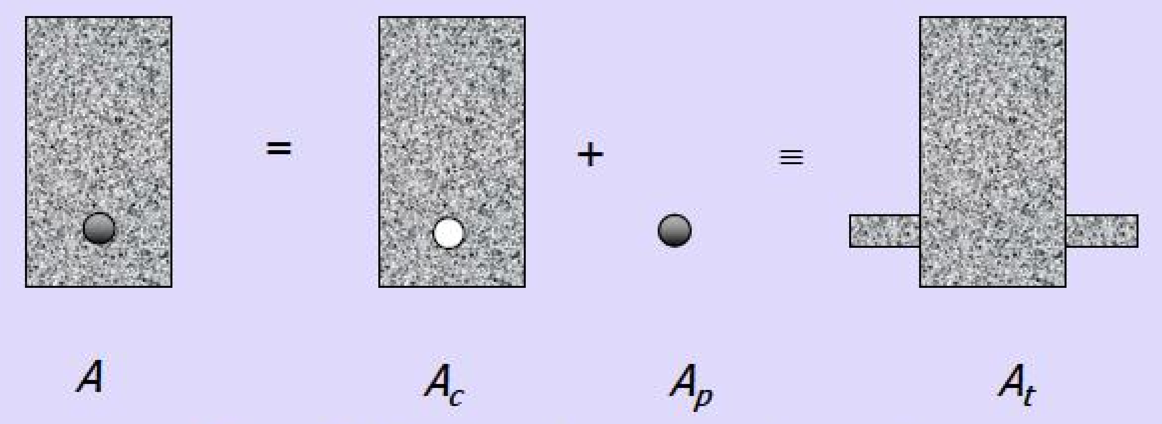


Figure 2-1.1 Areas for prestressed members

CGC = Centroid of concrete
= Centroid of the gross section. The CGC may lie outside the concrete (Figure 2-1.2).
CGS = Centroid of prestressing steel
= Centroid of the tendons. The CGS may lie outside the tendons
I = Moment of inertia of prestressed member
= Second moment of area of the gross section about the CGC.
I_t = Moment of inertia of transformed section
= Second moment of area of the transformed section about the centroid of the transformed section.
e = Eccentricity of CGS with respect to CGC

Load Variables

Pi = Initial prestressing force
= The force which is applied to the tendons by the jack.
*P*0 = Prestressing force after immediate losses
= The reduced value of prestressing force after elastic shortening, anchorage slip and loss due to friction.
Pe = Effective prestressing force after time-dependent losses
= The final value of prestressing force after the occurrence of creep, shrinkage and relaxation.

Introduction

In prestressed concrete applications, the most important variable is the prestressing force. In the early days, it was observed that the prestressing force does not stay constant, but reduces with time. Even during prestressing of the tendons and the transfer of prestress to the concrete member, there is a drop of the prestressing force from the recorded value in the jack gauge. The various reductions of the prestressing force are termed as the losses in prestress.

The losses are broadly classified into two groups, immediate and time-dependent. The immediate losses occur during prestressing of the tendons and the transfer of prestress to the concrete member. The time-dependent losses occur during the service life of the prestressed member. The losses due to elastic shortening of the member, friction at the tendon-concrete interface and slip of the anchorage are the immediate losses. The losses due to the shrinkage and creep of the concrete and relaxation of the steel are the time-dependent losses. The causes of the various losses in prestress are shown in the following chart.

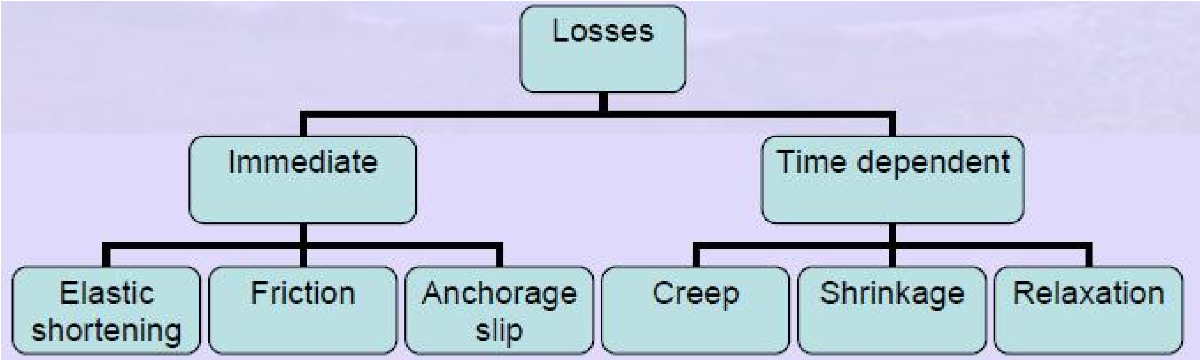


Figure 2-1.3 Causes of the various losses in prestress

Elastic Shortening

Pre-tensioned Members

When the tendons are cut and the prestressing force is transferred to the member, the concrete undergoes immediate shortening due to the prestress. The tendon also shortens by the same amount, which leads to the loss of prestress.

Post-tensioned Members

If there is only one tendon, there is no loss because the applied prestress is recorded after the elastic shortening of the member. For more than one tendon, if the tendons are stretched sequentially, there is loss in a tendon during subsequent stretching of the other tendons.

The elastic shortening loss is quantified by the drop in prestress (Δf_p) in a tendon due to the change in strain in the tendon ($\Delta\varepsilon_p$). It is assumed that the change in strain in the tendon is equal to the strain in concrete (ε_c) at the level of the tendon due to the prestressing force. This assumption is called **strain compatibility** between concrete and steel. The strain in concrete at the level of the tendon is calculated from the stress in concrete (f_c) at the same level due to the prestressing force. A linear elastic relationship is used to calculate the strain from the stress.

For simplicity, the loss in all the tendons can be calculated based on the stress in concrete at the level of CGS. This simplification cannot be used when tendons are stretched sequentially in a post-tensioned member. The calculation is illustrated for the following types of members separately.

1. Pre-tensioned Axial Members
2. Pre-tensioned Bending Members
3. Post-tensioned Axial Members
4. Post-tensioned Bending Members

1. Pre-tensioned Axial Members

The following figure shows the changes in length and the prestressing force due to elastic shortening of a pre-tensioned axial member.

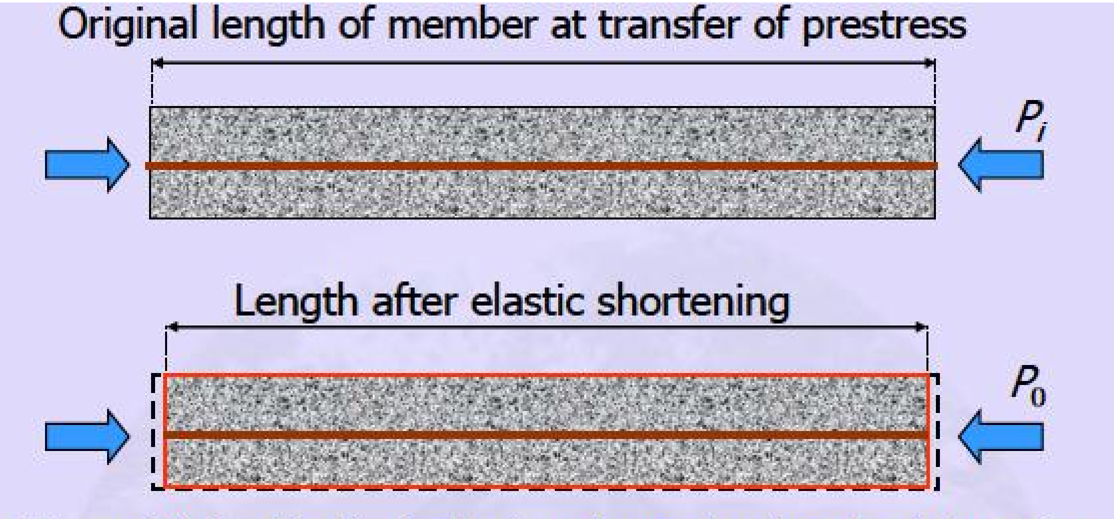


Figure 2-1.4 Elastic shortening of a pre-tensioned axial member

The loss can be calculated as per Eqn. (2-1.1) by expressing the stress in concrete in terms of the prestressing force and area of the section as follows.*t*

Δf =mf

Note that the stress in concrete due to the prestressing force after immediate losses (P_0/A_c) can be equated to the stress in the transformed section due to the initial prestress (P_i/A_t). This is derived below. Further, the transformed area A_t of the prestressed member can be approximated to the gross area A.

The following figure shows that the strain in concrete due to elastic shortening (ε_c) is the difference between the initial strain in steel (ε_{pi}) and the residual strain in steel (ε_{p0}).

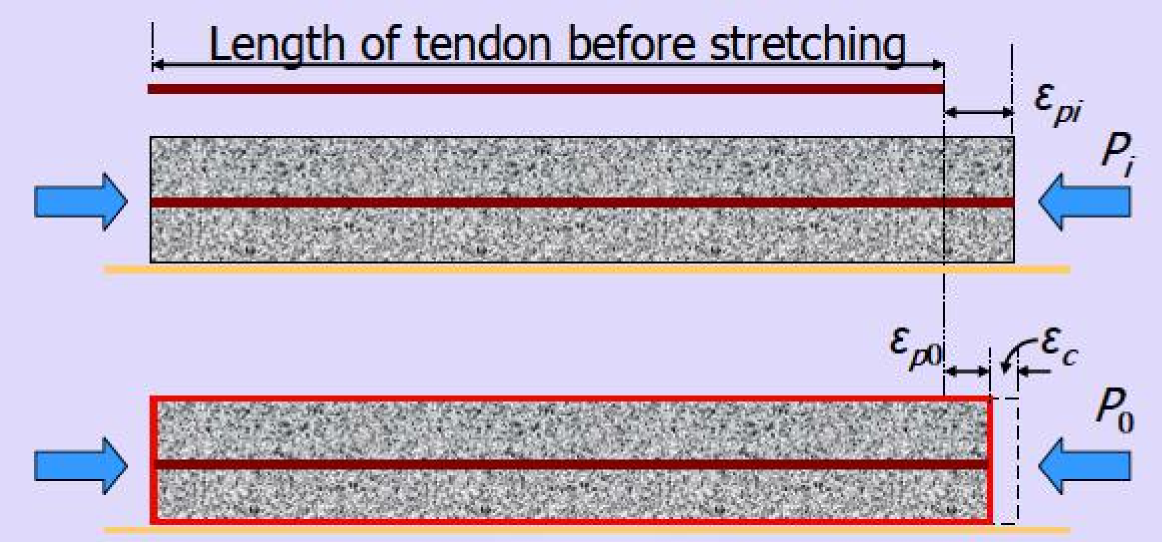


Figure 2-1.5 Strain variables in elastic shortening

The following equation relates the strain variables.

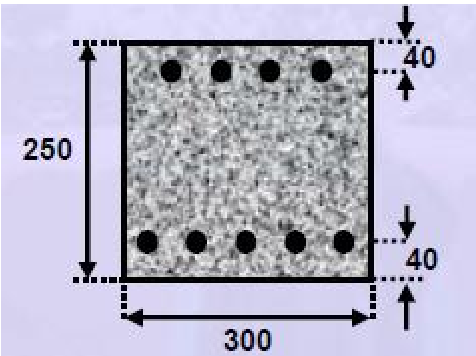
$\varepsilon_c = \varepsilon_{pi} - \varepsilon_{p0}$

The strains can be expressed in terms of the prestressing forces as follows.

$P\ \varepsilon = AE$

Example 2-1.1

A prestressed concrete sleeper produced by pre-tensioning method has a rectangular cross-section of 300mm × 250 mm ($b \times h$). It is prestressed with 9 numbers of straight 7mm diameter wires at 0.8 times the ultimate strength of 1570 N/mm^2. Estimate the percentage loss of stress due to elastic shortening of concrete. Consider m = 6.



a) Approximate solution considering gross section

The sectional properties are calculated as follows.

Area of a single wire, $A_w = \pi/4 \times 7^2$

$= 38.48\ mm^2$

Area of total prestressing steel, $A_p = 9 \times 38.48$

$= 346.32\ mm^2$

Area of concrete section, $A = 300 \times 250$

$= 75 \times 10^3\ mm^2$

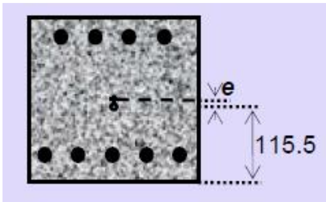
Moment of inertia of section, $I = 300 \times 250^3/12$

$= 3.91 \times 10^8$ mm4

Distance of centroid of steel area (CGS) from the soffit,

=(4×38.48×250- 40+5×38.48×40)

y = 9×38.48 =115.5 mm



Prestressing force, $P_i = 0.8 \times 1570 \times 346.32$ N

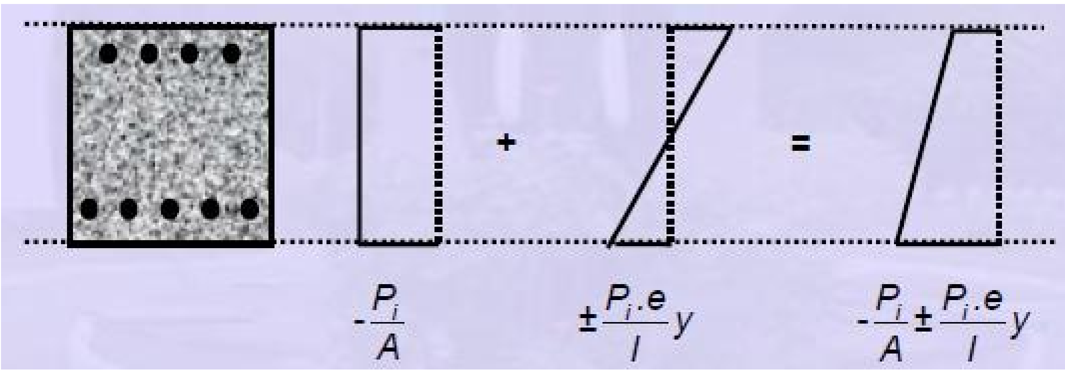
= 435 kN

Eccentricity of prestressing force,

$e = (250/2) - 115.5$

= 9.5 mm

The stress diagrams due to P_i are shown.



Since the wires are distributed above and below the CGC, the losses are calculated for the top and bottom wires separately.

Stress at level of top wires ($y = y_t = 125 - 40$)

Loss of prestress in top wires = *mfcAp*
(in terms of force) = 6 × 4.9 × (4 × 38.48)
= 4525.25 N
Loss of prestress in bottom wires = 6 × 6.7 × (5 × 38.48)
= 7734.48 N
Total loss of prestress = 4525 + 7735
= 12259.73 N
≈ 12.3 kN
Percentage loss = (12.3 / 435) × 100%
= 2.83%
b) Accurate solution considering transformed section.
Transformed area of top steel,
$A1$ = (6 – 1) 4 × 38.48
= 769.6 mm2
Transformed area of bottom steel,
$A2$ = (6 – 1) 5 × 38.48
= 962.0 mm2
Total area of transformed section,
$AT = A + A1 + A2$
= 75000.0 + 769.6 + 962.0
= 76731.6 mm2

Centroid of the section (CGC)
y = A× + A × + A ×A
125 1 (250- 40) 2 40
= 124.8 mm from soffit of beam
Moment of inertia of transformed section,
$IT = Ig + A(0.2)2 + A1(210 – 124.8)2 + A2(124.8 – 40)2$
= 4.02 × 108mm4
Eccentricity of prestressing force,
e = 124.8 – 115.5
= 9.3 mm
Stress at the level of bottom wires,
= - 435×10 - (435×10 ×9.3)84.8
76.73×10 4.02×10
= -5.67-0.85
= -6.52 N/mm
(fc)b
Stress at the level of top wires,
= - 435×10 + (435×10 ×9.3)85.2
76.73×10 4.02×10

= -5.67+0.86
= -4.81 N/mm
(fc)t
Loss of prestress in top wires = 6 × 4.81 × (4 × 38.48)
= 4442 N
Loss of prestress in bottom wires = 6 × 6.52 × (5 × 38.48)
= 7527 N
Total loss = 4442 + 7527
= 11969 N
≈ 12 kN
Percentage loss = (12 / 435) × 100%
= 2.75 %

Pre-tensioned Bending Members

The following figure shows the changes in length and the prestressing force due to elastic shortening of a pre-tensioned bending member

Due to the effect of self-weight, the stress in concrete varies along length (Figure 2-1.6). The loss can be calculated by Eqn. (2-1.1) with a suitable evaluation of the stress in concrete. To have a conservative estimate of the loss, the maximum stress at the level of CGS at the mid-span is considered.

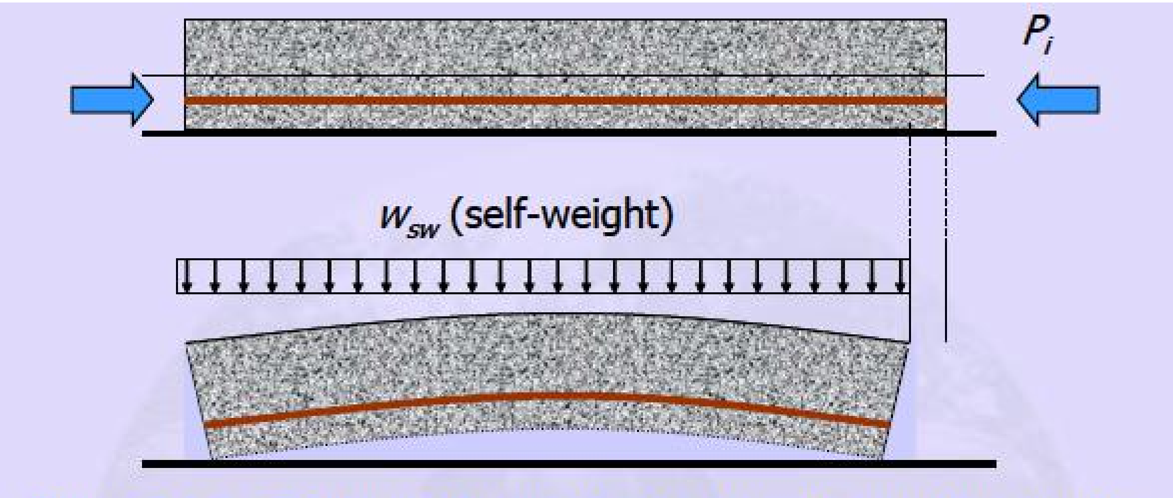


Figure 2-1.6 Elastic shortening of a pre-tensioned bending member

Post-tensioned Axial Members

For more than one tendon, if the tendons are stretched sequentially, there is loss in a tendon during subsequent stretching of the other tendons. The loss in each tendon can be calculated in progressive sequence. Else, an approximation can be used to calculate the losses.

The loss in the first tendon is evaluated precisely and half of that value is used as an average loss for all the tendons. Σ *p pcn i,j j= Δf = Δf mf*

Post-tensioned Bending Members

The calculation of loss for tendons stretched sequentially, is similar to post-tensioned axial members. For curved profiles, the eccentricity of the CGS and hence, the stress in concrete at the level of CGS vary along the length. An average stress in concrete can be considered.

Losses in Prestress (Part II)

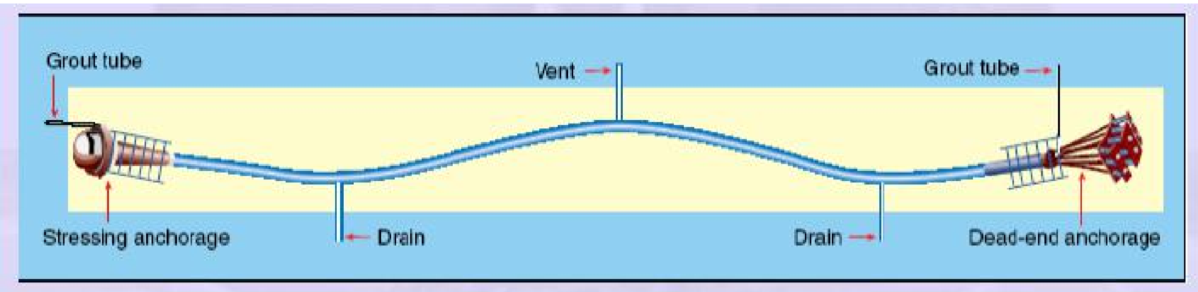
This section covers the following topics

- **Friction**
- **Anchorage Slip**
- **Force Variation Diagram**

Friction

The friction generated at the interface of concrete and steel during the stretching of a curved tendon in a post-tensioned member, leads to a drop in the prestress along the member from the stretching end. The loss due to friction does not occur in pre-tensioned members because there is no concrete during the stretching of the tendons.

The friction is generated due to the curvature of the tendon and the vertical component of the prestressing force. The following figure shows a typical profile (laying pattern) of the tendon in a continuous beam.



In addition to friction, the stretching has to overcome the **wobble** of the tendon. The wobble refers to the change in position of the tendon along the duct. The losses due to friction and wobble are grouped together under friction.

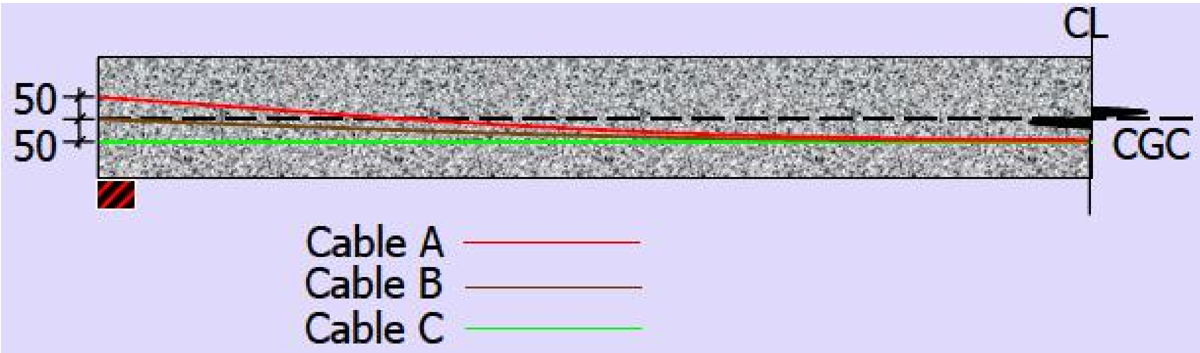
The derivation of the expression of *P* is based on a circular profile. Although a cable profile is parabolic based on the bending moment diagram, the error induced is insignificant.

The friction is proportional to the following variables.

1. Coefficient of friction (μ) between concrete and steel.
2. The resultant of the vertical reaction from the concrete on the tendon (N) generated due to curvature.

Example 2-2.1

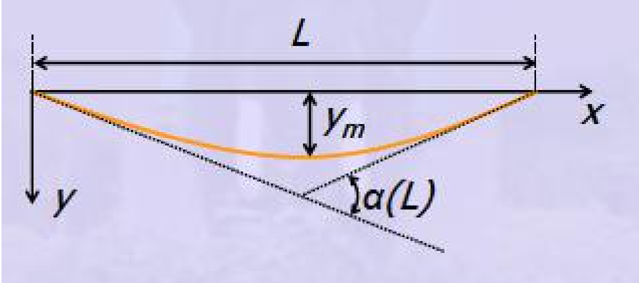
A post-tensioned beam 100 mm × 300 mm ($b \times h$) spanning over 10 m is stressed by successive tensioning and anchoring of 3 cables A, B, and C respectively as shown in figure. Each cable has cross section area of 200 mm^2 and has initial stress of 1200 MPa. If the cables are tensioned from one end, estimate the percentage loss in each cable due to friction at the anchored end. Assume μ = 0.35, k = 0.0015 / m.



Solution

Prestress in each tendon at stretching end = 1200 × 200
= 240 kN.
To know the value of *α(L),* the equation for a parabolic profile is required.

$$\frac{dy}{dx} = \frac{4y_m}{L^2}(L - 2x)$$



y_m = displacement of the CGS at the centre of the beam from the ends

L = length of the beam

x = distance from the stretching end

y = displacement of the CGS at distance x from the ends.

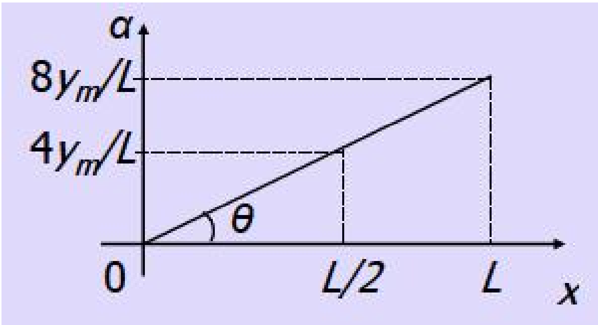
An expression of $\alpha(x)$ can be derived from the change in slope of the profile. The slope of the profile is given as follows.

$$\frac{dy}{dx} = \frac{4y_m}{L^2}(L - 2x)$$

At $x = 0$, the slope $dy/dx = 4y_m/L$. The change in slope $\alpha(x)$ is proportional to x.

The expression of $\alpha(x)$ can be written in terms of x as $\alpha(x) = \theta.x$,

where, $\theta = 8y_m/L^2$. The variation is shown in the following sketch.

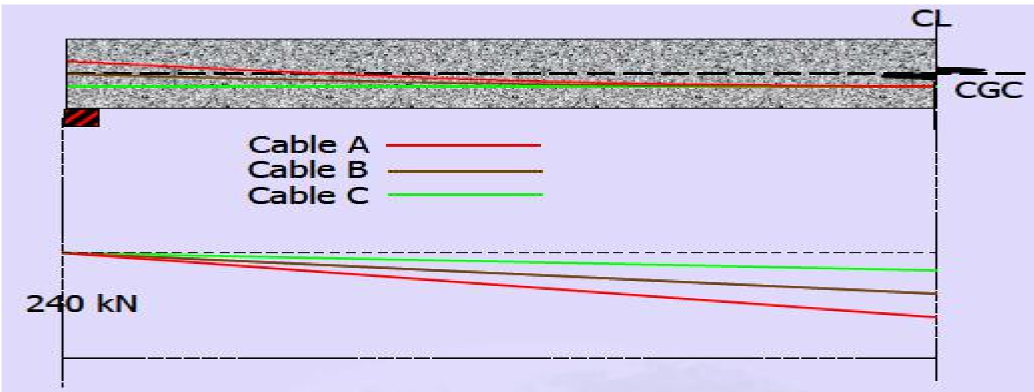


The total subtended angle over the length L is $8y_m/L$.

The prestressing force P_x at a distance x is given by

$$P_x = P_0 e^{-(\mu\alpha + kx)} = P_0 e^{-\eta x}$$

$$\eta x = \mu\alpha + kx$$



Variation of prestressing forces

The loss due to friction can be considerable for long tendons in continuous beams with changes in curvature. The drop in the prestress is higher around the intermediate supports where the curvature is high. The remedy to reduce the loss is to apply the stretching force from both ends of the member in stages.

2-2.2 Anchorage Slip

In a post-tensioned member, when the prestress is transferred to the concrete, the wedges slip through a little distance before they get properly seated in the conical space. The anchorage block also moves before it settles on the concrete. There is loss of prestress due to the consequent reduction in the length of the tendon.

The total anchorage slip depends on the type of anchorage system. In absence of manufacturer's data, the following typical values for some systems can be usedDue to the setting of the anchorage block, as the tendon shortens, there is a reverse friction. Hence, the effect of anchorage slip is present up to a certain length (Figure 2- 2.4). Beyond this **setting length**, the effect is absent. This length is denoted as l_{set}.

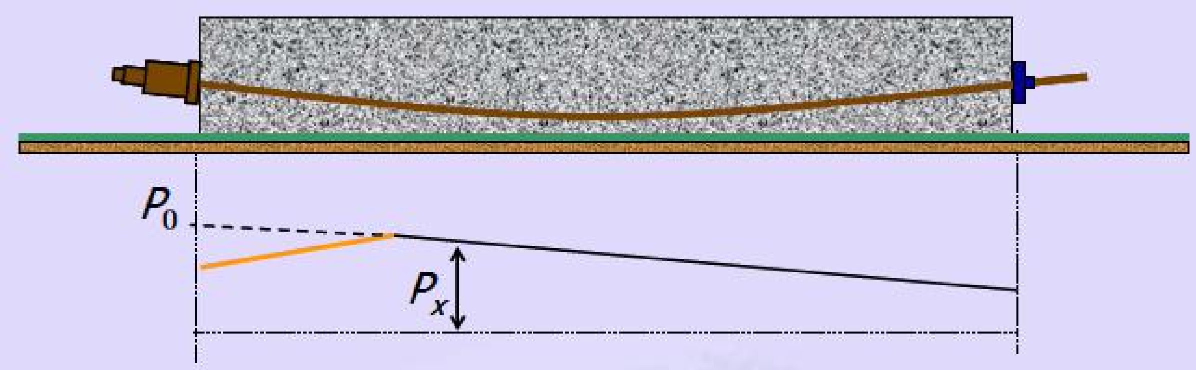


Figure 2-2.4 Variation of prestressing force after anchorage slip

Force Variation Diagram

The magnitude of the prestressing force varies along the length of a post-tensioned member due to friction losses and setting of the anchorage block. The diagram representing the variation of prestressing force is called the force variation diagram.

Considering the effect of friction, the magnitude of the prestressing force at a distance *x* from the stretching end is given as follows.

$P= Pe\ 0$

Here, $\eta x = \mu\alpha + kx$ denotes the total effect of friction and wobble. The plot of P_x gives the force variation diagram.

The initial part of the force variation diagram, up to length l_{set} is influenced by the setting of the anchorage block. Let the drop in the prestressing force at the stretching end be ΔP. The determination of ΔP and l_{set} are necessary to plot the force variation diagram including the effect of the setting of the anchorage block.

Losses in Prestress (Part III)

This section covers the following topics.

- Creep of Concrete
- Shrinkage of Concrete
- Relaxation of Steel
- Total Time Dependent Losses

2.3.1 Creep of Concrete

Creep of concrete is defined as the increase in deformation with time under constant load. Due to the creep of concrete, the prestress in the tendon is reduced with time.

The creep of concrete is explained in Section 1.6, Concrete (Part II). Here, the information is summarised. For stress in concrete less than one-third of the characteristic strength, the ultimate creep strain ($\varepsilon_{cr,ult}$) is found to be proportional to the elastic strain (ε_{el}). The ratio of the ultimate creep strain to the elastic strain is defined as the ultimate creep coefficient or simply creep coefficient θ.

The following considerations are applicable for calculating the loss of prestress due to creep.

1) The creep is due to the sustained (permanently applied) loads only.
Temporary loads are not considered in the calculation of creep.
2) Since the prestress may vary along the length of the member, an average value of the prestress can be considered.
3) The prestress changes due to creep and the creep is related to the instantaneous prestress. To consider this interaction, the calculation of creep can be iterated over small time steps.

Shrinkage of Concrete

Shrinkage of concrete is defined as the contraction due to loss of moisture. Due to the shrinkage of concrete, the prestress in the tendon is reduced with time. The shrinkage of concrete was explained in details in the Section 1.6, Concrete (Part II).

IS:1343 - 1980 gives guidelines to estimate the shrinkage strain in **Section 5.2.4**. It is a simplified estimate of the ultimate shrinkage strain (ε_{sh}). Curing the concrete adequately and delaying the application of load provide long term benefits with regards to durability and loss of prestress. In special situations detailed calculations may be necessary to monitor shrinkage strain with time. Specialised literature or international codes can provide guidelines for such calculations.

Relaxation of Steel

Relaxation of steel is defined as the decrease in stress with time under constant strain.
Due to the relaxation of steel, the prestress in the tendon is reduced with time. The relaxation depends on the type of steel, initial prestress (*fpi*) and the temperature. To
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calculate the drop (or loss) in prestress (Δfp), the recommendations of **IS:1343 - 1980** can be followed in absence of test data.