MODULE-5

STIFFNESS MATRIX METHOD

The systematic development of slope deflection method in the matrix form has lead to Stiffness matrix method. The method is also called Displacement method. Since the basic unknowns are the displacement at the joint.

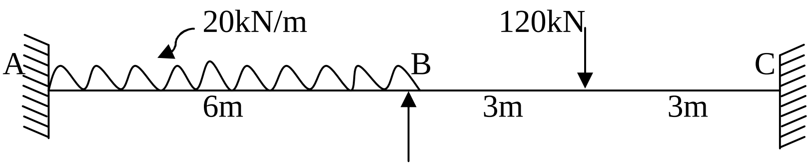
The stiffness matrix equation is given by

$[\Delta]\ [K] = \{[P] - [P_L]\}$

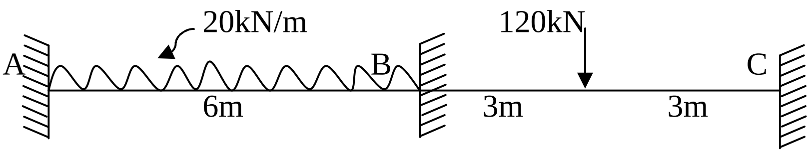
$[\Delta] = [K]^{-1}\{[P] - [P_L]\}$

Where,
[P] = Redundant in matrix form
[F] = Stiffness matrix
[P] = Final force at the joints in matrix form
$[P_L]$= force at the joints due to applied load in matrix form

1. Analyse the continuous beam by Stiffness method Sketch the BMD



Kinematic Indeterminacy KI = 1 (θ_B)

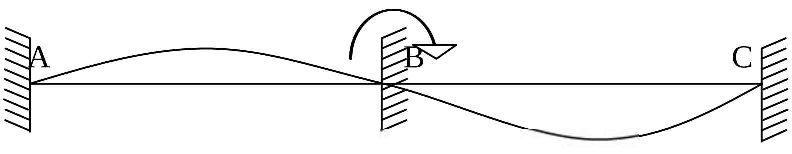


$[P_L] = M_{FBA} + M_{FBC}$

$$= \frac{wl^2}{12} + \left(-\frac{wl}{8}\right) = \frac{20\ X 6^2}{12} - \frac{120\ X 6}{8}$$

$[P_L] = -30\text{kN-m}$

Apply unit displacement at joint B.



$$[K] = \frac{4EI\theta}{l} + \frac{4EI\theta}{l} = \frac{4EI}{6} + \frac{4EI}{6} = 1.33EI \qquad (\theta=1)$$

By condition of equilibrium at joint B

$$[P] = 0$$

$$[\Delta] = [K]^{-1}\{[P] - [P_L]\}$$

$$= \frac{1}{K}\{[P] - [P_L]\}$$

$$\theta_B = \frac{1}{1.33EI}\{[0] - [-30]\} = \frac{22.56}{EI}$$

Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B)$$

$$= -60 + \frac{2\,EI}{6}\left(2\overset{0}{\cancel{\theta_A}} + \frac{22.5}{EI}\right) \qquad (\theta_A = 0 \text{ due to fixity at support A})$$

$$M_{AB} = -52.5\text{kN-m}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_B + \theta_A)$$

$$= 60 + \frac{2\,EI}{6}\left(2 \text{ X } \frac{22.5}{EI} + \overset{0}{\cancel{\theta_A}}\right)$$

$$M_{BA} = 75.04\text{kN-m}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C)$$

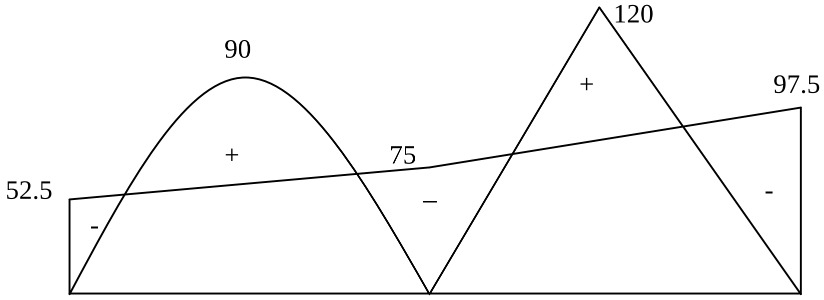
$$= -90 + \frac{2EI}{6}\left(2 \text{ X } \frac{22.5}{EI} + 0\right)$$

$$M_{BC} = -75\text{kN-m}$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}(2\theta_C + \theta_B)$$

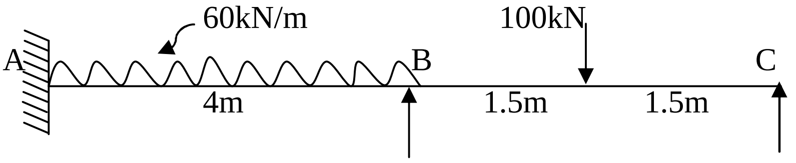
$$= 90 + \frac{2EI}{6}\left(0 + \frac{22.5}{EI}\right)$$

$$M_{CB} = 97.52\text{kN-m}$$

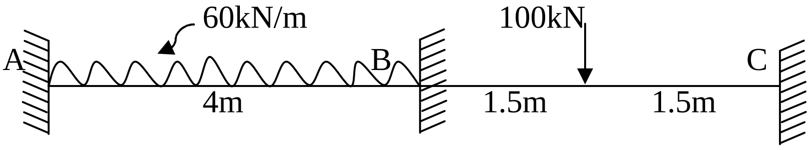


BMD

2. Analyse the continuous beam by Stiffness method Sketch the BMD



Kinematic Indeterminacy KI = 2 (θ_B & θ_C)



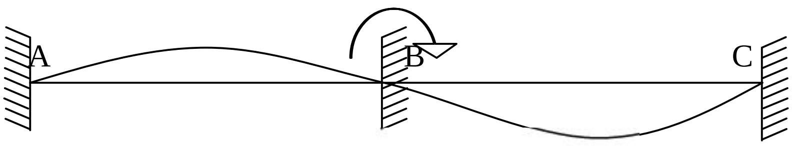
$[P_{1L}] = M_{FBA} + M_{FBC}$

$$= \frac{wl^2}{12} + \left(-\frac{wl}{8}\right) = \frac{60\,X\,4^2}{12} - \frac{100\,X\,3}{8} = 42.5\text{kN-m}$$

$$[P_{2L}] = M_{FCB} = \frac{wl}{8} = \frac{100\,X\,3}{8} = 37.5\text{kN-m}$$

$$[P_L] = \begin{pmatrix} P_{1L} \\ P_{2L} \end{pmatrix} = \begin{pmatrix} 42.5 \\ 37.5 \end{pmatrix} \text{kN-m}$$

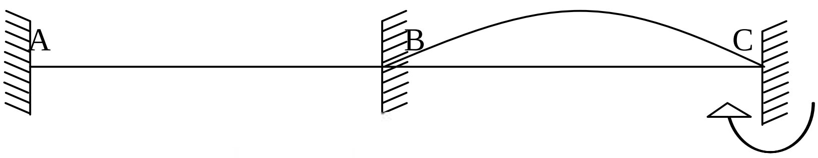
Apply unit displacement at joint B.



$$K_{11} = \frac{4EI\theta}{l} + \frac{4EI\theta}{l} = \frac{4EI}{4} + \frac{4EI}{3} = 2.33EI \qquad (\theta=1)$$

$$K_{21} = \frac{2EI\theta}{l} = \frac{2EI}{3} = 0.67EI$$

Apply unit displacement at joint B.



$$K_{12} = \frac{2EI\theta}{l} = \frac{2EI}{3} = 0.67EI$$

$$K_{22} = \frac{4EI\theta}{l} = \frac{4EI}{3} = 1.33EI$$

By condition of equilibrium at joint B

$$[P] = 0$$

$$[\Delta] = [K]^{-1}\{[P] - [P_L]\}$$

$$[\Delta] = \frac{1}{EI}\begin{pmatrix} 2.33 & 0.67 \\ 0.67 & 1.33 \end{pmatrix}^{-1} \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 42.5 \\ 37.5 \end{pmatrix} \right\}$$

$$\begin{pmatrix} \theta_B \\ \theta_C \end{pmatrix} = \frac{1}{EI} \begin{pmatrix} -11.88 \\ -22.19 \end{pmatrix}$$

Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B)$$

$$= -80 + \frac{2\,EI}{4}\left(2\cancelto{0}{\theta_A} - \frac{11.88}{EI}\right)$$ ($\theta_A = 0$ due to fixity at support A)

$$M_{AB} = -85.94\text{kN-m}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_B + \theta_A)$$

$$= 80 + \frac{2\,EI}{4}\left(2 \text{ X } \frac{-11.88}{EI} + \cancelto{0}{\theta_A}\right)$$

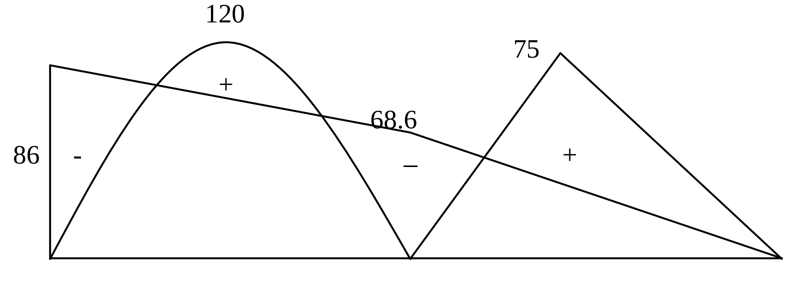
$$M_{BA} = 68.12\text{kN-m}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C)$$

$$= -37.5 + \frac{2EI}{6}\left(2 \text{ X } \frac{-11.88}{EI} + \frac{-22.9}{EI}\right)$$

$$M_{BC} = -68.6\text{kN-m}$$

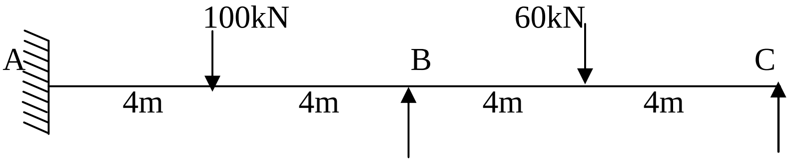
$$M_{CB} = 0$$



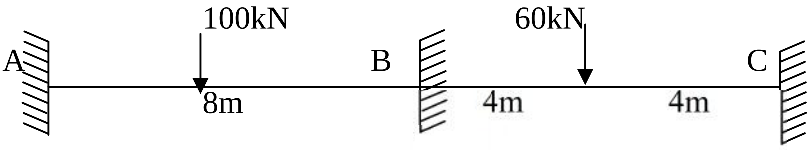
BMD

<u>Sinking of support</u>

1. Analyse the continuous beam shown in figure by stiffness method. Support B sinks by 300/EI units and support C sinks by 200/EI units



Kinematic Indeterminacy KI = 2 (θ_B & θ_C)



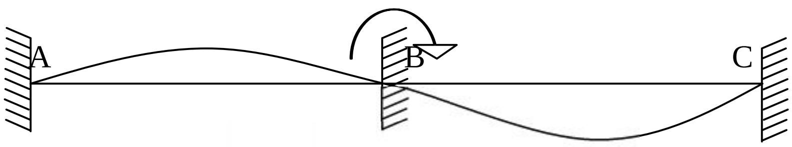
$$[P_{1L}] = M_{FBA} + M_{FBC} - \frac{6EI\delta}{l^2} - \frac{6EI\delta}{l^2}$$

$$= \frac{100\ X\ 8}{8} - \frac{60\ X8}{8} - \frac{6\ X\ 300}{8^2} + \frac{6\ X\ 100}{8^2} = 21.25\text{kN-m}$$

$$[P_{2L}] = M_{FCB} - \frac{6EI\delta}{l^2} = \frac{60\ X8}{8} + \frac{6\ X\ 100}{8^2} = 69.38\text{kN-m}$$

$$[P_L] = \begin{pmatrix} P_{1L} \\ P_{2L} \end{pmatrix} = \begin{pmatrix} 21.25 \\ 69.38 \end{pmatrix} \text{kN-m}$$

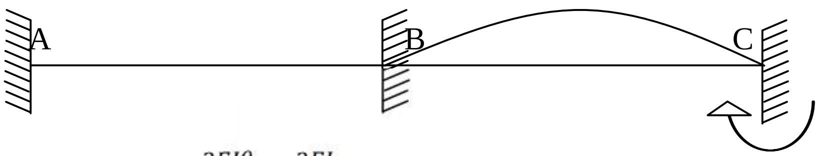
Apply unit displacement at joint B.



$$K_{11} = \frac{4EI\theta}{l} + \frac{4EI\theta}{l} = \frac{4EI}{8} + \frac{4EI}{8} = EI \qquad (\theta=1)$$

$$K_{21} = \frac{2EI\theta}{8} = \frac{2EI}{3} = 0.25EI$$

Apply unit displacement at joint B.



$$K_{12} = \frac{2EI\theta}{l} = \frac{2EI}{8} = 0.25EI$$

$$K_{22} = \frac{4EI\theta}{l} = \frac{4EI}{8} = 0.50EI$$

By condition of equilibrium at joint B

$$[P] = 0$$

$$[\Delta] = [K]^{-1}\{[P] - [P_L]\}$$

$$[\Delta] = \frac{1}{EI}\begin{pmatrix} 1 & 0.25 \\ 0.25 & 0.50 \end{pmatrix}^{-1} \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 21.25 \\ 69.38 \end{bmatrix} \right\}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI} \begin{pmatrix} 15.36 \\ -146.44 \end{pmatrix}$$

Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3(+\delta)}{l}\right)$$

$$= -100 + \frac{2\,EI}{8}\left(\overset{0}{\cancel{2\theta_A}} + \frac{15.36}{EI} - \frac{3EI(300/EI)}{8}\right)$$

($\theta_A = 0$ due to fixity at support A)

$M_{AB} = -124.29$kN-m

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\delta}{l}\right)$$

$$= 100 + \frac{2\,EI}{8}\left(\overset{0}{\cancel{\theta_A}} + 2\,X\,\frac{15.36}{EI} - \frac{3EI(300/EI)}{8}\right)$$

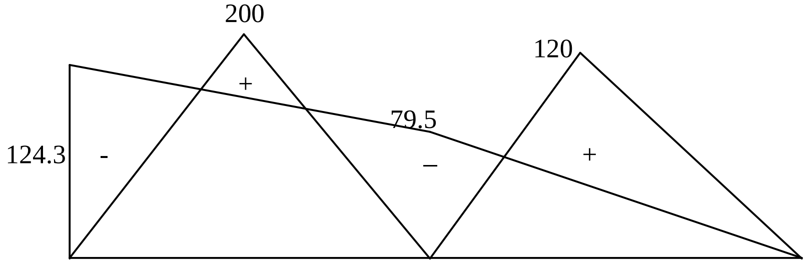
$M_{BA} = 79.55$kN-m

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\delta}{l}\right)$$

$$= -60 + \frac{2\,EI}{8}\left(2\,X\,\frac{15.36}{EI} + \frac{-146.44}{EI} - \frac{3EI(-100/EI)}{5}\right)$$

$M_{BC} = -79.55$kN-m

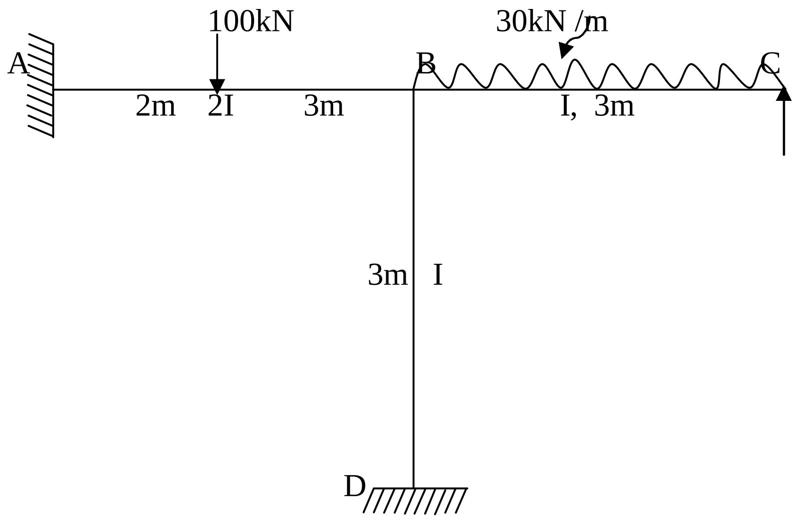
$M_{CB} = 0$



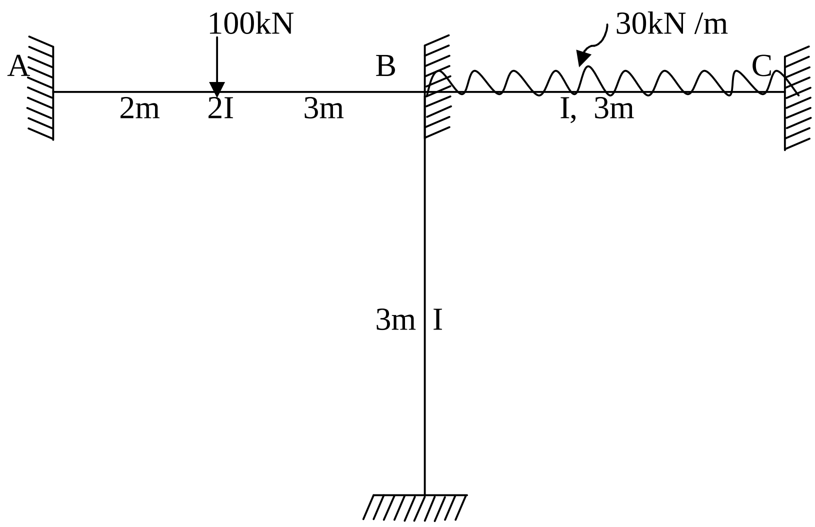
BMD

Analysis of frames

1. Analyse the frame by stiffness method



Kinematic Indeterminacy KI = 2 (θ_B & θ_C)



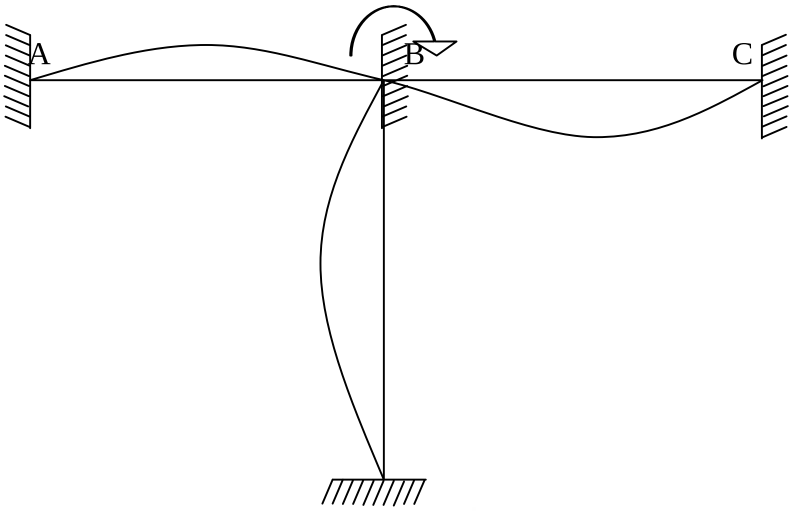
$[P_{1L}] = M_{FBA} - M_{FBC} + M_{FCD}$

$$= \frac{100 \times 2 \times 3^2}{5^2} - \frac{30 \times 3^2}{12} = 25.5\text{kN-m}$$

$$[P_{2L}] = M_{FCB} = \frac{30 \times 3^2}{12} = 22.5\text{kN-m}$$

$$[P_L] = \begin{pmatrix} P_{1L} \\ P_{2L} \end{pmatrix} = \begin{pmatrix} 25.5 \\ 22.5 \end{pmatrix} \text{kN-m}$$

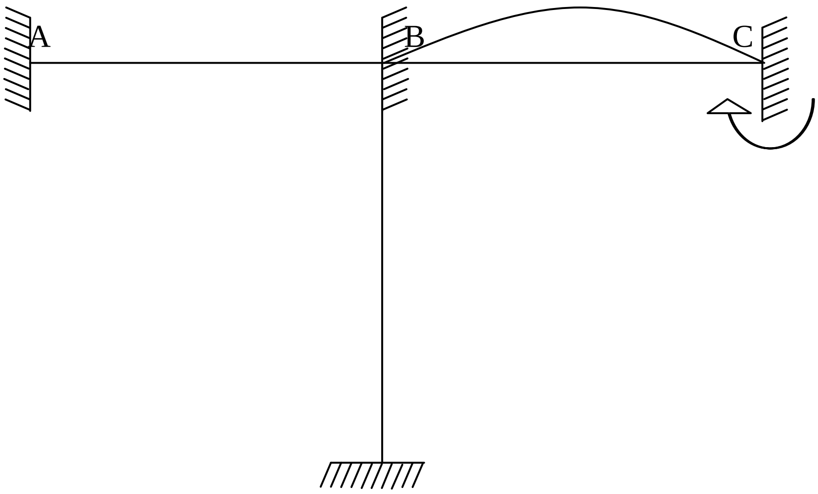
Apply unit displacement at joint B.



$$K_{11} = \frac{4EI\theta}{l} + \frac{4EI\theta}{l} + \frac{4EI\theta}{l} = \frac{4\,X 2EI}{5} + \frac{4EI}{3} + \frac{4EI}{3} = 4.267EI \qquad (\theta=1)$$

$$K_{21} = \frac{2EI\theta}{l} = \frac{2EI}{3} = 0.67EI$$

Apply unit displacement at joint C.



$$K_{12} = \frac{2EI\theta}{l} = \frac{2EI}{3} = 0.67EI$$

$$K_{22} = \frac{4EI\theta}{l} = \frac{4EI}{3} = 1.33EI$$

By condition of equilibrium at joint B

$$[P] = 0$$

$$[\Delta] = [K]^{-1}\{[P] - [P_L]\}$$

$$[\Delta] = \frac{1}{EI}\begin{pmatrix} 4.267 & 0.67 \\ 0.67 & 1.33 \end{pmatrix}^{-1} \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 25.5 \\ 22.5 \end{bmatrix} \right\}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} -3.604 \\ -15.01 \end{bmatrix}$$

Slope deflection equation

$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B)$

$= -72 + \frac{2\ X\ 2EI}{5}\left(\cancelto{0}{2\theta_A} + \frac{-3.604}{EI}\right)$ $\qquad$ ($\theta_A = 0$ due to fixity at support A)

$M_{AB} = -74.88$kN-m

$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_A + \theta_B)$

$= 72 + \frac{2\ X\ 2EI}{5}\left(\cancelto{0}{\theta_A} + 2\ X\frac{-3.604}{EI}\right)$

$M_{BA} = 42.23$kN-m

$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C)$

$= -22.5 + \frac{2\ EI}{3}\left(2\ X\frac{-3.604}{EI} + \frac{-15.1}{EI}\right)$

$M_{BC} = -37.37$kN-m

$M_{BD} = M_{FBD} + \frac{2EI}{l}(2\theta_B + \theta_D)$

$= 0 + \frac{2\ EI}{3}\left(2\ X\frac{-3.604}{EI} + 0\right)$

$M_{BD} = -4.81$kN-m

$M_{DB} = M_{FDB} + \frac{2EI}{l}(2\theta_D + \theta_B)$

$= 0 + \frac{2\ EI}{3}\left(2\ X 0 + \frac{-3.604}{EI}\right)$

$M_{DB} = -2.402$kN-m

$M_{CB} = 0$