MODULE-4

FLEXIBILITY MATRIX METHOD

The systematic development of consistent deformation method in the matrix form has lead to flexibility matrix method. The method is also called force method. Since the basic unknowns are the redundant forces in the structure.

This method is exactly opposite to stiffness matrix method.

The flexibility matrix equation is given by

$[P]\ [F] = \{[\Delta] - [\Delta_L]\}$

$[P] = [F]^{-1}\{[\Delta] - [\Delta_L]\}$

Where,

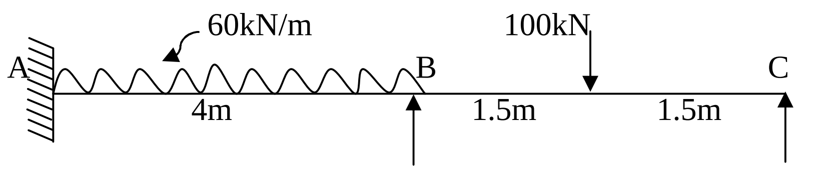
[P] = Redundant in matrix form

[F] = Flexibility matrix

[Δ] = Displacement at supports

$[\Delta_L]$= Displacement due to load

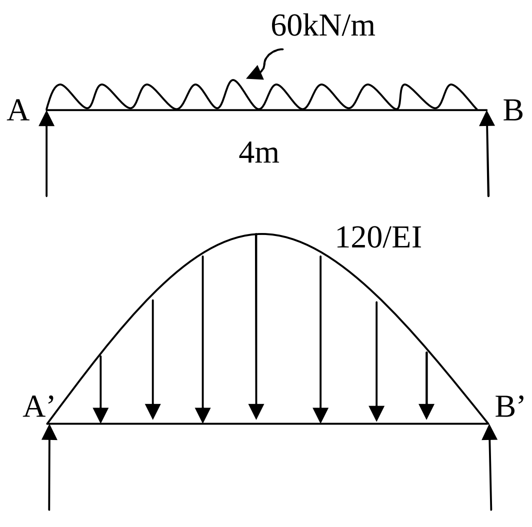
1. Analyse the continuous beam shown in the figure by flexibility matrix method, draw BMD

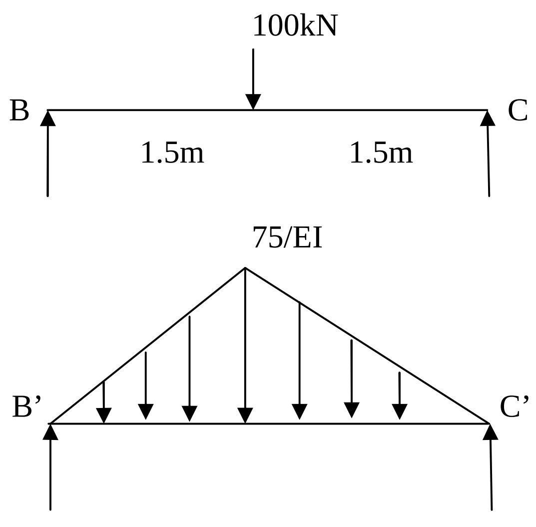


Static Indeterminacy SI = 2 (M_A and M_B)

M_A and M_B are the redundant

Let us remove the redundant to get primary determinate structure





$$[\Delta_L] = \begin{pmatrix} \Delta_{1L} \\ \Delta_{2L} \end{pmatrix}$$

Δ_{1L} = Rotation at A = SF at A'

$\Delta_{1L} = ½ [2/3 \text{ X } 4 \text{ X } \frac{120}{EI}]$

$\Delta_{1L} = \frac{160}{EI}$

Δ_{2L} = Rotation at A = SF at B'

= V_{B1}' + V_{B2}'

$\Delta_{2L} = ½ [2/3 \text{ X } 4 \text{ X } \frac{120}{EI}] + ½ [1/2 \text{ X } 3 \text{ X } \frac{75}{EI}]$

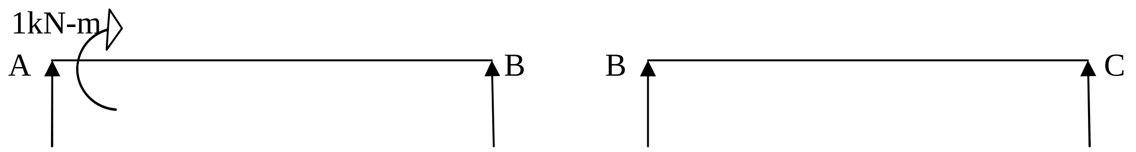
$\Delta_{2L} = \frac{216.25}{EI}$

$$[\Delta_L] = \frac{1}{EI} \begin{pmatrix} 160 \\ 216.25 \end{pmatrix}$$

Note: The rotation due to sagging is taken as positive. The moments producing due to sagging are also taken as positive.

<u>To get Flexibility Matrix</u>

Apply unit moment to joint A

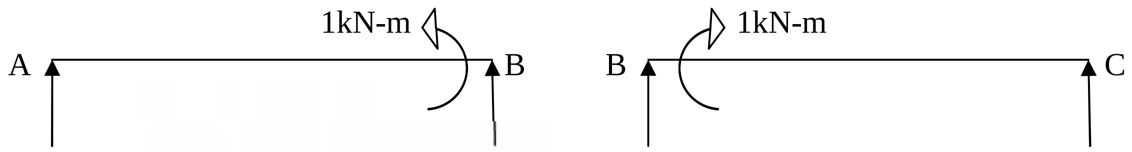


$$[F] = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix}$$

$$\delta_{11} = \frac{ml}{3EI} = \frac{1\ X\ 4}{3EI} = \frac{1.33}{EI}$$

$$\delta_{21} = \frac{ml}{6EI} = \frac{1\ X\ 4}{6EI} = \frac{0.67}{EI}$$

Apply unit moment to joint A



$$\delta_{12} = \frac{ml}{6EI} = \frac{1\ X\ 4}{6EI} = \frac{0.67}{EI}$$

$$\delta_{22} = \frac{ml}{3EI} + \frac{ml}{3EI} = \frac{1\ X\ 4}{3EI} + \frac{1\ X\ 3}{EI} = \frac{2.33}{EI}$$

$$[F] = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix} = \frac{1}{EI} \begin{pmatrix} 1.33 & 0.67 \\ 0.67 & 1.33 \end{pmatrix}$$

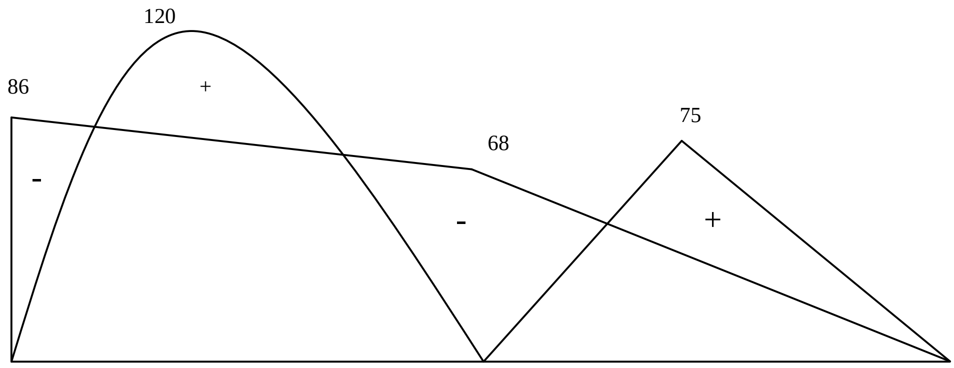
Apply the flexibility equation

$$[P] = [F]^{-1}\{[\Delta] - [\Delta_L]\}$$

$$[\Delta] = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

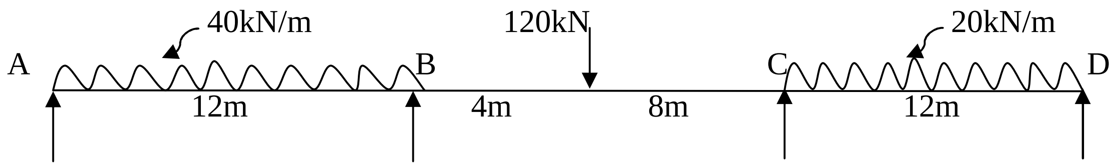
$$[P] = EI \begin{pmatrix} 1.33 & 0.67 \\ 0.67 & 1.33 \end{pmatrix}^{-1} \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \frac{1}{EI} \begin{pmatrix} 160 \\ 216.25 \end{pmatrix} \right\}$$

$$[P] = \begin{pmatrix} M_{AB} \\ M_{BA} \end{pmatrix} = \begin{pmatrix} -86.00 \\ -68.08 \end{pmatrix} \text{kN-m}$$

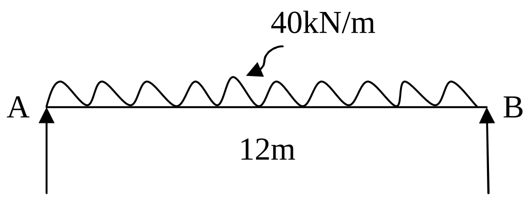


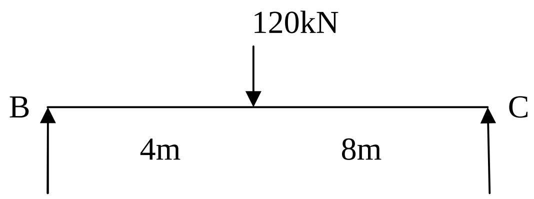
BMD

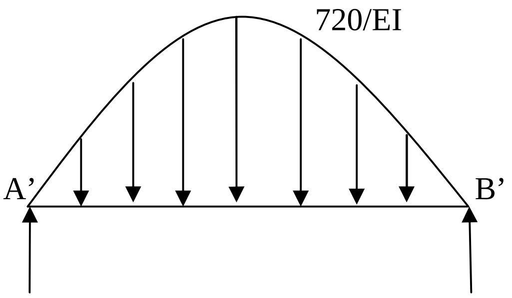
2. Analyse the continuous beam shown in the figure by flexibility matrix method, draw BMD

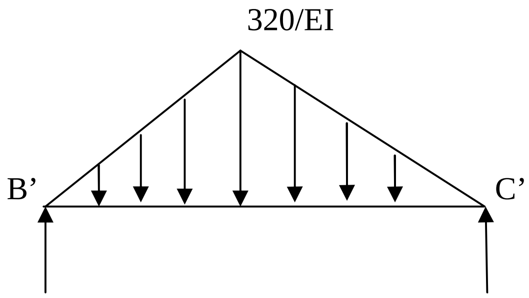


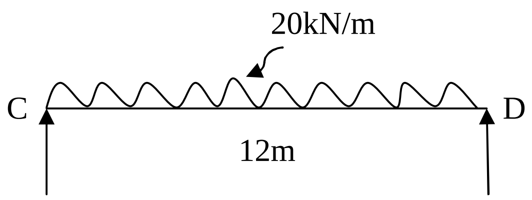
Static Indeterminacy SI = 2 (M_B and M_C)
M_B and M_C are the redundant
Let us remove the redundant to get primary determinate structure

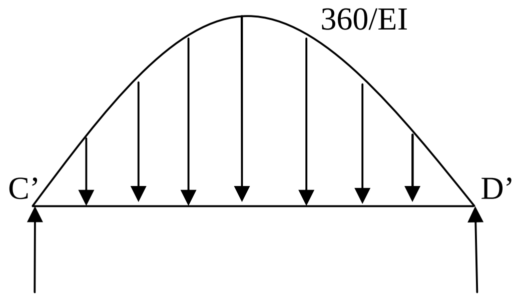












$$[\Delta_L] = \begin{pmatrix} \Delta_{1L} \\ \Delta_{2L} \end{pmatrix}$$

Δ_{1L} = Rotation at B = SF at B’

$= V_{B1}' + V_{B2}'$

$$\Delta_{1L} = \frac{3946.67}{EI}$$

Δ_{2L} = Rotation at C = SF at C’

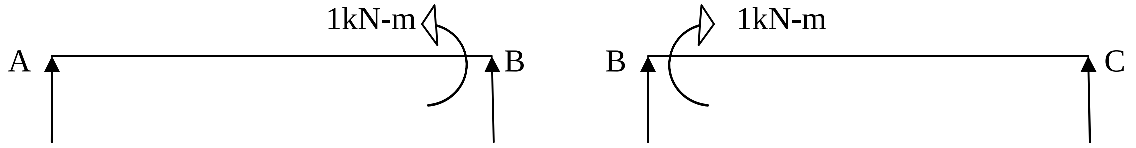
$= V_{C1}' + V_{C2}'$

$$\Delta_{2L} = \frac{2293.33}{EI}$$

$$[\Delta_L] = \frac{1}{EI} \begin{pmatrix} 3946.67 \\ 2293.33 \end{pmatrix}$$

To get Flexibility Matrix

Apply unit moment to joint A

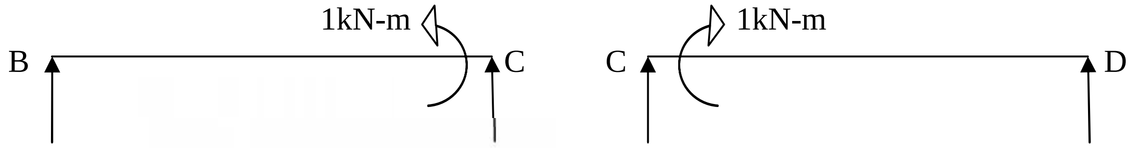


$$[F] = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix}$$

$$\delta_{11} = \frac{ml}{3EI} + \frac{ml}{3EI} = \frac{1 X 12}{3EI} + \frac{1 X 12}{3EI} = \frac{8}{EI}$$

$$\delta_{21} = \frac{ml}{6EI} = \frac{1 X 12}{6EI} = \frac{2}{EI}$$

Apply unit moment to joint A



$$\delta_{12} = \frac{ml}{6EI} = \frac{1 X 12}{6EI} = \frac{2}{EI}$$

$$\delta_{22} = \frac{ml}{3EI} + \frac{ml}{3EI} = \frac{1 X 12}{3EI} + \frac{1 X 12}{EI} = \frac{8}{EI}$$

$$[F] = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix} = \frac{1}{EI}\begin{pmatrix} 8 & 2 \\ 2 & 8 \end{pmatrix}$$

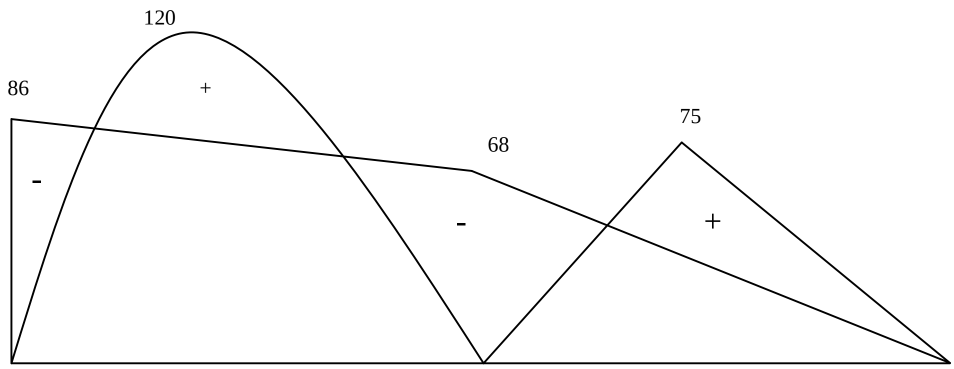
Apply the flexibility equation

$$[P] = [F]^{-1}\{[\Delta] - [\Delta_L]\}$$

$$[\Delta] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$[P] = EI \begin{pmatrix} 8 & 2 \\ 2 & 8 \end{pmatrix}^{-1} \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \frac{1}{EI} \begin{bmatrix} 3946 \\ 2293 \end{bmatrix} \right\}$$

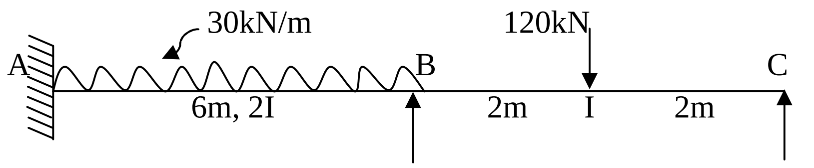
$$[P] = \begin{bmatrix} M_{AB} \\ M_{BA} \end{bmatrix} = \begin{bmatrix} -449.97 \\ -174.22 \end{bmatrix} \text{ kN-m}$$



BMD

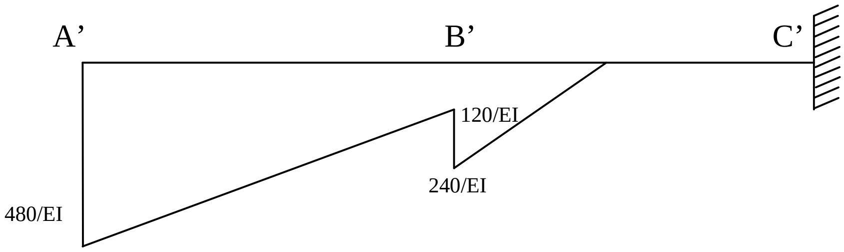
SINKING OF SUPPORT

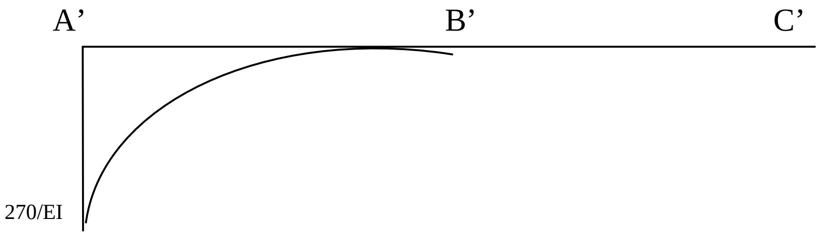
1. Analyse the continuous beam by flexibility method, support B sinks by 5mm. Sketch the BMD and EC given EI = 15 X 10^3 kN-m^2



NOTE: In this case of example with sinking of supports, the redundant should be selected as the vertical reaction.

Static indeterminacy is equal to 2. Let V_B and V_C be the redundant, remove the redundant to get the primary structure.





$$[\Delta_L] = \begin{pmatrix} \Delta_{1L} \\ \Delta_{2L} \end{pmatrix}$$

Δ_{1L} = Displacement at B in primary determinate structure = BM at B’ in conjugate beam

$$\Delta_{1L} = [\ \tfrac{1}{2} \text{ X } 6 \text{ X } \tfrac{360}{EI} \text{ X } (2/3 \text{ X } 6)\ ] + (6 \text{ X } \tfrac{120}{EI} \text{ X } 6/2) + [\ \tfrac{1}{3} \text{ X } 6 \text{ X } \tfrac{270}{EI} \text{ X } (3/4 \text{ X } 6)\ ]$$

$$\Delta_{1L} = \frac{8910}{EI}$$

Δ_{2L} = Displacement at C in primary determinate structure = BM at C’ in conjugate beam

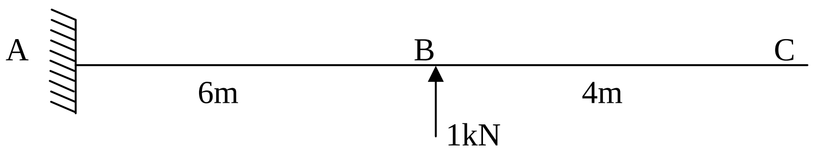
$$\Delta_{2L} = [\ \tfrac{1}{2} \text{ X } 6 \text{ X } \tfrac{360}{EI} \text{ X } (2/3 \text{ X } 6 + 4)\ ] + (6 \text{ X } \tfrac{120}{EI} \text{ X } 6/2 + 4) + [\ \tfrac{1}{3} \text{ X } 6 \text{ X } \tfrac{270}{EI} \text{ X } (3/4 \text{ X } 6 + 4)\ ]$$

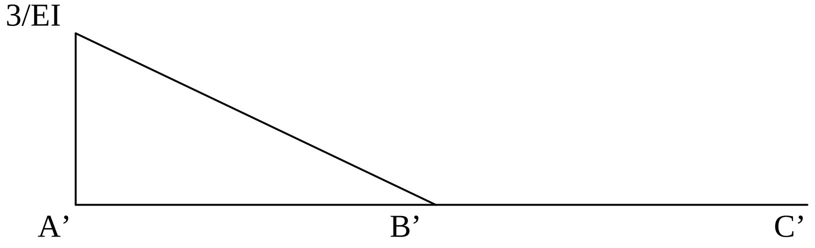
$$\Delta_{2L} = \frac{19070}{EI}$$

$$[\Delta_L] = \frac{1}{EI} \begin{pmatrix} 8910 \\ 19070 \end{pmatrix}$$

<u>To get Flexibility Matrix</u>

Apply unit Load at B



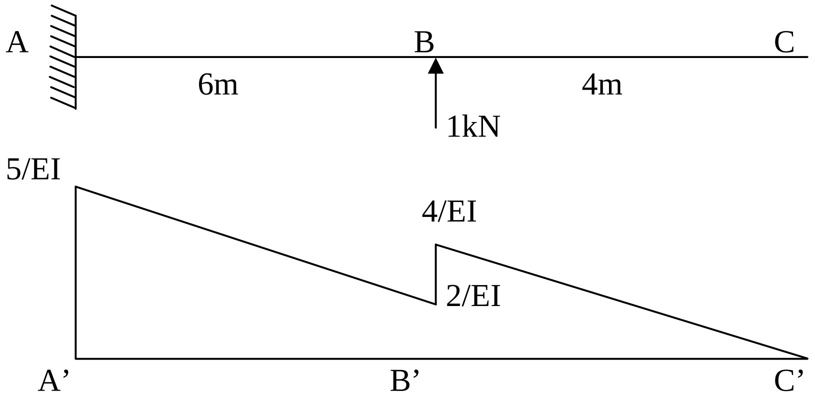


$$[F] = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix}$$

$\delta_{11} = -\frac{1}{2} \text{ X } 6 \text{ X } \frac{3}{EI} \text{ X } (2/3 \text{ X } 6) = \frac{-36}{EI}$

$\delta_{21} = -\frac{1}{2} \text{ X } 6 \text{ X } \frac{3}{EI} \text{ X } (2/3 \text{ X } 6 + 4) = \frac{-72}{EI}$

Apply unit load at C



$\delta_{12} = -\frac{1}{2} \text{ X } 6 \text{ X } \frac{3}{EI} \text{ X } (2/3 \text{ X } 6) - [6 \text{ X } \frac{2}{EI} \text{ X } (6/2)] = \frac{-72}{EI}$

$\delta_{22} = -\frac{1}{2} \text{ X } 6 \text{ X } \frac{3}{EI} \text{ X } (2/3 \text{ X } 6 + 4) - [6 \text{ X } \frac{2}{EI} \text{ X } (6/2 + 4)] - \frac{1}{2} \text{ X } 4 \text{ X } \frac{4}{EI} \text{ X } (2/3 \text{ X } 4) = \frac{-177.33}{EI}$

$$[F] = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix} = \frac{1}{EI} \begin{pmatrix} -36 & -72 \\ -72 & -177.33 \end{pmatrix}$$

Apply the flexibility equation

$[P] = [F]^{-1}\{[\Delta] - [\Delta_L]\}$

$$[\Delta] = \begin{pmatrix} 0.005 \\ 0 \end{pmatrix}$$

$$[P] = EI\begin{pmatrix} -36 & -72 \\ -72 & -177.33 \end{pmatrix}^{-1} \left\{ \begin{pmatrix} 0.005 \\ 0 \end{pmatrix} - \frac{1}{EI}\begin{pmatrix} 8910 \\ 19070 \end{pmatrix} \right\}$$

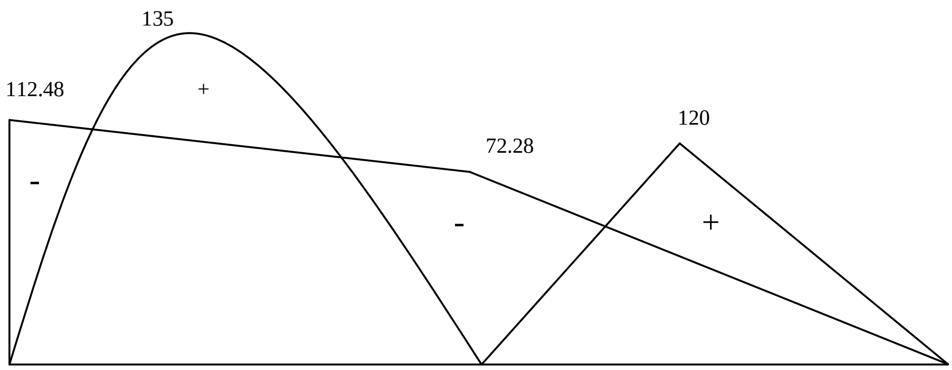
$$[P] = \begin{pmatrix} V_B \\ V_C \end{pmatrix} = \begin{pmatrix} 161.43 \\ 41.98 \end{pmatrix} \text{kN-m}$$

Support Reaction

$V_A = 96.64\text{kN}$, $V_{B1} = 83.36\text{kN}$, $V_{B2} = 78.07\text{kN}$, $V_C = 41.98\text{kN}$

$V_B = V_{B1} + V_{B2} = 161.43\text{kN}$

$$\begin{pmatrix} M_A \\ M_B \end{pmatrix} = \begin{pmatrix} 112.48 \\ 72.28 \end{pmatrix} \text{kN-m}$$



BMD