MODULE 3: STEAM AND REACTION TURBINE

3.1 Introduction:

Steam and gas turbines are power generating machines in which the pressure energy of the fluid is converted into mechanical energy. This conversion is due to the dynamic action of fluid flowing over the blades. These blades are mounted on the periphery of a rotating wheel in the radial direction. Today the steam turbine stands as one of the most important prime movers for power generation. It converts thermal energy into mechanical work by expanding high pressure and high temperature steam. The thermal efficiency of steam turbine is fairly high compared to steam engine. The uniform speed of steam turbine at wide loads makes it suitable for coupling it with generators, centrifugal pumps, centrifugal gas compressors, etc.

3.2 Classification of Steam Turbines:

Based on the action of steam on blades, steam turbines are classified into impulse turbines and reaction turbines (or impulse reaction turbines).

3.2.1 Impulse Steam Turbine: Impulse or impetus means sudden tendency of action without reflexes. A single-stage impulse turbine consists of a set of nozzles and moving blades as shown in figure 3.1. High pressure steam at boiler pressure enters the nozzle and expands to low condenser pressure in the nozzle. Thus, the pressure energy is converted into kinetic energy increasing the velocity of steam. The high velocity steam is then directed on a series of blades where the kinetic energy is absorbed and converted into an impulse force by changing the direction of flow of steam which gives rise to a change in momentum and therefore to a force. This causes the motion of blades. The velocity of steam decreases as it flows over the blades but the pressure remains constant, i.e. the pressure at the outlet side of the blade is equal to that at the inlet side. Such a turbine is termed as impulse turbine. Examples: De-Laval, Curtis, Moore, Zoelly, Rateau etc.

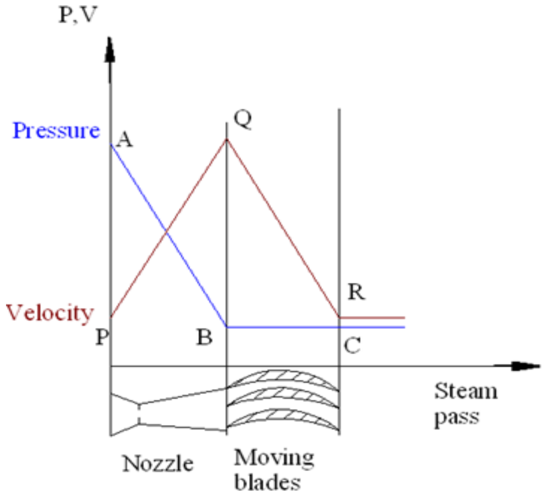

Fig. 3.1 Impulse turbine

3.2.2 Impulse Reaction Steam Turbine: In the impulse reaction turbine, power is generated by the combination of impulse action and reaction by expanding the steam in both fixed blades (act as nozzles) and moving blades as shown in figure 3.2. Here the pressure of the steam drops partially in fixed blades and partially in moving blades. Steam enters the fixed row of blades, undergoes a small drop in pressure and increases in velocity. Then steam enters the moving row of blades, undergoes a change in direction and momentum (impulse action), and a small drop in pressure too (reaction), giving rise to increase in kinetic energy. Hence, such a turbine is termed as impulse reaction turbine. Examples: Parson, Ljungstrom etc.

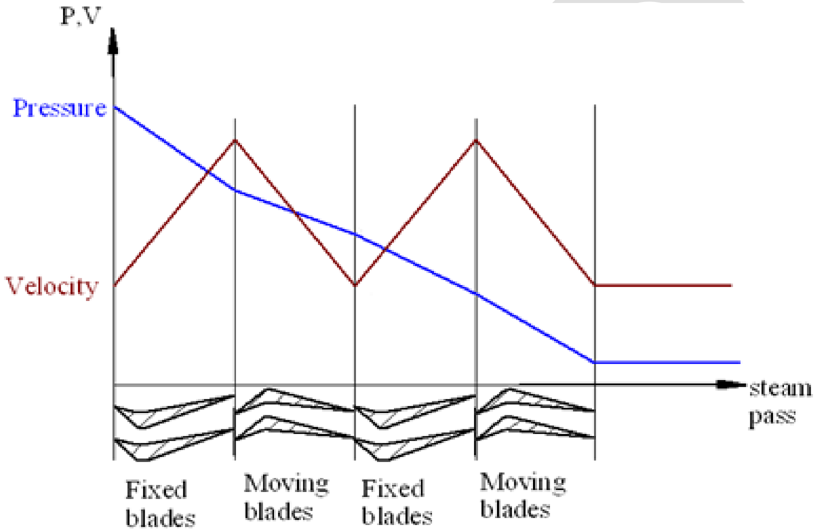

Fig. 3.2 Impulse reaction turbine

3.3 Difference between Impulse and Reaction Turbines:

The differences between impulse and reaction turbines are as follows:

Impulse Turbine	Reaction Turbine
✓ Complete expansion of the steam take place in the nozzle, hence steam is ejected with very high kinetic energy.	✓ Partial expansion of the steam takes place in the fixed blade (acts as nozzle) and further expansion takes place in the rotor blades.
✓ Blades are symmetrical in shape.	✓ Blades are non-symmetrical in shape, i.e. aerofoil section.
✓ Pressure remains constant between the ends of the moving blade. Hence relative velocity remains constant i.e., $V_{r1} = V_{r2}$	✓ Pressure drops from inlet to outlet of the moving blade. Hence relative velocity increases from inlet to outlet i.e., $V_{r2} > V_{r1}$
✓ Steam velocity at the inlet of machine is very high, hence needs compounding.	✓ Steam velocity at the inlet of machine is moderate or low, hence doesn't need compounding.
✓ Blade efficiency is comparatively low.	✓ Blade efficiency is high.

✓ Few number of stages required for given pressure drop or power output, hence machine is compact.	✓ More number of stages required for given pressure drop or power output, hence machine is bulky.
✓ Used for small power generation.	✓ Used for medium and large power generation.
✓ Suitable, where the efficiency is not a matter of fact.	✓ Suitable, where the efficiency is a matter of fact.

3.4 Need for Compounding of Steam Turbines:

Question No 3.1: What is the need for compounding in steam turbines? Discuss any two methods of compounding. ***(VTU, Jul/Aug-05, Dec-03/Jan-07, Dec-09/Jan-10, Dec-13/Jan-14)***

Answer: If the steam pressure drops from boiler pressure to condenser pressure in a single stage, exit velocity of steam from the nozzle will become very high and the turbine speed will be of the order of 30,000 rpm or more. As turbine speed is proportional to steam velocity, the carryover loss or leaving loss will be more (10% to 12%). Due to this very high speed, centrifugal stresses are developed on the turbine blades resulting in blade failure. In order to overcome all these difficulties it is necessary to reduce the turbine speed by the method of compounding. *Compounding is the method of reducing blade speed for a given overall pressure drop.*

3.5 Methods of Compounding of Steam Turbine:

Question No 3.2: What are the various methods of compounding of steam turbines? Explain any one of them. ***(VTU, Jul/Aug-02)***

Answer: Following are the methods of compounding of steam turbines:

1. Velocity compounding
2. Pressure compounding
3. Pressure and velocity compounding

3.5.1 Velocity Compounding:

Question No 3.3: Explain with the help of a schematic diagram a two row velocity compounded turbine stage. ***(VTU, Jan/Feb-05, Jul-03, Dec-12)***

Answer: A simple velocity compounded impulse turbine is shown in figure 3.3. It consists of a set of nozzles and a rotating wheel fitted with two or more rows of moving blades. One row of fixed blades fitted between the rows of moving blades. The function of the fixed blade is to direct the steam coming from the first row of moving blades to the next row of moving blades without appreciable change in velocity.

Steam from the boiler expands completely in the nozzle, hence whole of the pressure energy converts into kinetic energy. The kinetic energy of steam gained in the nozzle is successively absorbed by rows of moving blades and steam is exited from the last row axially with very low velocity. Due to this, the rotor speed decreases considerably. The velocity compounded impulse turbine is also called the Curtis turbine stage.

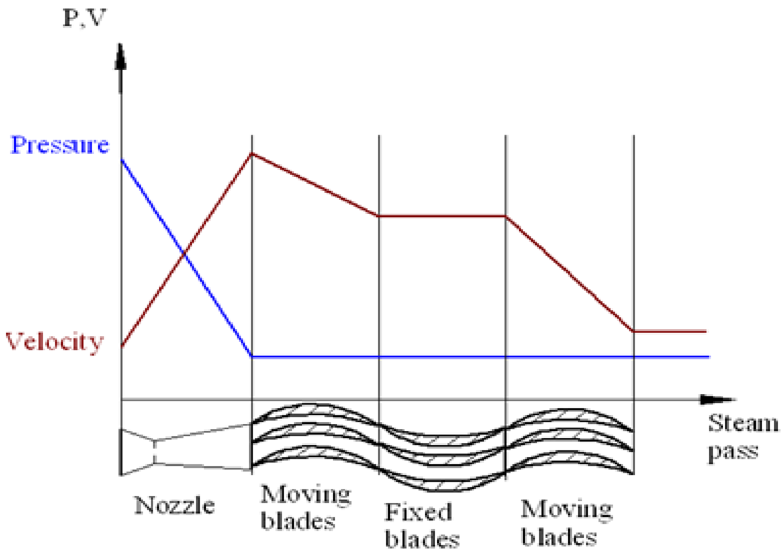

Fig. 3.3 Two stage velocity compounded impulse turbine.

3.5.2 Pressure Compounding:

Question No 3.4: Explain briefly a two stage pressure compounded impulse turbine and show the pressure and velocity variations across the turbine. ***(VTU, Jul-07, May/Jun-10, Dec-12)***

Answer: If a number of simple impulse stages arranged in series is known as pressure compounding. The arrangement contains one set of nozzles (fixed blades) at the entry of each row of moving blades. The total pressure drop doesn't take place in the first row of nozzles, but divided equally between all the nozzles as shown in figure 3.4.

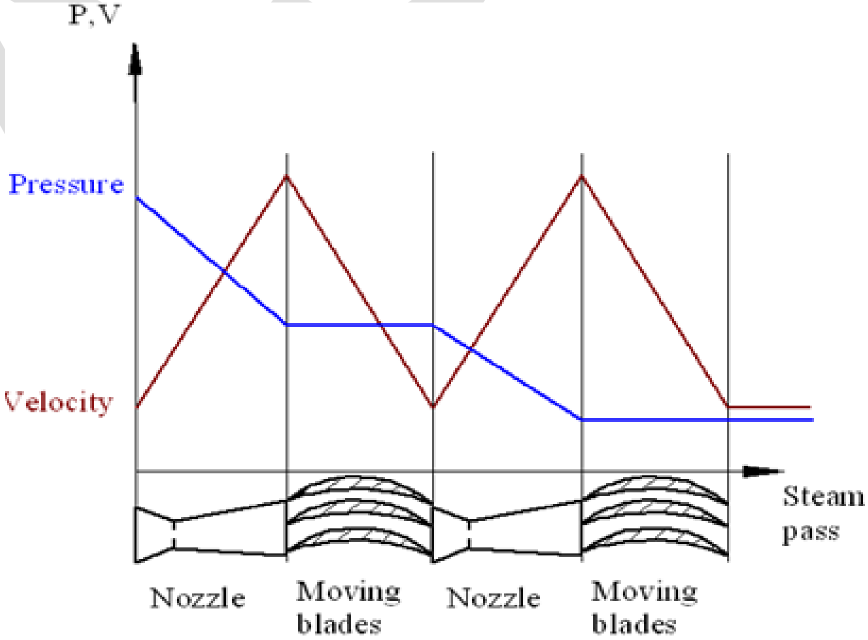

Fig. 3.4 Two stage pressure compounded impulse turbine.

The steam from the boiler is passed through the first set of nozzles in which it is partially expanded. Steam then passes over the first row of moving blades where almost all its velocity is absorbed. This completes expansion of steam in one stage. In the next stage, steam again enters the second set of nozzles and partially expands and enters the moving blades. Again the steam velocity is absorbed. This process continues till steam reaches the condenser pressure. Due to pressure compounding, smaller transformation of heat energy into kinetic energy takes place. Hence steam velocities become much lower and rotor speed decrease considerably. The pressure compounded impulse turbine is also called the Rateau turbine stage.

3.5.3 Pressure-Velocity Compounding:

Question No 3.5: Explain with a neat sketch pressure-velocity compounding. ***(Dec-03/Jan-07, Jun/Jul-13)***

Answer: If pressure and velocity are both compounded using two or more number of stages by having a series arrangement of simple velocity compounded turbines on the same shaft, it is known as pressure-velocity compounding. In this type of turbine both pressure compounding and velocity compounding methods are used. The total pressure drop of the steam is dividing into two stages and the velocity obtained in each stage is also compounded. Pressure drop occurs only in nozzles and remains constant in moving and fixed blades. As pressure drop is large in each stage only a few stages are necessary. This makes the turbine more compact than the other two types. Pressure-velocity compounding is used in Curtis turbine.

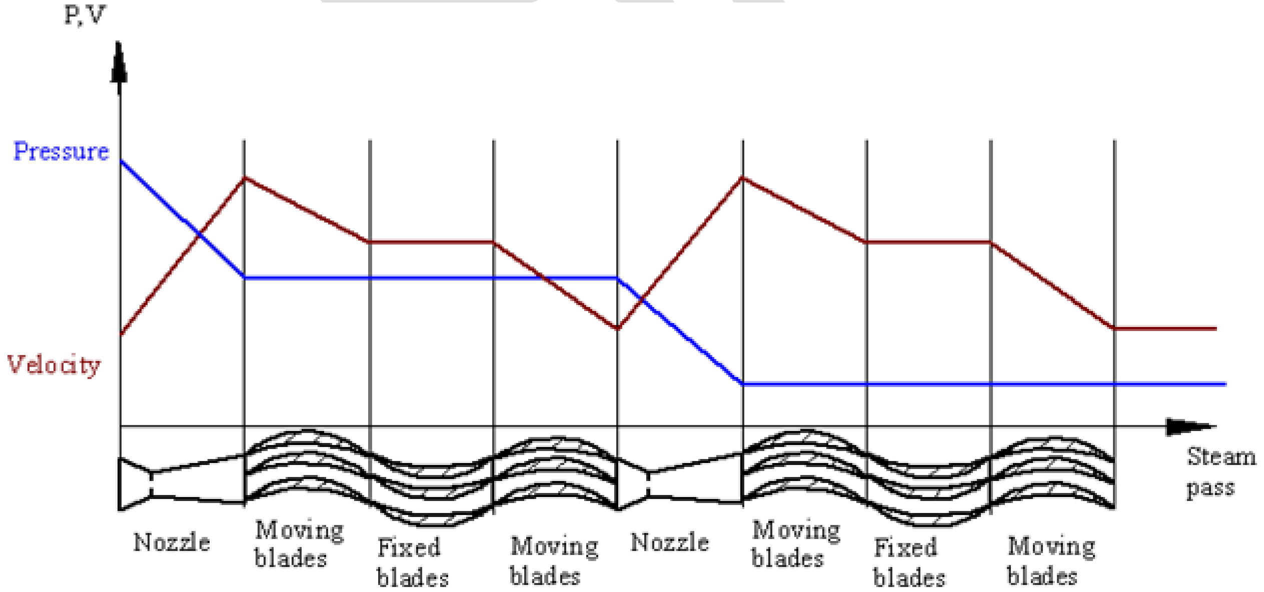

Fig. 3.5 Pressure-velocity compounded impulse turbine.

3.3 Efficiencies of Steam Turbine:

Question No 3.3: Define and explain (i) blade coefficient (ii) nozzle efficiency (iii) diagram efficiency (iv) stage efficiency. ***(VTU, Dec-11, Dec-12)***

Answer: Some performance parameters of steam turbines are as follows:

(i) Blade coefficient: It is also known as nozzle velocity coefficient. The losses in the flow over blades are due to friction, leakage and turbulence. Blade coefficient is the ratio of the velocity at the exit to the velocity at the inlet of the blade. i.e.,

$$C_b = \frac{V_{r2}}{V_{r1}} = \frac{V_2}{V_1}$$

(ii) Nozzle efficiency: It is defined as the ratio of actual enthalpy change per kg of steam to the isentropic enthalpy change per kg of steam. i.e.,

$$\eta_n = \frac{\Delta h}{\Delta h'}$$

For impulse turbine,

$$\eta_n = \frac{\frac{1}{2}V_1^2}{\Delta h'}$$

For reaction turbine the stator efficiency is,

$$\eta_p = \frac{\frac{1}{2}V_1^2 - \frac{1}{2}(V_{r1}^2 - V_{r2}^2)}{\Delta h'}$$

(iii) Diagram efficiency: It is also known as blade efficiency or rotor efficiency. It is defined as the ratio of work done per kg of steam by the rotor to the energy available at the inlet per kg of steam. i.e.,

$$\eta_b = \frac{w}{e_a} = \frac{U\Delta V_u}{e_a}$$

For impulse turbine, $e_a = \frac{1}{2}V_1^2$

For reaction turbine, $e_a = \frac{1}{2}V_1^2 - \frac{1}{2}(V_{r1}^2 - V_{r2}^2)$

(iv) Stage efficiency: It is defined as the ratio of work done per kg of steam by the rotor to the isentropic enthalpy change per kg of steam in the nozzle. i.e.,

$$\eta_s = \frac{w}{\Delta h'}$$

For impulse turbine,

$$\eta_s = \frac{U\Delta V_u}{\frac{1}{2}V_1^2} \times \frac{\frac{1}{2}V_1^2}{\Delta h'}$$

Or, $$\eta_s = \eta_b \times \eta_n$$

For reaction turbine,

$$\eta_s = \frac{U\Delta V_u}{\frac{1}{2}V_1^2 - \frac{1}{2}(V_{r1}^2 - V_{r2}^2)} \times \frac{\frac{1}{2}V_1^2 - \frac{1}{2}(V_{r1}^2 - V_{r2}^2)}{\Delta h'}$$

Or, $$\eta_s = \eta_b \times \eta_p$$

3.7 De' Laval Turbine (Single Stage Axial Flow Impulse Turbine):

Question No 3.7: Show that for a single stage axial flow impulse turbine the rotor efficiency is given by, $\eta_b = 2(\varphi cos\alpha_1 - \varphi^2)\left[1 + C_b\frac{cos\beta_2}{cos\beta_1}\right]$, where $C_b = \frac{V_{r2}}{V_{r1}}$, φ is speed ratio, β_1 and β_2 are rotating blade angles at inlet and exit, V_{r1} and V_{r2} are relative velocities at inlet and exit.

(VTU, Feb-03, Jun/Jul-14)

Answer: The combined velocity diagram for an axial flow impulse turbine is as shown in figure 3.3.

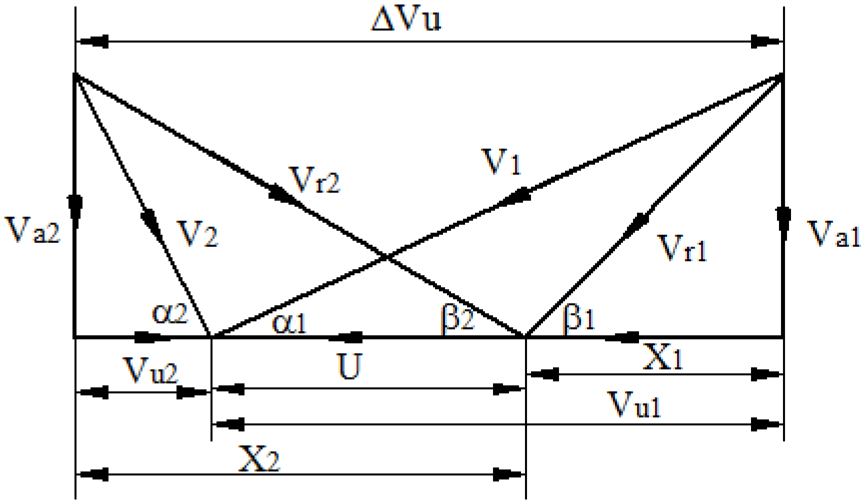

Fig. 3.3 Combined velocity diagram for an axial flow impulse turbine

Energy transfer for an axial flow turbine,

$$e = w = U\Delta V_u = U(V_{u1} + V_{u2})$$

From velocity diagram, $V_{u1} + V_{u2} = x_1 + U + x_2 - U = x_1 + x_2$

Or,
$$V_{u1} + V_{u2} = V_{r1}cos\beta_1 + V_{r2}cos\beta_2$$

$$V_{u1} + V_{u2} = V_{r1}cos\beta_1\left[1 + \frac{V_{r2}cos\beta_2}{V_{r1}cos\beta_1}\right] = x_1\left[1 + C_b\frac{cos\beta_2}{cos\beta_1}\right]$$

Where $C_b = \frac{V_{r2}}{V_{r1}}$, blade velocity coefficient

$$V_{u1} + V_{u2} = (V_{u1} - U)\left[1 + C_b\frac{cos\beta_2}{cos\beta_1}\right] = (V_1cos\alpha_1 - U)\left[1 + C_b\frac{cos\beta_2}{cos\beta_1}\right]$$

Then,

$$w = U(V_1cos\alpha_1 - U)\left[1 + C_b\frac{cos\beta_2}{cos\beta_1}\right]$$

Blade or rotor efficiency is given by,

$$\eta_b = \frac{w}{e_a} = \frac{U(V_1cos\alpha_1 - U)}{\frac{1}{2}V_1^2}\left[1 + C_b\frac{cos\beta_2}{cos\beta_1}\right]$$

$$\eta_b = 2\left(\frac{U}{V_1}cos\alpha_1 - \frac{U^2}{V_1^2}\right)\left[1 + C_b\frac{cos\beta_2}{cos\beta_1}\right]$$

$$\boldsymbol{\eta_b = 2(\varphi cos\alpha_1 - \varphi^2)\left[1 + C_b \frac{cos\beta_2}{cos\beta_1}\right]}$$

Where $\varphi = \frac{U}{V_1}$, blade speed ratio

Question No 3.8: Find the condition of maximum blade efficiency in a single stage impulse turbine. ***(VTU, Jan/Feb-03)***

Answer: The blade efficiency for single stage impulse turbine is given by,

$$\eta_b = 2(\varphi cos\alpha_1 - \varphi^2)\left[1 + C_b \frac{cos\beta_2}{cos\beta_1}\right]$$

The variation of blade efficiency vs. speed ratio is shown in figure 3.7.

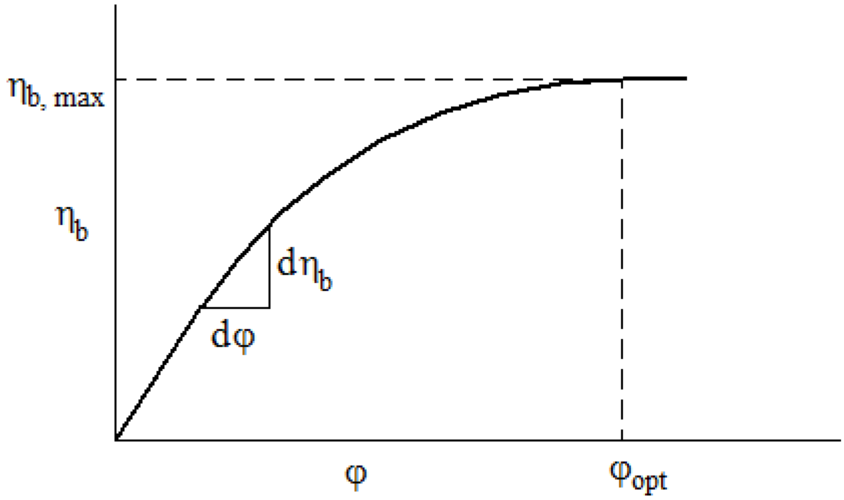

Fig. 3.7 Variation of blade efficiency vs. speed ratio

The slope for maximum blade efficiency is,

$$\frac{d\eta_b}{d\varphi} = 0$$

$$\frac{d}{d\varphi}\left\{2(\varphi cos\alpha_1 - \varphi^2)\left[1 + C_b \frac{cos\beta_2}{cos\beta_1}\right]\right\} = 0$$

$$2(cos\alpha_1 - 2\varphi)\left[1 + C_b \frac{cos\beta_2}{cos\beta_1}\right] = 0$$

$$\boldsymbol{\varphi_{opt} = \frac{cos\alpha_1}{2}}$$

The optimum speed ratio is the speed ratio at which the blade efficiency is the maximum.

Question No 3.9: For a single stage impulse turbine, prove that the maximum blade efficiency is given by $\boldsymbol{\eta_{b,max} = \frac{cos^2\alpha_1}{2}\left[1 + C_b \frac{cos\beta_2}{cos\beta_1}\right]}$***, where*** $\boldsymbol{C_b = \frac{V_{r2}}{V_{r1}}}$***,*** α_1 ***is speed ratio,*** β_1 ***and*** β_2 ***are rotating blade angles at inlet and exit,*** V_{r1} ***and*** V_{r2} ***are relative velocities at inlet and exit.*** ***(VTU, Dec-08/Jan-09)***

Answer: The blade efficiency for single stage impulse turbine is given by,

$$\eta_b = 2(\varphi cos\alpha_1 - \varphi^2)\left[1 + C_b \frac{cos\beta_2}{cos\beta_1}\right]$$

When $\varphi = \frac{cos\alpha_1}{2}$, the blade efficiency is the maximum, therefore

$$\eta_{b,max} = 2\left[\left(\frac{cos\alpha_1}{2}\right)cos\alpha_1 - \left(\frac{cos\alpha_1}{2}\right)^2\right]\left[1 + C_b\frac{cos\beta_2}{cos\beta_1}\right]$$

$$\boldsymbol{\eta_{b,max} = \frac{cos^2\alpha_1}{2}\left[1 + C_b\frac{cos\beta_2}{cos\beta_1}\right]}$$

Question No 3.10: Prove that the maximum blade efficiency for a single stage impulse turbine with equiangular rotor blades is given by $\eta_{b,max} = \frac{cos^2\alpha_1}{2}[1 + C_b]$, where α_1 is the nozzle angle and C_b is blade velocity coefficient. ***(VTU, Dec-10) Or,***

Prove that the maximum blade efficiency for a single stage impulse turbine with equiangular rotor blades is given by $\eta_{b,max} = cos^2\alpha_1$, where α_1 is the nozzle angle. ***(VTU, Jun/Jul-09, Jun/Jul-13)***

Answer: The maximum blade efficiency for a single stage impulse turbine is,

$$\eta_{b,max} = \frac{cos^2\alpha_1}{2}\left[1 + C_b\frac{cos\beta_2}{cos\beta_1}\right]$$

For equiangular rotor blades, β_1=β_2

$$\boldsymbol{\eta_{b,max} = \frac{cos^2\alpha_1}{2}[1 + C_b]}$$

If no losses due to friction, leakage and turbulence in the flow over blades, V_{r1}=V_{r2} (i.e. C_b=1)

$$\eta_{b,max} = \frac{cos^2\alpha_1}{2}[1 + 1]$$

$$\boldsymbol{\eta_{b,max} = cos^2\alpha_1}$$

Above equation conclude that, if the flow over blades doesn't have any losses due to friction, leakage and turbulence then for a single stage impulse turbine with equiangular rotor blades maximum blade efficiency is same as maximum utilization factor.

3.8 Curtis Turbine (Velocity Compounded Axial Flow Impulse Turbine):

The velocity diagrams for first and second stages of a Curtis turbine (velocity compounded impulse turbine) are as shown in figure 3.8. The tangential speed of blade for both the rows is same since all the moving blades are mounted on the same shaft. Assume equiangular stator and rotors blades and blade velocity coefficients for stator and rotors are same.

The work done by first row of moving blades is,

$$w_1 = U\Delta V_{u1} = U(V_{u1} + V_{u2})$$

From first stage velocity diagram,

$$V_{u1} = V_1cos\alpha_1$$

And also,

$$V_{u2} = x_2 - U = V_{r2}cos\beta_2 - U$$

$$V_{u2} = C_b V_{r1} cos\beta_1 - U$$
$$(Because\ \beta_1 = \beta_2\ and\ V_{r2} = C_b V_{r1})$$
$$V_{u2} = C_b x_1 - U = C_b(V_{u1} - U) - U$$
$$(Because\ V_{r1} cos\beta_1 = x_1 = V_{u1} - U)$$
$$\boldsymbol{V_{u2} = C_b(V_1 cos\alpha_1 - U) - U}$$
$$(Because\ V_{u1} = V_1 cos\alpha_1)$$

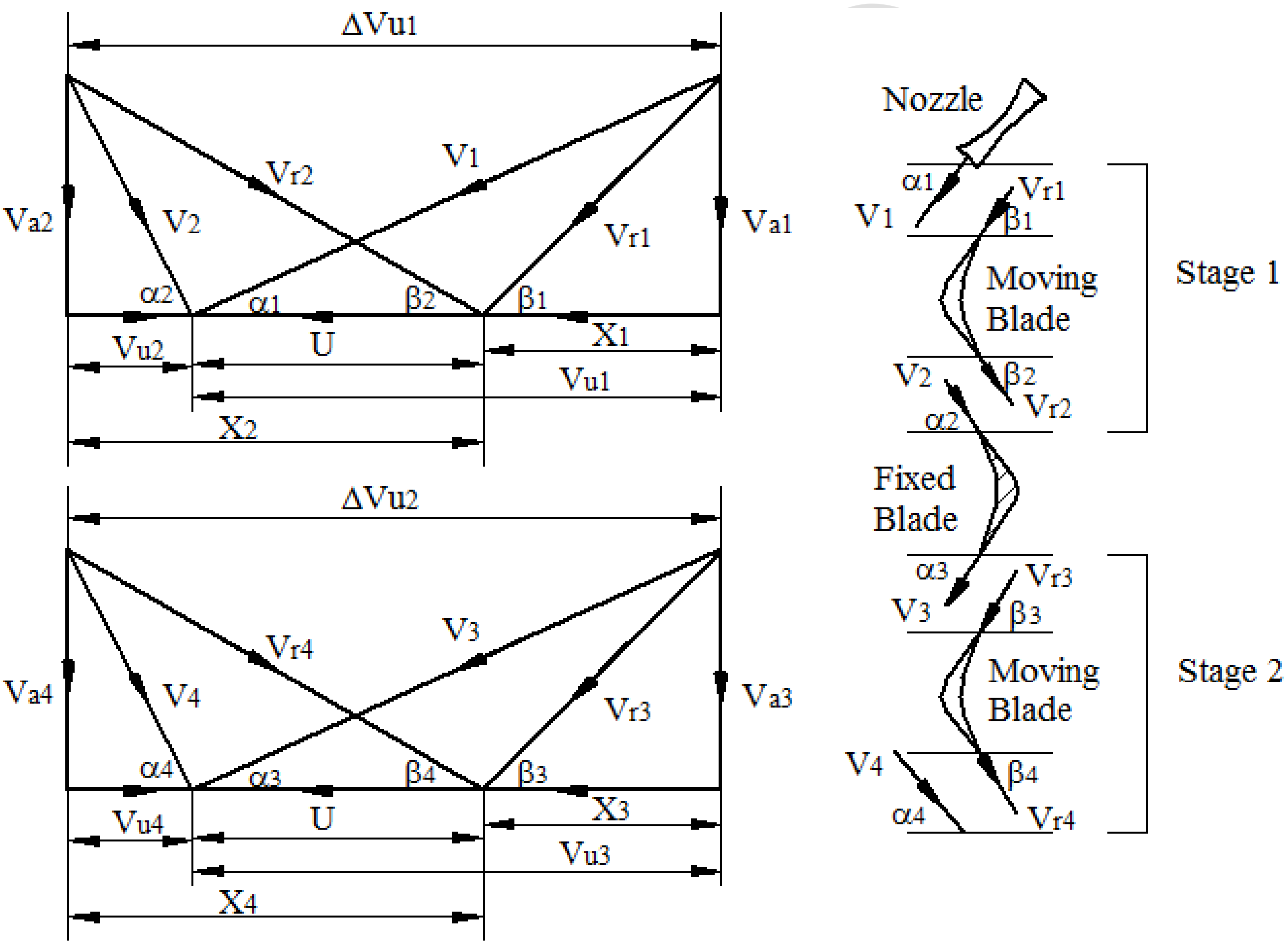

Fig. 3.8 Velocity diagrams for first and second stages of a Curtis turbine.

Then,

$$w_1 = U(V_1 cos\alpha_1 + C_b(V_1 cos\alpha_1 - U) - U)$$
$$w_1 = UV_1 cos\alpha_1 + C_b UV_1 cos\alpha_1 - C_b U^2 - U^2$$
$$w_1 = (1 + C_b)UV_1 cos\alpha_1 - (1 + C_b)U^2$$
$$\boldsymbol{w_1 = (1 + C_b)[UV_1 cos\alpha_1 - U^2]}$$

Similarly work done by second stage is,

$$w_2 = (1 + C_b)[UV_3 cos\alpha_3 - U^2]$$
$$w_2 = (1 + C_b)[C_b UV_2 cos\alpha_2 - U^2]$$
$$(Because\ \alpha_3 = \alpha_2\ and\ V_3 = C_b V_2)$$

$$w_2 = (1 + C_b)[C_b U V_{u2} - U^2]$$

$$(Because\ V_2 cos\alpha_2 = V_{u2})$$

$$w_2 = (1 + C_b)[C_b U\{C_b(V_1 cos\alpha_1 - U) - U\} - U^2]$$

$$(Because\ V_{u2} = C_b(V_1 cos\alpha_1 - U) - U)$$

$$w_2 = (1 + C_b)\left[{C_b}^2 U V_1 cos\alpha_1 - {C_b}^2 U^2 - C_b U^2 - U^2\right]$$

$$\boldsymbol{w_2 = (1 + C_b)\left[{C_b}^2 U V_1 cos\alpha_1 - U^2\left(1 + C_b + {C_b}^2\right)\right]}$$

The total work done by the Curtis turbine is, $w_T = w_1 + w_2$

$$w_T = (1 + C_b)\left[U V_1 cos\alpha_1 - U^2\right] + (1 + C_b)\left[{C_b}^2 U V_1 cos\alpha_1 - U^2\left(1 + C_b + {C_b}^2\right)\right]$$

$$w_T = (1 + C_b)\left[\left(1 + {C_b}^2\right) U V_1 cos\alpha_1 - U^2\left(2 + C_b + {C_b}^2\right)\right]$$

$$Let, C_b' = (1 + C_b)\left(1 + {C_b}^2\right)\ and\ C_b'' = (1 + C_b)\left(2 + C_b + {C_b}^2\right)$$

Then,

$$\boldsymbol{w_T = \left[C_b' U V_1 cos\alpha_1 - C_b'' U^2\right]}$$

Blade or rotor efficiency is given by,

$$\eta_b = \frac{w_T}{e_a} = \frac{\left(C_b' U V_1 cos\alpha_1 - C_b'' U^2\right)}{\frac{1}{2} V_1^2}$$

$$\eta_b = 2\left(C_b'\left(\frac{U}{V_1}\right) cos\alpha_1 - C_b''\left(\frac{U^2}{V_1^2}\right)\right)$$

$$\boldsymbol{\eta_b = 2\left(C_b' \varphi cos\alpha_1 - C_b'' \varphi^2\right)}$$

The slope for maximum blade efficiency is,

$$\frac{d\eta_b}{d\varphi} = 0$$

$$\frac{d}{d\varphi}\left\{2\left(C_b' \varphi cos\alpha_1 - C_b'' \varphi^2\right)\right\} = 0$$

$$2\left(C_b' cos\alpha_1 - 2C_b'' \varphi\right) = 0$$

$$\boldsymbol{\varphi_{opt} = \left(\frac{C_b'}{C_b''}\right)\frac{cos\alpha_1}{2}}$$

The maximum blade efficiency is,

$$\eta_{b,max} = 2\left(C_b'\left(\left(\frac{C_b'}{C_b''}\right)\frac{cos\alpha_1}{2}\right) cos\alpha_1 - C_b''\left(\left(\frac{C_b'}{C_b''}\right)\frac{cos\alpha_1}{2}\right)^2\right)$$

$$\eta_{b,max} = 2\left(\frac{(C_b')^2}{C_b''}\left(\frac{cos^2\alpha_1}{2}\right) - \frac{(C_b')^2}{C_b''}\left(\frac{cos^2\alpha_1}{4}\right)\right)$$

$$\boldsymbol{\eta_{b,max} = \frac{(C_b')^2}{C_b''}\left(\frac{cos^2\alpha_1}{2}\right)}$$

Note: If blade velocity coefficient, $\boldsymbol{C_b = 1}$

Then, $C_b' = 4$ and $C_b'' = 8$

For single stage impulse turbine,

$$w = 2[UV_1 cos\alpha_1 - U^2]$$

$$\varphi_{opt} = \frac{cos\alpha_1}{2}$$

$$\eta_{b,max} = cos^2\alpha_1$$

For Curtis (two stage velocity compounded) turbine,

$$w_T = 4\left[UV_1 cos\alpha_1 - 2U^2\right]$$

$$\varphi_{opt} = \frac{cos\alpha_1}{4}$$

$$\eta_{b,max} = cos^2\alpha_1$$

Similarly for *'n'* stage Curtis (velocity compounded) turbine,

$$w = 2n[UV_1 cos\alpha_1 - U^2]$$

$$\varphi_{opt} = \frac{cos\alpha_1}{2n}$$

$$\eta_{b,max} = cos^2\alpha_1$$

For all Curtis turbines the maximum blade efficiency remains same irrespective of their number of stages.

3.9 Parson's Turbine (50% Axial Flow Reaction Turbine):

Question No 3.12: Show that for an axial flow reaction turbine, the degree of reaction is given by $\boldsymbol{R = \left(\frac{V_a}{2U}\right)[cot\beta_2 - cot\beta_1]}$ and also show that for axial flow 50% reaction turbine the blade speed is given by $\boldsymbol{U = V_a[cot\beta_2 - cot\beta_1]}$, where β_1 and β_2 are inlet and outlet rotor blade angles. Assume velocity of flow or axial velocity to be constant. ***(VTU, Jun-12)***

Answer: The combined velocity diagram for an axial flow reaction turbine is as shown in figure 3.9. From data given in the problem, $V_{a1}=V_{a2}=V_a$.

Degree of reaction for axial flow turbine,

$$R = \frac{\frac{1}{2}(V_{r2}^2 - V_{r1}^2)}{e} = \frac{(V_{r2}^2 - V_{r1}^2)}{2e}$$

From velocity diagram, $(V_{u1} + V_{u2}) = (x_1 + U + x_2 - U)$

$$= (x_1 + x_2) = (V_{a1}cot\beta_1 + V_{a2}cot\beta_2)$$

Or, $(V_{u1} + V_{u2}) = V_a(cot\beta_1 + cot\beta_2)$

From velocity diagram, $sin\beta_2 = \frac{V_{a2}}{V_{r2}} \Longrightarrow V_{r2} = \frac{V_{a2}}{sin\beta_2}$

$$V_{r2} = V_a cosec\beta_2$$

Similarly, $sin\beta_1 = \frac{V_{a1}}{V_{r1}} \Longrightarrow V_{r1} = \frac{V_{a1}}{sin\beta_1}$

$$V_{r1} = V_a cosec\beta_1$$

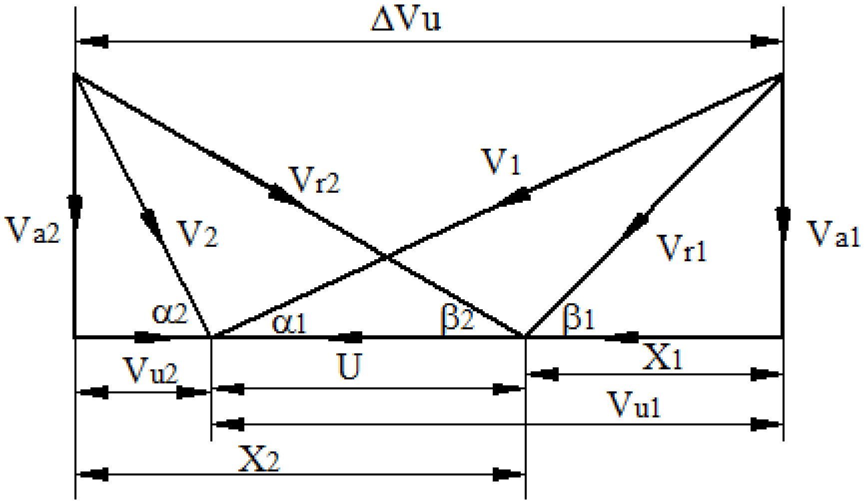

Fig. 3.9 Combined velocity diagram for an axial flow reaction turbine

Then, $e = U(V_{u1} + V_{u2}) \Longrightarrow \boldsymbol{e = UV_a(cot\beta_1 + cot\beta_2)}$

And, $(V_{r2}^2 - V_{r1}^2) = (V_a^2 cosec^2\beta_2 - V_a^2 cosec^2\beta_1) \Longrightarrow \boldsymbol{(V_{r2}^2 - V_{r1}^2) = V_a^2(cosec^2\beta_2 - cosec^2\beta_1)}$

Therefore,

$$R = \frac{V_a^2(cosec^2\beta_2 - cosec^2\beta_1)}{2UV_a(cot\beta_1 + cot\beta_2)}$$

$$R = \frac{V_a[(1 + cot^2\beta_2) - (1 + cot^2\beta_1)]}{2U(cot\beta_1 + cot\beta_2)}$$

$$R = \frac{V_a[(cot^2\beta_2) - (cot^2\beta_1)]}{2U(cot\beta_1 + cot\beta_2)}$$

$$R = \frac{V_a[(cot\beta_2 - cot\beta_1)(cot\beta_2 + cot\beta_1)]}{2U(cot\beta_1 + cot\beta_2)}$$

$$\boldsymbol{R = \left(\frac{V_a}{2U}\right)[cot\beta_2 - cot\beta_1]}$$

For an axial flow 50% reaction turbine, R=0.5

$$0.5 = \frac{1}{2} = \left(\frac{V_a}{2U}\right)[cot\beta_2 - cot\beta_1]$$

$$\boldsymbol{U = V_a[cot\beta_2 - cot\beta_1]}$$

Alternate method:

From velocity diagram, $U = V_{u1} - x_1 = V_{a1}cot\alpha_1 - V_{a1}cot\beta_1$

For an axial flow 50% reaction turbine, $\alpha_1=\beta_2$ and $\alpha_2=\beta_1$ and also $V_1=V_{r2}$ and $V_2=V_{r1}$

$$\boldsymbol{U = V_a[cot\beta_2 - cot\beta_1]}$$

Question No 3.13: For a 50% reaction steam turbine, show that α_1=β_2 and α_2=β_1, where α_1 and β_1 are the inlet angles of fixed and moving blades, α_2 and β_2 are the outlet angles of fixed and moving blades. ***(VTU, Dec-12)***

Answer: The general equation of degree of reaction for axial flow reaction steam turbine is,

$$R = \left(\frac{V_a}{2U}\right)[cot\beta_2 - cot\beta_1]$$

$$R = \left(\frac{V_a}{2U}\right)[(cot\beta_2 - cot\alpha_1) + (cot\alpha_1 - cot\beta_1)]$$

From velocity diagram (fig.3.9), $U = V_{u1} - x_1 = V_{a1}cot\alpha_1 - V_{a1}cot\beta_1$

Assume velocity of flow or axial velocity to be constant, $V_{a1}=V_{a2}=V_a$

$$U = V_a(cot\alpha_1 - cot\beta_1)$$

Or,

$$\frac{U}{V_a} = (cot\alpha_1 - cot\beta_1)$$

Then,

$$R = \left(\frac{V_a}{2U}\right)\left[(cot\beta_2 - cot\alpha_1) + \frac{U}{V_a}\right]$$

$$\boldsymbol{R = \frac{1}{2}\left(\frac{V_a}{U}\right)(cot\beta_2 - cot\alpha_1) + \frac{1}{2}}$$

For a 50% reaction steam turbine, $R = {}^1/_2$

Therefore, $0 = cot\beta_2 - cot\alpha_1 \Rightarrow cot\alpha_1 = cot\beta_2$

Or, $\boldsymbol{\alpha_1 = \beta_2}$

From velocity diagram (fig.3.9), $U = x_2 - V_{u2} = V_{a2}cot\beta_2 - V_{a2}cot\alpha_2$

Assume velocity of flow or axial velocity to be constant, $V_{a1}=V_{a2}=V_a$

$$U = V_a(cot\beta_2 - cot\alpha_2)$$

Then, $U = V_a(cot\alpha_1 - cot\beta_1) = V_a(cot\beta_2 - cot\alpha_2)$

But, $\alpha_1 = \beta_2$

$$V_a(cot\beta_2 - cot\beta_1) = V_a(cot\beta_2 - cot\alpha_2)$$

$$cot\beta_1 = cot\alpha_2$$

Or, $\boldsymbol{\alpha_2 = \beta_1}$

From velocity diagram (fig.3.9), $V_a = V_1cos\alpha_1 = V_{r2}cos\beta_2$

But, $\alpha_1 = \beta_2$

$$\boldsymbol{V_1 = V_{r2}}$$

$$V_a = V_2cos\alpha_2 = V_{r1}cos\beta_1$$

But, $\alpha_2 = \beta_1$

$$\boldsymbol{V_2 = V_{r1}}$$

These relations show that the velocity triangles at the inlet and outlet of the rotor of a 50% reaction stage are symmetrical.

Question No 3.14: What is meant by reaction staging? Prove that the maximum blade efficiency of Parson's (axial flow 50% reaction) turbine is given by $\boldsymbol{\eta_{b,max} = \frac{2cos^2\alpha_1}{1+cos^2\alpha_1}}$.

(VTU, Jan/Feb-04, Jun/Jul-08, May/Jun-10, Dec-14/Jan-15)

Answer: In reaction staging the expansion of steam and enthalpy drop occurs both in fixed and moving blades. Due to the effect of continuous expansion during flow over the moving blades, the relative velocity of steam increases i.e., $V_{r2}>V_{r1}$.

For Parson's (axial flow 50% reaction) turbine, $\alpha_1=\beta_2$ and $\alpha_2=\beta_1$ and also $V_1=V_{r2}$ and $V_2=V_{r1}$, then the velocity triangles are symmetric (refer figure 3.9).

Work done by Parson's turbine,

$$w = U\Delta V_u = U(V_{u1} + V_{u2})$$

From velocity diagram,

$$w = U(V_{u1} + x_2 - U) = U(V_1 cos\alpha_1 + V_{r2} cos\beta_2 - U)$$

But, $\alpha_1=\beta_2$ and $V_1=V_{r2}$

Then,

$$w = U(V_1 cos\alpha_1 + V_1 cos\alpha_1 - U) = 2UV_1 cos\alpha_1 - U^2$$

Or,

$$w = V_1^2\left[\frac{2UV_1 cos\alpha_1}{V_1^2} - \frac{U^2}{V_1^2}\right]$$

But, blade speed ratio $\varphi = \frac{U}{V_1}$

$$\boldsymbol{w = V_1^2[2\varphi cos\alpha_1 - \varphi^2]}$$

For reaction turbine energy available at rotor inlet,

$$e_a = \frac{1}{2}V_1^2 - \frac{1}{2}(V_{r1}^2 - V_{r2}^2)$$

But $V_1=V_{r2}$,

$$e_a = \frac{1}{2}V_1^2 - \frac{1}{2}(V_{r1}^2 - V_1^2) = V_1^2 - \frac{V_{r1}^2}{2}$$

From velocity diagram,

$$V_{r1}^2 = V_1^2 + U^2 - 2UV_1 cos\alpha_1 \qquad \text{(By cosine rule)}$$

Then,

$$e_a = V_1^2 - \frac{1}{2}[V_1^2 + U^2 - 2UV_1 cos\alpha_1]$$

$$e_a = \frac{1}{2}[V_1^2 + 2UV_1 cos\alpha_1 - U^2] = \frac{V_1^2}{2}\left[1 + \frac{2UV_1 cos\alpha_1}{V_1^2} - \frac{U^2}{V_1^2}\right]$$

But, blade speed ratio $\varphi = \frac{U}{V_1}$

$$\boldsymbol{e_a = \frac{V_1^2}{2}[1 + 2\varphi cos\alpha_1 - \varphi^2]}$$

Blade efficiency of reaction turbine,

$$\eta_b = \frac{w}{e_a} = \frac{V_1^2[2\varphi cos\alpha_1 - \varphi^2]}{\frac{V_1^2}{2}[1 + 2\varphi cos\alpha_1 - \varphi^2]}$$

$$\boldsymbol{\eta_b = \frac{2[2\varphi cos\alpha_1 - \varphi^2]}{[1 + 2\varphi cos\alpha_1 - \varphi^2]}}$$

Or,

$$\eta_b = \frac{2[1 + 2\varphi cos\alpha_1 - \varphi^2] - 2}{[1 + 2\varphi cos\alpha_1 - \varphi^2]}$$

$$\eta_b = 2 - \frac{2}{[1 + 2\varphi cos\alpha_1 - \varphi^2]} = 2 - 2[1 + 2\varphi cos\alpha_1 - \varphi^2]^{-1}$$

The slope for maximum blade efficiency is (refer figure 3.7),

$$\frac{d\eta_b}{d\varphi} = 0$$

$$\frac{d}{d\varphi}\{2 - 2[1 + 2\varphi cos\alpha_1 - \varphi^2]^{-1}\} = 0$$

$$2[1 + 2\varphi cos\alpha_1 - \varphi^2]^{-2}[2cos\alpha_1 - 2\varphi] = 0$$

$$[2cos\alpha_1 - 2\varphi] = 0$$

$$\boldsymbol{\varphi_{opt} = cos\alpha_1}$$

When $\varphi = cos\alpha_1$, the blade efficiency is the maximum, therefore

$$\eta_{b,max} = \frac{2[2cos^2\alpha_1 - cos^2\alpha_1]}{[1 + 2cos^2\alpha_1 - cos^2\alpha_1]}$$

$$\boldsymbol{\eta_{b,max} = \frac{2cos^2\alpha_1}{1 + cos^2\alpha_1}}$$