# MODULE 4 INTRODUCTION AND FREE VIBRATION



## 4.1 INTRODUCTION

In earlier units, you have studied various mechanisms and machines. The IC engine is one of them which converts thermal energy of fossil fuels to power. It produces highly fluctuating torque. Even the machines having rotating parts are never completely balanced. From static and dynamic analysis of such machines, it is known that these machines transmit forces to the ground through structure. These forces are periodic in nature.

You know that in a simple pendulum, bob starts to and fro motion or we can say oscillations when bob is disturbed from its equilibrium position. It executes oscillations at natural frequency. It keeps on oscillating until its motion dies out. If such a system is subjected to the periodic forces it responds to the impressed frequency which makes system to execute forced vibration at forcing frequency. If impressed frequency is equal to the natural frequency, resonance occurs which results in large oscillations and due to this it results in excessive dynamic stresses.

This unit deals with oscillatory behaviour of the dynamic systems. All the bodies having mass and elasticity are capable of vibration. In studying mechanical vibrations, the bodies are treated as elastic bodies instead of rigid bodies. The bodies have mass also. Because of mass it they can possess kinetic energy by virtue of their velocity. They can possess elastic strain energy which is comparable to the potential energy. The change of potential energy into kinetic energy and vice-versa keeps the body vibrating without external excitation (force or disturbance). If the cause of vibration is known, the remedy to control it can be made.

Vibration of a system is undesirable because of unwanted noise, high stresses, undesirable wear, etc. It is of great importance also in diagnostic maintenance.

### Objectives

After studying this unit, you should be able to

- analyse a system for mechanical vibration, determine degree of freedom of a system,

- determine natural frequency of a system,
- analyse and study dynamical behaviour of a system, and
- control vibration in a system.

## 4.2 DEFINITIONS

**Periodic Motion**

The motion which repeats after a regular interval of time is called periodic motion.

**Frequency**

The number of cycles completed in a unit time is called frequency. Its unit is cycles per second (cps) or Hertz (Hz).

**Time Period**

Time taken to complete one cycle is called periodic time. It is represented in seconds/cycle.

**Amplitude**

The maximum displacement of a vibrating system or body from the mean equilibrium position is called amplitude.

**Free Vibrations**

When a system is disturbed, it starts vibrating and keeps on vibrating thereafter without the action of external force. Such vibrations are called free vibrations.

**Natural Frequency**

When a system executes free vibrations which are undamped, the frequency of such a system is called natural frequency.

**Forced Vibrations**

The vibrations of the system under the influence of an external force are called forced vibrations. The frequency of forced vibrations is equal to the forcing frequency.

**Resonance**

When frequency of the exciting force is equal to the natural frequency of the system it is called resonance. Under such conditions the amplitude of vibration builds up dangerously.

**Degree of Freedom**

The degree of freedom of a vibrating body or system implies the number of independent coordinates which are required to define the motion of the body or system at given instant.

**Simple Harmonic Motion**

It is a to and fro periodic motion of a particle in which :

(a) acceleration is proportional to the displacement from the mean position.

(b) Acceleration is always directed towards a fixed point which is the mean equilibrium position.

It can be represented by an expression having a periodic function like sine or cosine.

$$x = X \sin \omega t$$

where $X$ is the amplitude.

Diagramatically it can be represented as shown in Figure 7.1.

when $\omega t = 0, \pi$ or $2\pi \Rightarrow x = 0$

when $\omega t = \frac{\pi}{2}, \Rightarrow x = X$

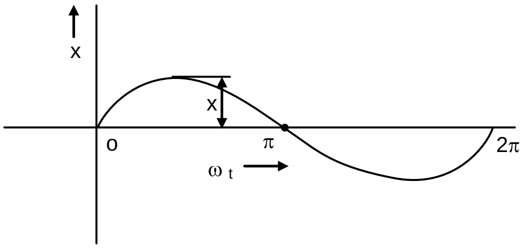



Figure : Simple Harmonic Motion

## SAQ 1

At which phase angle, amplitude occurs for a sinusoidal function?

# 4.3 ANALYSIS OF A SINGLE DEGREE OF **FREEDOM SYSTEMS FOR FREE VIBRATIONS**

A practical system is very complicated. Therefore, before proceeding to analyse the system it is desirable to simplify it by modeling the system. The modeling of the system is carried over in such a manner that the result is acceptable within the desirable accuracy. Instead of considering distributed mass, a lumped mass is easier to analyse, whose dynamic behaviour can be determined by one independent principal coordinate, in a single degree freedom system. It is important to study the single degree freedom system for a clear understanding of basic features of a vibration problem.

## 4.3.1 Elements of Lumped Parameter Vibratory System

The elements constituting a lumped parameter vibratory system are :

**The Mass**

The mass is assumed to be rigid and concentrated at the centre of gravity.

**The Spring**

It is assumed that the elasticity is represented by a helical spring. When deformed it stores energy. The energy stored in the spring is given by

$$PE = \frac{1}{2} k\, x^2$$

where *k* is stiffness of the spring. The force at the spring is given by

$$F = k\, x$$

The springs work as energy restoring element. They are treated massless.

**The Damper**

In a vibratory system the damper is an element which is responsible for loss of energy in the system. It converts energy into heat due to friction which may be either sliding friction or viscous friction. A vibratory system stops vibration because of energy conversion by damper. There are two types of dampers.

*Viscous Damper*

A viscous damper consists of viscous friction which converts energy into heat due to this. For this damper, force is proportional to the relative velocity.

$$F_d \ \alpha \ \text{relative velocity } (v)$$

$$\therefore \qquad F_d = c\,v$$

where $c$ is constant of proportionality and it is called coefficient of damping.

The coefficient of viscous damping is defined as the force in '*N*' when velocity is 1 m/s.

*Coulumb's Damper*

The dry sliding friction acts as a damper. It is almost a constant force but direction is always opposite to the sliding velocity. Therefore, direction of friction changes due to change in direction of velocity.

**The Excitation Force**

It is a source of continuous supply of energy to the vibratory system. It is an external periodic force which acts on the vibratory system.

It is important to study the single degree freedom system for a clear understanding of basic features of a vibration problem.

## 4.3.2 Undamped Free Vibration

There are several methods to analyse an undapmed system.

**Methodology**

*Method Based on Newton's II Law*

According to the Newton's II law, the rate of change of linear momentum is proportional to the force impressed upon it

$$\frac{d}{dt}(mv) \ \alpha \ \text{Net force in direction of the velocity}$$

Using $\qquad v = \frac{dx}{dt} = \dot{x}$

$$\therefore \qquad \frac{d\dot{x}}{dt} = (m\ddot{x}) = c \sum F$$

where $c$ is constant of proportionality.

or $\qquad m\ddot{x} = c \sum F$

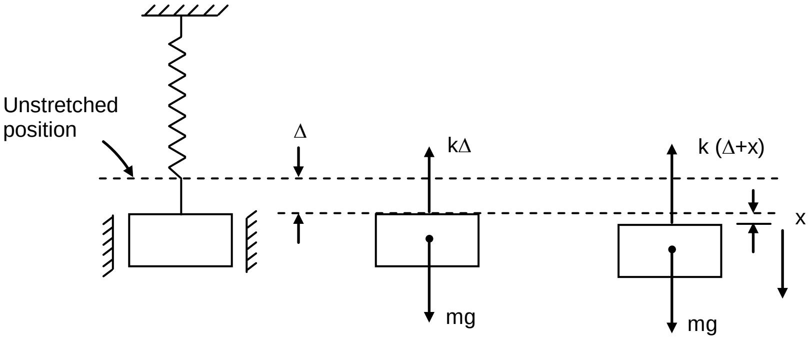
For proper units in a system $c = 1$

$$m\ddot{x} = \sum F$$

The direction of forces $m\ddot{x}$ and $\sum F$ are same. A model which represents undamped single degree of freedom system shall have two elements, i.e. helical spring and mass. The mass is constrained to move only in one direction as shown in Figure 7.2. The mass is in static condition in Figure 7.2(a). The free body diagram of the mass is shown in

Figure 7.2(b). The body is in equilibrium under the action of the two forces. Here 'Δ' is the extension of the spring after suspension of the mass on the spring.

Therefore, $k\,\Delta = mg$ . . . (7.1)




**(a) Spring Mass (b) Static Condition (c) Dynamic Condition**

Figure : Undamped Free Vibration

figure represents the dynamic condition of the body. In this case, the body is moving down with acceleration '$\ddot{x}$' also in downward direction, therefore,

$$m\ddot{x} = \sum F \text{ in direction of } \ddot{x}$$

or $$m\dot{x} = mg - k\,(x + \Delta)$$

Incorporating Eq. (7.1) in Eq. (7.2)

$$m\ddot{x} = -\,k\,x$$

or $$m\ddot{x} + k\,x = 0$$

*Method Based on D'Alembert's Principle*

The free body diagram of the mass in dynamic condition can be drawn as follows :

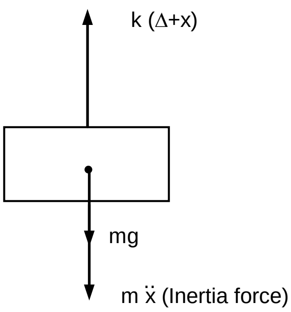



Figure : Free Body Diagram

The free body diagram of mass is shown in Figure 7.3. The force equation can be written as follows :

$$m\ddot{x} + mg = k\,(x + \Delta)$$

Incorporating Eq. (7.1) in Eq. (7.4), the following relation is obtained.

$$m\ddot{x} + kx = 0$$

This equation is same as we got earlier.

*Energy Method*

This method is applicable to only the conservative systems. In conservative systems there is no loss of energy and therefore total energy remains constant. When a mechanical system is in motion, the total energy of the system is partly kinetic and partly potential (elastic strain energy). The kinetic energy is due to the mass ($m$) and velocity ($\dot{x}$). The potential energy is due to spring stiffness and relative movement between the two ends of the spring.

$$\text{Energy } (E) = T + U = \text{constant } (C)$$

where $T$ = Kinetic energy of the system, and'

$U$ = Elastic strain energy.

Since total energy remains constant

$$\therefore \qquad \frac{dE}{dt} = 0 \quad \text{or} \quad \frac{d}{dt}(T + U) = 0$$

$$T = \frac{1}{2} m (\dot{x})^2$$

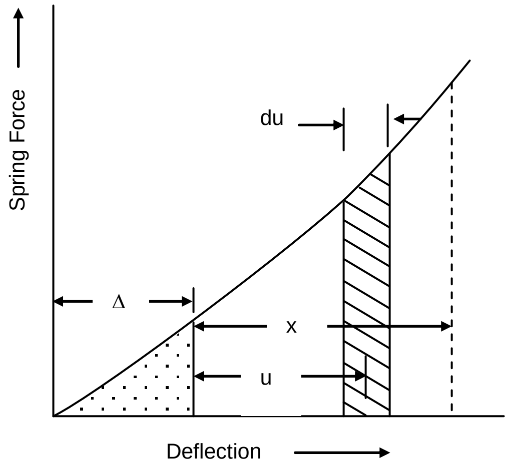



Figure : Spring Force **Deflection Diagram**

The potential energy of the system consists of two points :

(a) loss/gain in PE of mass, and

(b) strain energy of spring.

Consider an infinitesimal element $du$ at $x = u$.

From Figure 7.4

$$\text{Spring force } (F_u) = k (u + \Delta)$$

$$\text{Work done } dW = k (u + \Delta) \times du$$

$$\therefore \qquad U = \int_0^x dW - \text{loss of PE of mass}$$

$$= \int_0^x k (u + \Delta)\, du - mg\, x$$

$$\therefore \qquad U = \int_0^x (ku + mg)\, du - mg\, x \qquad [\because k\Delta = mg]$$

$$\text{or} \qquad U = \frac{1}{2}(kx^2) + mg\, x - mg\, x$$

$$\text{or} \qquad U = \frac{1}{2} kx^2$$

$$\frac{d}{dt}\left(\frac{1}{2} m\dot{x} + \frac{1}{2} kx^2\right) = 0$$

$\therefore$ $$\frac{1}{2} m \times 2\dot{x} \times \ddot{x} + \frac{1}{2} k \times 2x + \dot{x} = 0$$

or $$m\ddot{x} + kx = 0$$

This is the same equation as we got earlier.

*Rayleigh's Method*

It is a modified energy method. It may be noted that in a conservative system potential energy is maximum when kinetic energy is minimum and vice-versa. Therefore, equating maximum kinetic energy with maximum potential energy.

$\therefore$ $$\frac{1}{2} m\,(\dot{x}_{max})^2 = \frac{1}{2} k\,(x_{max})^2$$

and $$x_{max} = X$$

$\therefore$ $$\frac{1}{2} m\,(X\,\omega)^2 = \frac{1}{2} k\,X^2$$

or $$\omega = \sqrt{\frac{k}{m}}$$

**Solution of Differential Equation**

The differential equation of single degree freedom undamped system is given by

$$m\ddot{x} + kx = 0$$

or $$\ddot{x} + \left(\frac{k}{m}\right) x = 0$$

when coefficient of acceleration term is unity, the underroot of coefficient of $x$ is equal to the natural circular frequency, i.e. '$\omega_n$'

$\therefore$ $$\omega_n = \sqrt{\frac{k}{m}}$$

Therefore, Eq. (7.7) becomes

$$\ddot{x} + \omega_n^2\, x = 0$$

The equation is satisfied by functions sin $\omega_n\, t$ and cos $\omega_n\, t$. Therefore, solution of Eq. (7.9) can be written as

$$x = A \sin \omega_n t + B \cos \omega_n t$$

where $A$ and $B$ are constants. These constants can be determined from initial onditions. The system shown in Figure can be disturbed in two ways :

(a) by pulling mass by distance '$X$', and

(b) by hitting mass by means of a fast moving object with a velocity \ say '$V$'.

Considering case (a)

$$t = 0,\; x = X \text{ and } \dot{x} = 0$$

$\therefore$ $$X = B \text{ and } A = 0$$

$\therefore$ $$x = X \cos \omega_n t$$

Considering case (b)

$$t = 0,\ x = 0 \text{ and } \dot{x} = V$$

$$B = 0 \quad \text{and} \quad A = \frac{V}{\omega_n}$$

$\therefore$ $$x = \frac{V}{\omega_n} \sin \omega_n t$$

**Behaviour of Undamped System**

onsider the system shown in Figure . The system has been disturbed by pulling the mass by distance '*X*'. The solution of the system in this case is given by Eq. (7.11) which is

$$x = X \cos \omega_n t$$

$\therefore$ $$\dot{x} = -X \omega_n \sin \omega_n t = X \omega_n \cos \left(\omega_n t + \frac{\pi}{2}\right)$$

and $$\ddot{x} = -X \omega_n^2 \cos \omega_n t = X \omega_n^2 \cos (\omega_n t + \pi)$$

These expressions indicate that velocity vector leads displacement by $\frac{\pi}{2}$ and acceleration leads displacement by 'π'. The maximum velocity is ($X \omega_n$) and maximum acceleration is $(X \omega_n^2)$ .

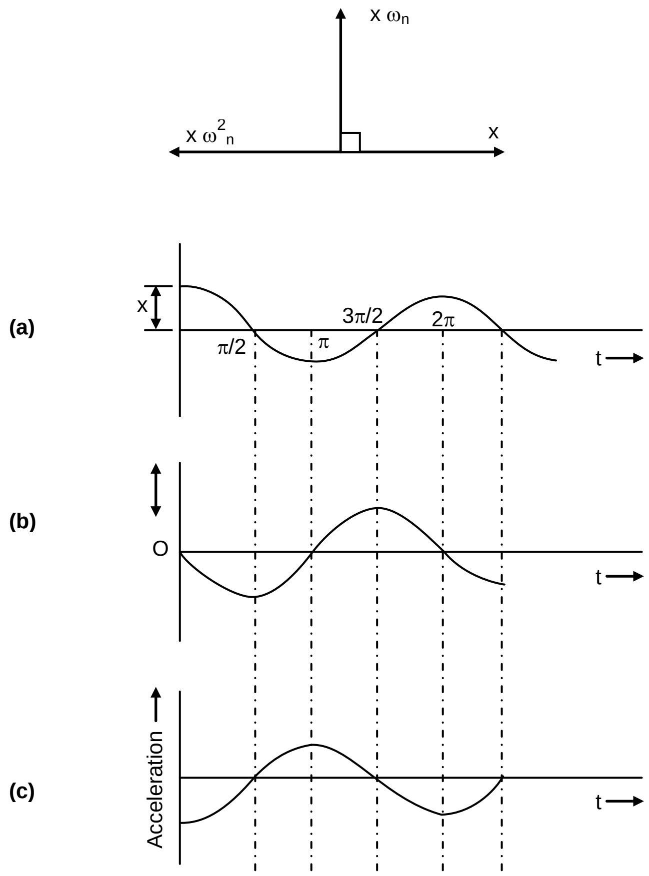



Figure : Plots of Displacement, Velocity and Acceleration

Figure 7.6 shows the plots of displacement, velocity and acceleration, with respect to time. The following observations can be made from these diagrams :

(a) A body, if disturbed, will never stop vibrating.

(b) When displacement is maximum, velocity is zero and acceleration is maximum in direction opposite to displacement.

(c) When displacement is zero, velocity is maximum and acceleration is zero.

## 4.3.3 Damped Free Vibration

In undamped free vibrations, two elements (spring and mass) were used but in damped third element which is damper in addition to these are used. The three element model is shown in Figure 7.7. In static equilibrium

$$k\,\Delta = mg$$

$$m\ddot{x} = mg - k\,(x + \Delta) - c\dot{x}$$

$\therefore$ $$m\ddot{x} = -\,kx - c\dot{x}$$

or $$m\ddot{x} + c\dot{x} + kx = 0$$

Let $$x = X\,e^{st}$$

$$m s^2 + c s + k = 0$$

or $$s^2 + \frac{c}{m}\,s + \frac{k}{m} = 0$$

$\therefore$ $$s_{1,2} = -\left(\frac{c}{2m}\right) \pm \frac{1}{2}\sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{k}{m}\right)}$$

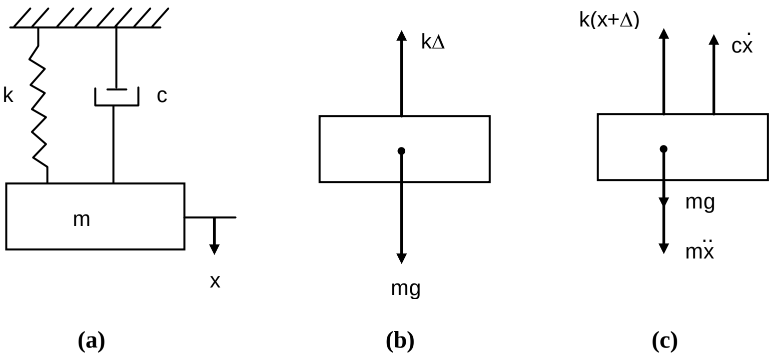



Figure : Damped Free Vibration

$$x = X_1\,e^{\left[-\left(\frac{c}{2m}\right) + \frac{1}{2}\sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{k}{m}\right)}\right]t} + X_2\,e^{\left[-\left(\frac{c}{2m}\right) - \frac{1}{2}\sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{k}{m}\right)}\right]t}$$

$$= e^{-\left(\frac{c}{2m}\right)t}\left[X_1\,e^{\frac{1}{2}\left\{\sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{k}{m}\right)}\right\}t} + X_2\,e^{-\frac{1}{2}\left\{\sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{k}{m}\right)}\right\}t}\right]$$

The nature of this solution depends on the term in the square root. There are three possible cases :

(a) $\left(\frac{c}{m}\right)^2 > 4\left(\frac{k}{m}\right)$ – Overdamped case

(b) $\left(\frac{c}{m}\right)^2 = 4\left(\frac{k}{m}\right)$ – Critically damped case

(c) $\left(\frac{c}{m}\right)^2 < 4\left(\frac{k}{m}\right)$ – Underdamped case

Let the critical damping coefficient be $C_c$, therefore,

$$\left(\frac{C_c}{m}\right)^2 = 4\left(\frac{k}{m}\right)$$

or $$C_c = 2\sqrt{km} = 2\sqrt{\frac{k}{m}\, m^2} = 2m\sqrt{m^2}\sqrt{\frac{k}{m}} = 2m\,\omega_n$$

or $$C_c = 2\sqrt{km} = 2m\,\omega_n$$

Almost all the systems are underdamped in practice.

Therefore, $$\sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{k}{m}\right)} = i\sqrt{4\left(\frac{k}{m}\right) - \left(\frac{c}{m}\right)^2}$$

The ratio of damping coefficient ($c$) to the critical damping coefficient is called damping factor '$\zeta$'.

$$\zeta = \frac{C}{C_c}$$

$$\sqrt{4\omega_n^2 - \left(\frac{c}{C_c} \times \frac{C_c}{m}\right)^2} = 4\sqrt{4\omega_n^2 - \zeta^2 \times \left(\frac{2m\,\omega_n}{m}\right)^2}$$

$$= 2\omega_n\sqrt{1-\zeta^2}$$

$\therefore$ $$x = e^{-\frac{c}{2m}t}\left[X_1\, e^{(i\,\omega_n\sqrt{1-\zeta^2})\,t} + X_2\, e^{(-\,i\,\omega_n\sqrt{1-\zeta^2})\,t}\right]$$

Let $$\omega_n\sqrt{1-\zeta^2} = \omega_d \quad \text{(say)}$$

where $\omega_d$ is natural frequency of the damped free vibrations.

Therefore, for under-damped case

$$x = e^{-\frac{c}{2m}}\left[X_1\, e^{i\,\omega_d\,t} + X_2\, e^{-\,i\,\omega_d\,t}\right]$$

For critically damped system

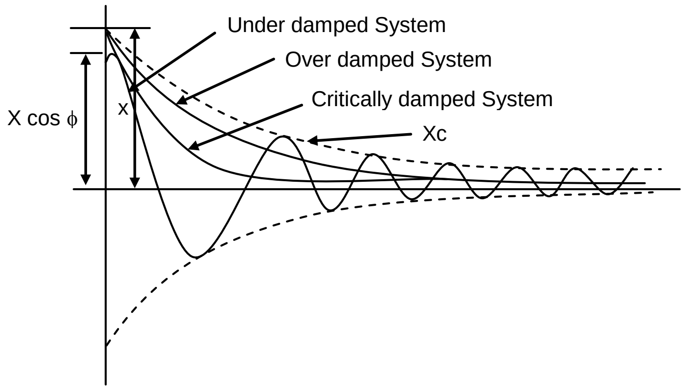
$$x = (X_1 + X_2\, t)\, e^{-\frac{c}{2m}t}$$

For overdamped system

$$x \quad e^{-\frac{c}{2m}t} \quad \lfloor \quad \sqrt{\quad} \qquad \sqrt{\quad} \quad \rfloor$$

$$\frac{C}{2m} = \frac{C}{C_c} \times \frac{C_c}{2m} = \zeta \times \frac{2m\,\omega_n}{2m} = \zeta\,\omega_n$$

$\therefore$ $$x \quad e^{-\zeta\,\omega_n\,t} \quad \lfloor \quad \sqrt{\quad} \qquad \sqrt{\quad} \quad \rfloor$$




Figure

The Eq. (7.19) can also be written as

$$x = X\, e^{-\zeta\, \omega_n\, t} \cos(\omega_d\, t + \phi)$$

where $X$ and $\phi$ are constants. $X$ represents amplitude and $\phi$ phase angle.

Let at $t = t,\quad x = x_0$.

$$\therefore \qquad x_0 = X\, e^{-\zeta\, \omega_n\, t} \cos(\omega_d\, t + \phi)$$

After one time period

$$t = t + t_p \quad \text{and} \quad x = x_1$$

$$\therefore \qquad x_1 = X\, e^{-\zeta\, \omega_n\, (t + t_p)} \cos\{\omega_d\, (t + t_p) + \phi\}$$

Dividing Eq. (7.24) by Eq. (7.25)

$$\frac{x_0}{x_1} = \frac{X\, e^{-\zeta\, \omega_n\, (t + t_p)} \cos \omega_d\, t + \phi}{X\, e^{-\zeta\, \omega_n\, (t + t_p)} \cos\{\omega_d\, (t + t_p)\ \phi\}}$$

Since $\quad t_p = \dfrac{1}{f_p} = \dfrac{2\pi}{\omega_d}$

or $\quad \omega_d\, t_p = 2\pi$

$$\therefore \qquad \frac{x_0}{x_1} = e^{\zeta\, \omega_n\, t_p}\ \frac{\cos(\omega_d\, t + \phi)}{\cos\{\omega_d\, t + 2\pi + \phi\}}$$

Since $\quad \cos\theta = \cos(2\pi + \theta)$

$$\therefore \qquad \cos(\omega_d\, t + \phi) = \cos\{\omega_d\, t + 2\pi + \phi\}$$

$$\therefore \qquad \frac{x_0}{x_1} = e^{\zeta\, \omega_n\, t_p}$$

or $$L_n \left(\frac{x_0}{x_1}\right) = \zeta\, \omega_n\, t_p = \zeta\, \omega_n\, \frac{2\pi}{\omega_d} = \frac{2\pi\, \omega_n\, \zeta}{\omega_n \sqrt{1 - \zeta^2}}$$

or $$L_n \left(\frac{x_0}{x_1}\right) = \frac{2\pi\, \zeta}{\sqrt{1 - \zeta^2}} \qquad \ldots (7.26)$$

$\dfrac{2\pi\, \zeta}{\sqrt{1 - \zeta^2}}$ is called logarithmic decrement.

If at $\quad t = t + n\, t_p$

It can be proved that

$$L_n \frac{x_0}{x_n} = \frac{2n\,\pi\,\zeta}{\sqrt{1-\zeta^2}}$$

If $\qquad \zeta < 0.3 \quad L_n \dfrac{x_0}{x_1} \approx 2\pi\,\zeta$

Figure 7.8 represents displacement time diagram for the above mentioned three cases. For over-damped and critically damped system mass returns to its original position slowly and there is no vibration. Vibration is possible only in the under-damped system because the roots of Eq. (7.14) are complex and solution consists of periodic functions (Eq. (7.22)).

### 4.3.4 Free Transverse Vibration due to a Point Load on a Simply **Supported Shaft**

In this type of vibration, all the particles vibrate along paths perpendicular to the shaft axis. The shaft may be having single to several supports. It may be carrying its own load, a single point load or several point loads come in this category. Now these cases are to be dealt with separately.

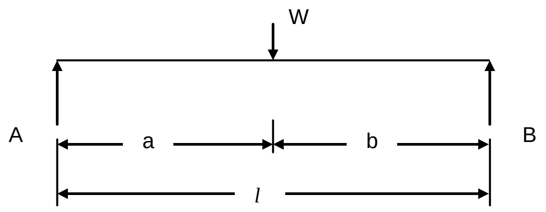



Figure : Free Transverse Vibration

Consider a very light shaft *AB* of length '*l*' carrying a point load '*W*' at a distance '*a*' from the support *A* and at a distance '*b*' from the support *B*.

$$a + b = l$$

and the deflection

$$\delta = \frac{W\,a^2\,b^2}{3E\,I\,l}$$

The natural circular frequency for the system is given by

$$\omega_n = \sqrt{\frac{k}{\left(\frac{W}{g}\right)}}$$

or $\qquad \omega_n = \sqrt{\dfrac{kg}{W}} = \sqrt{\dfrac{g}{\left(\frac{W}{k}\right)}}$

or $\qquad \omega_n = \sqrt{\dfrac{g}{\delta}}$

where $\qquad \delta = \dfrac{W}{k}$

$\therefore \qquad f_n = \dfrac{\omega_n}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{\delta}} = \dfrac{4.985}{\sqrt{\delta}}$ Hz

The mass of the beam was neglected for determination of the above mentioned natural frequency.

### 4.3.5 Free Torsional Vibration of a Single Rotor System

In torsional vibration, all the particles of the system vibrate along circular arcs having their centers along the axis of rotation. Figure 7.10 represents a single rotor systems. In both the cases (a) and (b), there is only one inertia '*I*'.

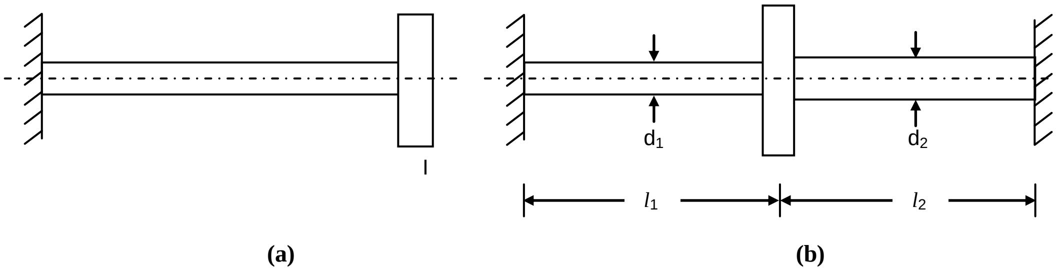



Figure : Free Torsional Vibration

In part (a) it is supported by one shaft segment and in part (b) it is supported by the two shaft segments.

The differential equation for the rotor shown in Figure 7.10(a) can be obtained by considering two couples, i.e. inertia couple and torsional elastic couple. If shaft is twisted slightly say by angle 'θ', the couple is given by

$$(k_t \ \theta)$$

where $k_t$ is torsional stiffness which is given by

$$k_t = \frac{T}{\theta} = \frac{G\,J}{l}$$

where $G$ is modulus of rigidity,

$J$ is polar moment of inertia, and

$l$ is length of shaft.

The differential equation for the rotor given in Figure 7.10(a) is

$$I\,\ddot{\theta} + k_t \ \theta = 0$$

or $$\ddot{\theta} + \frac{G\,J}{l} \ \theta = 0$$

∴ $$\omega_n = \sqrt{\frac{G\,J}{l}}$$

For the shaft shown in Figure 7.10(b), the two segments are acting like parallel springs. Therefore, the differential equation for this will be

$$I\,\ddot{\theta} + \left(\frac{G\,J_1}{l_1} + \frac{G\,J_2}{l_2}\right)\theta = 0$$

or $$\ddot{\theta} + \frac{G}{I}\left(\frac{J_1}{l_1} + \frac{J_2}{l_2}\right)\theta = 0$$

or $$\omega_n = \sqrt{\frac{G}{I}\left(\frac{J_1}{l_1} + \frac{J_2}{l_2}\right)}$$

## SAQ 2

(a) What is the difference between energy method and Rayleigh's method?

(b) By how much angle acceleration and velocity lead displacement?

(c) Along which curve amplitude decays in under-damped system?

## 4.4 CAUSES OF VIBRATION IN MACHINES

There are various sources of vibration in an industrial environment :

(a) Impact processes such as pile driving and blasting.

(b) Rotating or reciprocating machinery such as engines, compressors and motors.

(c) Transportation vehicles such as trucks, trains and aircraft.

(d) Flow of fluids through pipes and without pipes.

(e) Natural calamities such as earthquakes.

## 4.5 THE HARMFUL EFFECTS OF VIBRATIONS

There are various harmful effects of vibration :

(a) Excessive wear of bearings.

(b) Formation of cracks in machines, buildings and structure, etc.

(c) Loosening of fasteners in mechanical systems.

(d) Structural and mechanical failures in machines and buildings.

(e) Frequent and costly maintenance of machines.

(f) Electronic malfunctions through failure of solder joints.

(g) Abrasion of insulation around electric conductors, causing soots.

(h) The occupational exposure of humans to vibration leads to pain, discomfort and reduction in working efficiency.

## 4.6 VIBRATION CONTROL

The vibration can sometimes be eliminated on the basis of theoretical analysis. However, in eliminating the vibration may be too high. Therefore, a designer must compromise the manufacturing costs involved between an acceptable amount of vibration and a reasonable manufacturing cost. The following steps may be taken to control vibrations :

(a) The first group of methods attempts to reduce the excitation level at the source. The balancing of inertial forces, smoothening of fluid flows and proper lubrication at joints are effective methods and should be applied whenever possible.

(b) A suitable modification of parameters may also reduce the excitation level. The system parameters namely inertia, stiffness and damping are suitably chosen or modified to reduce the response to a given excitation.

(c) In this method, transmission of path of vibration is modified. It is popularly known as vibration isolation.

As mentioned above, the first attempt is made to reduce vibration at the source. In some cases, this can be easily achieved by either balancing or an increase in the precision of machine element. The use of close tolerances and better surface finish for machine parts make the machine less susceptible to vibration. This method may not be feasible in some cases like earthquake excitation, atmospheric turbulence, road roughness, engine combustion instability.

After reduction of excitation at the source, we need to look for a method to further control the vibration. Such a selection is guided by the factors predominantly governing the vibration level.

Example 4.1

Determine the natural frequency of spring mass pulley system shown in

**Solution**

**By Energy Method**

$$\text{Total energy } (E) = \frac{1}{2} m x_2 + \frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} k x^2$$

$$x = r\theta$$

or $$\dot{x} = r\dot{\theta}$$

$$\ddot{x} = r\ddot{\theta}$$

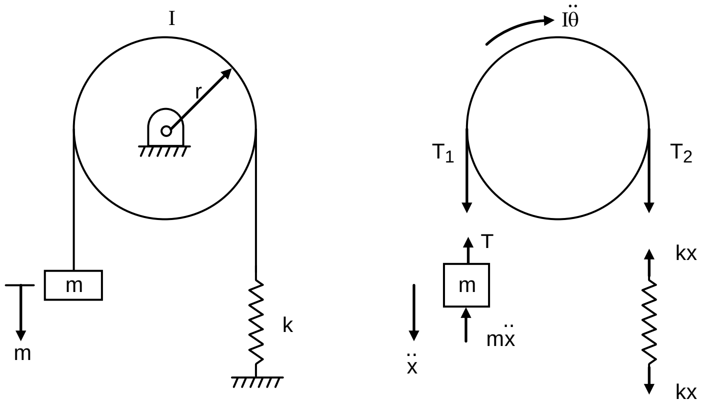
$\therefore$ $$E = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} \frac{I}{r^2} \dot{x}^2 + \frac{1}{2} k x^2$$

$$\frac{dE}{dt} = \frac{1}{2} m \, 2\dot{x} \, \ddot{x} + \frac{1}{2} \frac{I}{r^2} 2\dot{x} \, \ddot{x} + \frac{1}{2} k \, 2x \, \dot{x} = 0$$

or $$\left(m + \frac{1}{r^2}\right) \ddot{x} + k x = 0$$

or $$\ddot{x} + \frac{k}{\left(m + \frac{I}{r^2}\right)} x = 0$$

$\therefore$ $$\omega_n = \sqrt{\frac{k}{\left(m + \frac{1}{r^2}\right)}}$$




**By D'Alembert's Principle**

$$(T_1 - T_2) \, r = I \, \ddot{\theta} \quad \text{and} \quad r \, \ddot{\theta} = \ddot{x}$$

$$T_1 = -m\ddot{x} \quad \text{and} \quad T_2 = k x$$

$$-m\ddot{x} - k x = \frac{I}{r^2} \ddot{x}$$

or $$m\ddot{x} + \frac{I}{r^2} \ddot{x} + k x = 0$$

or $$\left(m + \frac{}{r^2}\right) \ddot{x} + k\,x = 0$$

or $$\ddot{x} + \frac{k}{m + \frac{I}{r^2}}\, x = 0$$

$\therefore$ $$\omega_n = \sqrt{\frac{k}{m + \frac{I}{r^2}}}$$

Example

Determine the effect of mass of the spring on the natural frequency of spring mass system.

**Solution**

Let $m_s$ be the mass in kg per unit length.

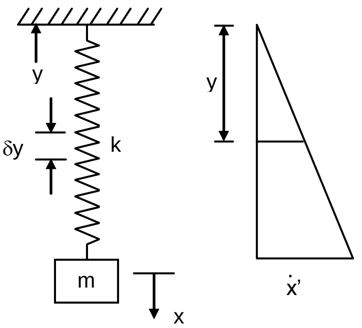
Figure 7.12 shows a spring mass system. Let the velocity distribution be linear therefore, the total energy '*E*' is given by

$$E = \frac{1}{2}\, m\,\dot{x}^2 + \frac{1}{2}\int_0^l (m_s\,\delta_y)\,\dot{y}^2 + \frac{1}{2}\,k\,x^2$$

$$= \frac{1}{2}\, m\,\dot{x}^2 + \frac{1}{2}\int_0^l m_s \left(\frac{\dot{x}\,y}{l}\right)^2 dy + \frac{1}{2}\,k\,x^2$$

$$= \frac{1}{2}\, m\,\dot{x}^2 + \frac{1}{2}\, m_s\,\dot{x}^2 \int_0^l \frac{y^2}{l^2}\, dy + \frac{1}{2}\,k\,x^2$$

$$= \frac{1}{2}\left(m + \frac{m_s\,l}{3}\right)\dot{x}^2 + \frac{1}{2}\,k\,x^2$$




$$\frac{dE}{dt} = \frac{1}{2}\left(m + \frac{m_s\,l}{3}\right)\ddot{x} \times 2\dot{x} + \frac{1}{2}\,k\;2x\;\dot{x} = 0$$

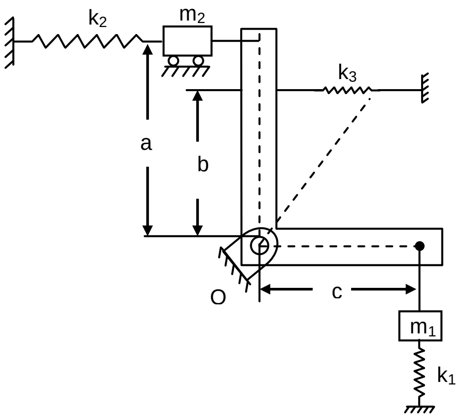
or $$\left(m + \frac{m_s\,l}{3}\right)\ddot{x} + k\,x = 0$$

or $$\ddot{x} + \frac{k}{\left(m + \frac{m_s\,l}{3}\right)}\, x = 0$$

$$\omega_n = \sqrt{\frac{k}{\left(m + \frac{m_s\,l}{3}\right)}}$$

Example

Figure 7.13 shows an indicator mechanisms. The bell crank arm is pivoted at *O* and has mass moment of inertia *I*. Find natural frequency of the system.




**Solution**

Let θ be the angular displacement of bell crank arm.

$$KE = \frac{1}{2} I \, \dot{\theta}^2 + \frac{1}{2} m_2 \, (a \, \dot{\theta})^2 + \frac{1}{2} m_1 \, (c \, \dot{\theta})^2$$

$$PE = \frac{1}{2} k_1 \, (c \, \theta)^2 + \frac{1}{2} k_2 \, (a \, \theta)^2 + \frac{1}{2} k_3 \, (b \, \theta)^2$$

Total energy (*E*) = *KE* + *PE*

and $\qquad \dfrac{dE}{dt} = 0$

$\therefore \qquad (I + m_2 \, a^2 + m_1 \, c^2) \, \ddot{\theta} + (k_1 \, c^2 + k_2 \, a^2 + k_3 \, b^2) \, \theta = 0$

or $\qquad \ddot{\theta} + \left( \dfrac{k_1 \, c^2 + k_2 \, a^2 + k_3 \, b^2}{I + m_2 \, a^2 + m_1 \, c^2} \right) \theta = 0$

$\therefore \qquad \omega_n = \sqrt{\dfrac{k_1 \, c^2 + k_2 \, a^2 + k_3 \, b^2}{I + m_2 \, a^2 + m_1 \, c^2}}$ rad/sec.

Example

A damped system has following elements :

Mass = 4 kg; $\qquad$ *k* = 1 kN/m; $\qquad$ *C* = 40 N-sec/m

Determine :

(a) damping factor,

(b) natural frequency of damped oscillation,

(c) logarithmic decrement, and

(d) number of cycles after which the original amplitude is reduced to 20%.

**Solution**

Given data :

*m* = 4 kg; $\qquad$ *k* = 1 kN/m; $\qquad$ *C* = 40 N-sec/m

$$C_c = 2 \sqrt{k \, m} = 2 \sqrt{1000 \times 4} = 126.49 \text{ Ns/m}$$

(a) Damping factor

$$\varsigma = \frac{40}{126.49} = 0.316$$

(b) $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1000}{4}} = 15.8 \text{ r/s}$

$$\therefore \quad \omega_d = \omega_n \sqrt{1 - \zeta^2} = 15.8\sqrt{1 - (0.316)^2} = 14.99 \text{ r/s}$$

$$\therefore \quad f_d = \frac{\omega_d}{2\pi} = 2.386 \text{ cps or Hz}$$

(c) Logarithmic decrement ($\delta$) $= \frac{2\pi \zeta}{\sqrt{1 - \zeta^2}}$

$$= \frac{2\pi \times 0.316}{\sqrt{1 - 0.316^2}} = 2.0928$$

(d) $\delta = \ln \frac{x_1}{x_2} = \frac{1}{n} \ln \frac{x_1}{x_n}$

or $$2.0928 = \frac{1}{n} \ln 5$$

or $$n = \frac{\ln 5}{2.0928} = 0.769$$

## 4.7 SUMMARY

A system which has mass and elasticity can start vibrating if it is disturbed. The natural frequencies of a system depend on the degrees of freedom of a system. For a multi-degree of freedom system, there will be several natural frequencies. For a two-degree of freedom system, there will be two natural frequencies.

The vibration can be linear, transverse or rotational depending on the type of the system. The methods of analysis constitutes applications of Newton's law, D'Alembert's principle, energy method and Rayleigh's method. All the methods can in general be used to analyse the system but it can be easily analysed by using a particular method. Therefore, selection of a particular method is always desirable for a given system. The energy method and Rayleigh's method can be used for a conservative system where there is no energy loss but a practical system cannot be conservative in ideal sense. The cause of vibration, their harmful effects and remedies have also been mentioned for practical utility to control vibrations.

## 4.8 KEY WORDS

**Periodic Motion** : It is the motion which repeats after a regular interval of time.

**Frequency** : It is the number of cycles completed in a unit time.

**Time Period** : It is the time taken to complete one cycle.

**Amplitude** : It is maximum displacement of a vibrating system from the position of mean equilibrium position.

**Free Vibration** : It is the vibration of the system which takes place without any external force after the disturbance.

| | | |
|---|---|---|
| Natural Frequency | : | It is the frequency of vibration of a system which is undamped and without external excitation when it is disturbed. |
| **Forced Vibration** | : | It is the vibration of a system which is due to external excitation. |
| **Resonance** | : | When forcing frequency is equal to the natural frequency, resonance takes place. |
| **Degree of Freedom** | : | It is equal to the number of independent coordinates which are required to define the motion of the system. |
| **Mode of Vibration** | : | It is the way, the system vibrates in the free vibrations. |
| **Conservative System** | : | It is the system for which total energy remains constant. |
| **Damper** | : | It is the element which is responsible for decay in energy. |