

7.1 Introduction:

Stability of a large interconnected power system is its ability to return to normal or stable operation after having been subjected to some form of disturbance. Conversely, instability denotes a condition of loss of synchronization in the system. This will result in wild fluctuation of currents and voltages within the power system network which is obviously undesirable. Hence, stability considerations form an important aspect in the study of power systems.

7.2 Some definitions:

Stability:

Stability, when used with reference to a power system, is that attribute of the system, or part of the system, which enables it to develop restoring forces between the elements thereof equal to or greater than the disturbing forces so as to restore a state of equilibrium between the elements.

Steady state stability:

This is the stability of the system under consideration subjected to a gradual or relatively slow change in load.

Transient state stability:

This refers to the stability of the system subjected to a sudden large disturbance. The large disturbance may be brought about by a sudden large change in load, faults in system or loss of generation in the system.

Dynamic stability:

This denotes the artificial stability given to a system by the action of automatic control device like fast acting voltage regulators and governors.

Steady state stability limit (SSSL):

This refers to the maximum flow of power possible through a particular point in the system without loss of stability when the power is increased gradually.

Transient state stability stability (TSL):

This refers to the maximum flow of power possible through a particular point in the system without the loss of stability when a sudden disturbance occurs.

Infinite bus:



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A system having a constant voltage and a constant frequency regardless of the load on it is called an infinite bus-bar system or an infinite bus. Physically, it is impossible to have an infinite bus-bar system. This is just considered for the purpose of analysis.

7.3 Steady state stability:

The study of steady state stability is basically concerned with the determination of the maximum power flow possible through the power system, without loss of synchronism (stability). The formation of power angle equation plays a vital role in the study of steady state stability.

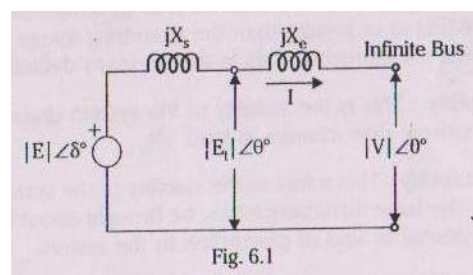
7.3.1 Power angle equation of synchronous machine:

The synchronous machine is one of the most important element of a power system. A synchronous generator converts mechanical power into electrical form and feeds it into the power system network. On the other hand, a synchronous motor draws electrical power from the network and converts it into mechanical form. There are basically two types of synchronous machines, the round rotor or Non-salient type and the salient pole type.

a) Power-angle equation of a non salient pole synchronous machine:

In the case of a non salient pole synchronous machine, the rotor consists of a cylindrical structure having a number of slots at intervals along the outer periphery of the cylinder for accommodating the field coils. Here, the air gap is uniform all along the rotor periphery and hence the flux linkage is also uniform. Therefore, the machine offers the same reactance for the flow of armature current at all places. This reactance is called as the synchronous reactance (X_s) of the machine.

The single phase equivalent reactance diagram of a non salient pole synchronous generator connected through a transmission line to an infinite bus is shown in fig 6.1.



Let,

$E \angle \delta$ = Generated voltage in the machine

δ = Load angle or Torque angle or power angle

X_s = Synchronous reactance of the machine

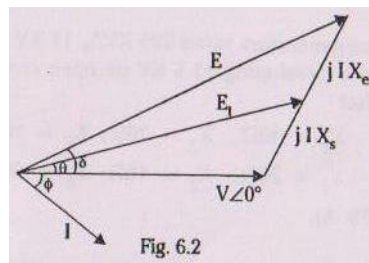
$E_t \angle \theta$ = Voltage at the terminals of the machine

X_e = reactance of the transmission line

$V \angle 0^\circ$ = Voltage at infinite bus (taken as reference)

I = load current.

Let us suppose that the machine is operating for a large power factor load, that is, the load current lags the infinite bus voltage (reference) by an angle ϕ . The corresponding phasor diagram of the system is shown in fig. 6.2.



Referring to the phasor diagram, we can relate the phasors as,

$$E = V + j \cdot I(X_s + X_e)$$

or

$$I = (E - V) / j(X_s + X_e) \dots\dots\dots 6.1$$

The net power delivered by the machine is given as

$$P = \text{Re}[V \cdot I^*]$$

Substituting Eq. 6.1 in the above equation, we get

$$\begin{aligned} P &= \text{Re} [V \cdot \{(E - V) / j(X_s + X_e)\}^*] \\ &= \text{Re} [|V| \angle 0^\circ \cdot \{ (|E| \angle \delta - |V| \angle 0^\circ) / (X_s + X_e) \angle 90^\circ \}^*] \\ &= \text{Re} [|V| \angle 0^\circ \cdot \{ (|E| \angle -\delta - |V|) / (X_s + X_e) \angle -90^\circ \}] \\ &= [\{ (|V| \cdot |E|) / (X_s + X_e) \} \cdot \cos(90^\circ - \delta)] - [\{ |V|^2 / (X_s + X_e) \} \cdot \cos 90^\circ] \\ &= [(|V| \cdot |E|) / (X_s + X_e)] \cdot \sin \delta \end{aligned}$$

Thus,

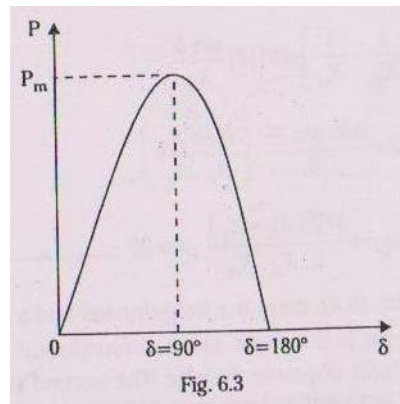
$$P = [(|V| \cdot |E|) / (X_s + X_e)] \cdot \sin \delta \dots\dots\dots 6.2$$

Eq. 6.2 shows that, the power transferred depends upon the generated voltage E, bus voltage V, system reactance and the torque angle δ . This equation is called as the power angle equation of a non salient pole synchronous machine. A graphical plot showing the variation of electrical power P against the load angle δ



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for fixed values of E,V and reactance is called as the power angle curve. This is shown in fig 6.3



The maximum power transfer occurs at $\delta=90^\circ$. The corresponding power is $P_m = (|V| \cdot |E|) / (X_s + X_e)$ 6.3

For values of $\delta > 90^\circ$, the power output of the machine reduces successively and finally the machine may stall. Hence, P_m at which maximum power transfer occurs is called as the steady state stability limit (SSSL) of the machine. The machine operation is stable in the region $0^\circ < \delta < 90^\circ$ i.e the slope of the curve $(dP/d\delta) > 0$. This term $(dP/d\delta)$ is called as synchronizing power coefficient or machine stiffness. The condition $(dP/d\delta) > 0$ is called as the stability criterion. The SSSL is reached when $(dP/d\delta) = 0$ and if $(dP/d\delta) < 0$ the system is unstable.

Note:

1) If the synchronous machine is concerned directly to an infinite bus, i.e $X_e = 0$, then eq. 6.2 becomes,

$$P = [(|V| \cdot |E|) / X_s] \cdot \sin \delta \quad \dots\dots\dots 6.4$$

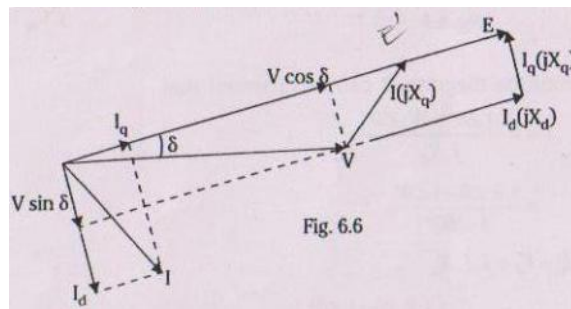
2) P_m , the maximum power is also called as pull out power of the machine.

b) Power angle equation of a salient pole synchronous machine.

A salient pole machine has a number of projecting (salient) poles. Hence, the air gap is non uniform along the rotor periphery. It is least along the axis of the main poles (called the direct axis) and maximum along the axis of the inter polar region (called the quadrature axis). Hence flux linkages is non uniform. Correspondingly, the machine offers a direct axis reactance X_d and quadrature axis reactance X_q for the flow of armature current. A circuit model of the machine cannot be easily drawn. However, the phasor diagram of the machine neglecting its armature resistance is shown in fig. 6.6.



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Here,

$E \angle \delta$ = Generated voltage in the machine

$V \angle 0^\circ$ = Voltage at infinite bus (taken as reference)

δ = Load angle or Torque angle or power angle

X_d = Synchronous reactance of the machine

X_q = quadrature axis synchronous reactance

X_e = reactance of the transmission line

I = current delivered at a lagging power factor of $\cos \phi$.

Referring to the diagram, we can write the expression for power developed as,

$$P = |V| \cos \delta \cdot |I_q| + |V| \sin \delta \cdot |I_d| \dots\dots\dots 6.5$$

also, from fig 6.6 we observe that,

$$|I_q \cdot X_q| = |V \cdot \sin \delta|$$

or,

$$|I_q| = V \cdot \sin \delta / X_q \dots\dots\dots 6.6$$

again,

$$|I_d \cdot X_d| = |E - V \cos \delta|$$

or,

$$|I_d| = |E - V \cos \delta| / X_d \dots\dots\dots 6.7$$

Using equations 6.6 and 6.7 in eq. 6.5 we get,

$$\begin{aligned} P &= [|V| \cdot \cos \delta \cdot \{(|V| \cdot \sin \delta) / X_q\}] + [(|V| \cdot \sin \delta) \cdot (|E| - |V| \cos \delta) / X_d] \\ &= [|V|^2 \cdot (\sin 2\delta / 2 \cdot X_q)] + [(|V| \cdot |E| \cdot \sin \delta) / X_d] - [(|V|^2 \cdot \sin 2\delta) / 2 \cdot X_d] \\ &= |V|^2 \cdot (\sin 2\delta / 2) \cdot ((1/X_q) - (1-X_d)) + |V| \cdot |E| \cdot (\sin \delta / X_d) \\ &= (|V| \cdot |E| / X_d) \cdot \sin \delta + |V|^2 \cdot (\sin 2\delta / 2) \cdot ((X_d - X_q) / (X_d \cdot X_q)) \end{aligned}$$

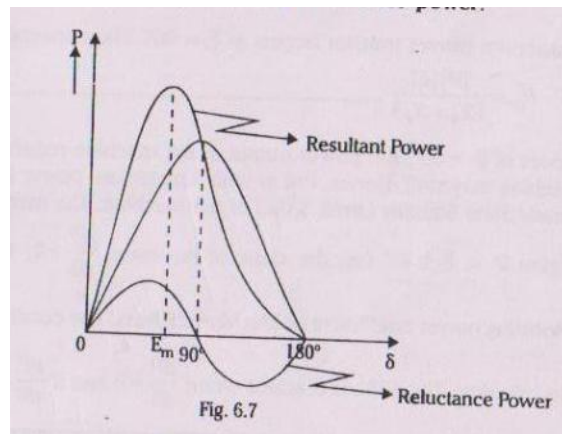
Thus,

$$P = [(|V| \cdot |E|) / X_d] \cdot \sin \delta + |V|^2 \cdot ((X_d - X_q) / 2 \cdot X_d \cdot X_q) \cdot \sin 2\delta \dots\dots\dots 6.8$$

As evident from eq.6.8, there is a fundamental and a second harmonic component of power. The first term is the same as far a round rotor machine with $X_s = X_d$. This constitutes the major part of power transfer. The second term is quite small (10-20%) compared to the first term and is known as reluctance power.



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The power angle curve of the machine is shown in fig 6.7. it is noticed that the maximum power output (SSSL) occurs at $\delta < 90^\circ$ (about 70°). This value of δ at which the power flow is maximum can be computed by equating the synchronizing power coefficient i.e $dP/d\delta$ to zero.

7.3.5 Methods of improving SSSL:

For a two machine system, we have

$$\text{SSSL} = |E_g| \cdot |E_m| / |X|$$

As indicated by the equation, the SSSL can be increased by

- i) increasing either of the voltages $|E_g|$ or $|E_m|$. This can be achieved by increasing the excitation to the generator or motor or both.
- ii) Reducing the reactance between the transmission and receiving points. If the transmission lines are of sufficiently high reactance, the stability limit can be raised by using two parallel lines which incidentally also increases the reliability of the system. Series capacitors are sometimes employed in lines to get better voltage regulation and to raise the SSSL by decreasing the line reactance. The use of bundled conductors is another method of reducing the line reactance and hence improving the SSSL.

7.4 Transient stability:

The transient stability refers to the maximum power flow possible through a point without losing stability with sudden and large changes in the power system. Following a sudden disturbance on a power system, the rotor speeds and rotor angular differences undergo fast changes whose magnitudes are dependent on the severity of disturbance. For a large disturbance, changes in angular differences δ may be so large that the machines may fall out of step. Thus, the transient



stability of a system predominantly depends upon the dynamics of the synchronous machine.

7.4.1 Dynamics of a synchronous machine:

The kinetic energy of a rotor is,

$$K.E = (1/2).I.\omega^2$$

Where,

I=moment of inertia in kg.m²

ω =angular speed in rad/sec

The angular momentum is,

$$M=I.\omega$$

therefore,

$$K.E=(1/2).M.\omega \text{ joules}$$

Rather than having the moment of inertia of a synchronous machine for dynamic studies, it is more convenient to use per unitized quantity called inertia constant H. This is defined as the ration stored energy of a machine at synchronous speed to the rated apparent power of the machine, i.e.

$$H=(\text{stored energy in megajoules})/ (\text{machine rating in mega volt-amperes}) \dots 6.19$$

Let G=rating of the machine in MVA, then

GH=stored energy in mega joules

Hence, GH=K.E

$$=(1/2).I.\omega^2$$

$$=(1/2)M.\omega \text{ joules (MJ)}$$

Where,

$$\omega = 2\pi f \text{ elec-rad/sec}$$

$$= 360.f \text{ elec.deg/sec}$$

or

$$GH=(1/2).M.(360.f)$$

therefore,

$$M=GH/180.f \text{ MJ-sec./elect.deg} \dots\dots\dots 6.20$$

M is also called as the inertial constant eq. 6.20 relates the two inertia constants of the machine. For stability studies it is necessary to determine M which depends upon the size and speed of the machine, but instead H has a characteristic value of range of values for each class of machines. Typical values of H are indicated below:

cylindrical rotor alternator: 4-10

salient pole alternators: 2-3

Salient pole synchronous motors: .5-2



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Modern power systems have many interconnected generators stations each with several generators and many loads. The machines located at any one point in a system usually act in unison. It is, therefore, common practice in stability studies to consider all machines at one point as a single equivalent machine, having a rating equal to the sum of the ratings of several machines considered to act together. The inertia constant M of the equivalent machines is the sum of the inertia constants of the individual machines, i.e.

$$M_{eq} = M_1 + M_2 + \dots + M_n$$

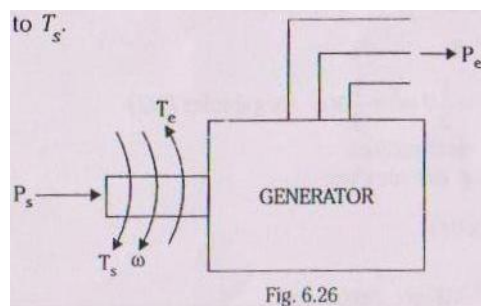
$$\text{or } H_{eq} \cdot G_{base} = H_1 G_1 + H_2 G_2 + \dots + H_n G_n$$

$$\text{or } H_{eq} = H_1 G_1 / G_{base} + H_2 G_2 / G_{base} + \dots + H_n G_n / G_{base} \quad \dots \dots \dots 6.21$$

7.4.2 Swing equation:

The load angle or the torque angle δ depends upon the loading of the machine. Larger the loading, larger is the value of the torque angle. If some load is added or removed from the shaft of the synchronous machine, the rotor will decelerate or accelerate respectively with respect to the synchronously rotating stator field and a relative motion begins. It is said that the rotor is swinging with respect to the stator field. The equation describing the relative motion of the rotor (load angle δ) with respect to the stator field as a function of time is called as swing equation.

Consider the generator shown in fig 6.26. It receives mechanical power P_s at torque T_s and rotor speed ω via shaft from the prime mover. It delivers electrical power P_e to the power system network via the bus bars. The generator develops electromechanical torque T_e in opposition to T_s .



Assuming that winding and friction losses to be negligible, the accelerating torque on the rotor is given by'

$$T_a = T_s - T_e \quad \dots \dots \dots 6.22$$

Multiplying by ω on both sides, we get

$$\omega \cdot T_a = \omega \cdot T_s - \omega \cdot T_e$$



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but

$\omega \cdot T_a = P_a = \text{accelerating power}$

$\omega \cdot T_s = P_s = \text{mechanical power input}$

$\omega \cdot T_e = P_e = \text{electrical power output assuming that power loss is negligible.}$

Therefore, we get

$$P_a = P_s - P_e \dots\dots\dots 6.23$$

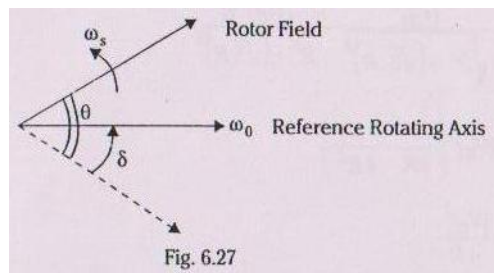
Under steady state conditions, $P_a = P_e$, so that $P_a = 0$.

When P_s, P_e balance is disturbed, the machine undergoes dynamics governed by

$$P_a = T_a \cdot \omega = I \cdot a \cdot \omega = M \cdot (d^2\theta/dt^2) \dots\dots\dots 6.24$$

where $d^2\theta/dt^2$ is the angular acceleration of the rotor.

Since the angular position of the motor is continually varying with time, it is more convenient to measure the angular position and velocity with respect to a synchronously rotating axis fig 6.27.



From the fig 6.25, it can be inferred that

$$\delta = \theta - \omega_0 t \dots\dots\dots 6.25$$

where, $\omega_0 = \text{angular velocity of the reference rotating axis.}$

$\delta = \text{rotor angular displacement with respect with respect to the stator field.}$

Taking time derivatives of eq. 6.25

$$(d\delta/dt) = (d\theta/dt) - \omega_0$$

$$\text{and } (d^2\delta/dt^2) = d^2\theta/dt^2 \dots\dots\dots 6.26$$

combining equation 6.23 and 6.24 and 6.26, we get

$$M \cdot (d^2\theta/dt^2) = P_a = P_s - P_e \dots\dots\dots 6.27$$

This equation is called as the swing equation of the synchronous machine. When the machine is connected to the infinite bus bars, then $P_e = [(|E| \cdot |V|) / X] \cdot \sin\delta = P_m \cdot \sin\delta$.

$$\text{Or } M \cdot (d^2\delta/dt^2) = P_s - P_m \cdot \sin\delta \dots\dots\dots 6.28$$

7.4.3 Swing curve:

The solution of swing equation gives the relation between rotor angle δ as a function of time t . The plot of δ versus t is called as swing curve. The exact



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solution of the swing equation is however a very tedious task. Normally, step by step method or any other numerical solution techniques like Euler's method, Runge-Kutta's method are used for solving the swing equation. The swing curve is used to determine the stability of the system. In case δ increases indefinitely, it indicates instability. Whereas if it reaches a maximum and starts decreasing, it shows that the system will not lose stability since the oscillations will be damped out with time. A sample swing curve is shown in fig 6.28.

For the stability of the system, $d\delta/dt=0$ 6.29

The system will be unstable if $d\delta/dt>0$ for a sufficiently long time (normally more than 1sec)

7.4.4 Equal area criterion (EAC):

This provides a qualitative assessment of transfer stability of a synchronous system. It helps in deciding whether a system is stable or not under transient conditions, without solving the swing equation.

Consider the swing equation of a single machine connected to an infinite bus,
 $M.(d^2\theta/dt^2)=P_a$

Multiplying both sides of the equation by $(2/M).(d\delta/dt)$, we get

$$2.(d\delta/dt).(d^2\delta/dt^2)=(2/M).P_a.(d\delta/dt) \quad \text{because } d(x^2)/dt=2.x. dx/dt$$

or

$$(d(d\delta/dt)^2/dt)=(2/M).P_a.(d\delta/dt)$$

Integrating with respect to t we obtain,

$$(d\delta/dt)^2=(2/M) \int_{\delta_0}^{\delta} P_a.(d\delta/dt). dt$$

$$=(2/M) \int_{\delta_0}^{\delta} P_a.d\delta$$

$$\text{or } (d\delta/dt)= \sqrt{\frac{2}{M} \int_{\delta_0}^{\delta} P_a.d\delta}$$

for the system to be stable, $(d\delta/dt)=0$

$$\text{i.e } \sqrt{\frac{2}{M} \int_{\delta_0}^{\delta} P_a.d\delta} =0$$

$$\text{or } \int_{\delta_0}^{\delta} P_a.d\delta =0 \quad \dots\dots\dots 6.30$$

The physical meaning of integration is the estimation of the area under the curve. The above integral indicates zero area. Thus, the system is stable if the area

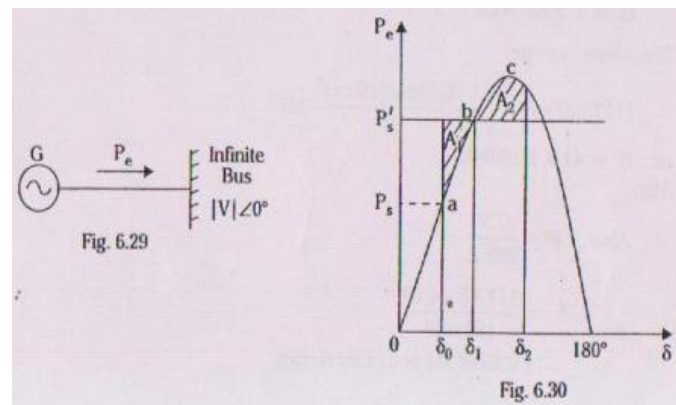


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under P_a - δ curve reduces to zero for some value of δ . This is possible only when P_a has both positive (accelerating) and negative (decelerating) powers. For a stable system, the positive area under P_a - δ curve must be equal to the negative area and hence, this is called Equal Area Criterion for stability.

7.4.5 Applications of Equal area criterion:

Fig 6.29 shows the one line diagram of a synchronous generator connected to an infinite bus.



Let us consider the case of sudden change (increase) in the mechanical input. Fig 6.30 shows the plot of P_e - δ , the power angle curve with the system operating at point a corresponding to input P_s . Let the mechanical input be suddenly increased to P_s' as shown. The accelerating power $P_a(=P_s'-P_e)$ causes the rotor to accelerate. Hence the rotor angle δ increases, the electrical power transfer increases, reducing P_a , till a point b at which $P_a=0$. The rotor angle δ , however, continues to increase because of the inertia of the rotor and P_a becomes negative

causing the rotor to decelerate. At some point c where $A_1 = \text{area } A_2$ or $\int_{\delta_0}^{\delta} P_a \cdot d\delta = 0$, the rotor velocity $d\delta/dt$ becomes zero (this corresponds to the synchronous speed) and then starts to become negative owing to continuing negative P_a . The rotor angle thus reaches the maximum value δ_2 and then starts to decrease.

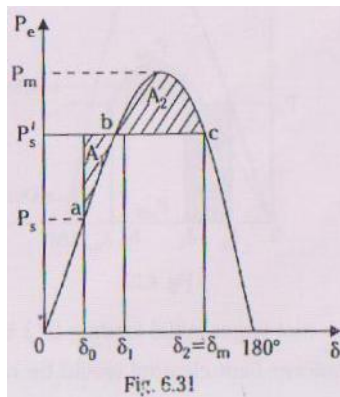
From fig 6.30 areas A_1 and A_2 are given by,

$$A_1 = \int_{\delta_0}^{\delta_1} (P_s' - P_e) \cdot d\delta$$

$$A_2 = \int_{\delta_1}^{\delta_2} (P_e - P_s') \cdot d\delta$$



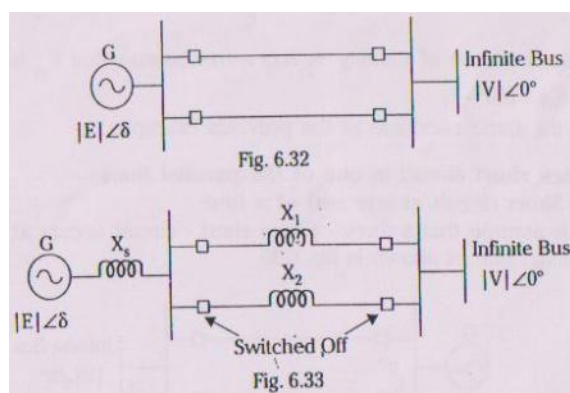
For the system to be stable, it should be possible to find angle δ_2 such that $A_1=A_2$ as P_s' is increased, a limiting condition is finally reached when area A_1 equals the entire area A_2 above the line P_s' as shown in the fig 6.31. Under this condition δ_2 acquires the maximum value δ_m such that,
 $\delta_2 = \delta_m = 180^\circ - \delta_1$ 6.31



Now, the system is said to be critically stable. Any further increase in P_s' means the area available for A_2 is less than A_1 , so that the system becomes unstable. It has thus been shown by the use of EAC that there is an upper limit to sudden increase in mechanical input ($P_s' - P_s$) for the system in question to remain stable. The power P_s' is transient stability Limit (TSL) of the system. As clearly visible from the fig 6.31, TSL (P_s') is less than SSSL (P_m) of the system.

b) Sudden loss of one of the parallel lines:

Let us consider a single machine connected to an infinite bus through two parallel lines as in fig 6.32. Circuit model of the system is given in fig 6.33.



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The transient stability of the system when one of the lines is suddenly switched OFF from the system, while operating under steady load conditions is now being considered.

Case i) Before switching OFF, the power angle equation is,

$$P_{e1} = [(|E| \cdot |V|) / \{X_s + (X_1 || X_2)\}] \cdot \sin \delta = P_{m1} \cdot \sin \delta \quad \dots\dots\dots \text{curve 1}$$

case ii) On switching OFF line -2, the power angle equation is

$$P_{e2} = [(|E| \cdot |V|) / (X_s + X_1)] \cdot \sin \delta = P_{m2} \cdot \sin \delta \quad \dots\dots\dots \text{curve 2}$$

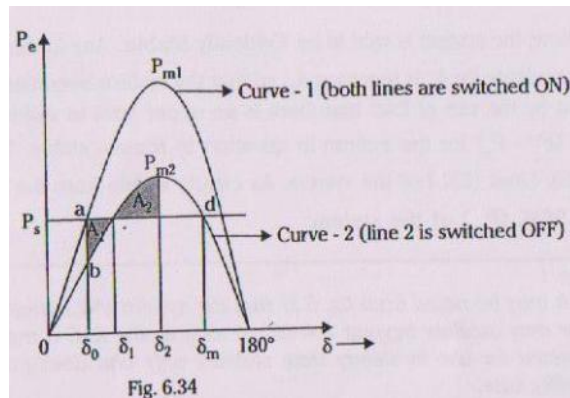


Fig 6.34 shows the two curves wherein $P_{m2} < P_{m1}$ as $(X_s + X_1) > (X_s + (X_1 || X_2))$. As soon as line-2 was switched OFF, the original operating point a on curve-1 is shifted to a point b on curve-2. Accelerating energy corresponding to area A_1 is put into rotor followed by decelerating energy. If an area A_2 equal to A_1 is found above the P_s line, the system will be stable, and finally operates at C corresponding to a new rotor angle $\delta_1 > \delta_0$.

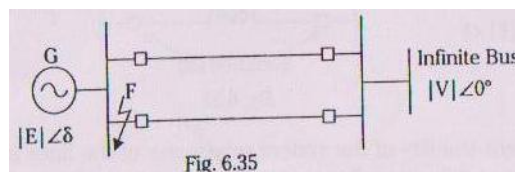
For the limiting case of stability, δ_2 has a maximum value δ_m is given by $\delta_2 = \delta_m = 180^\circ - \delta_1$

Which is the same condition as the previous example.

c) Sudden short circuit in one of the parallel lines:

case i) short circuit at one end of a line:

Let us assume that a three phase short circuit occurs at the end of line-2 of a double circuit line as shown in fig 6.35.



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The equivalent reactance diagrams before, during and after fault clearance are shown in fig 6.36a, 6.36b and 6.36c respectively.

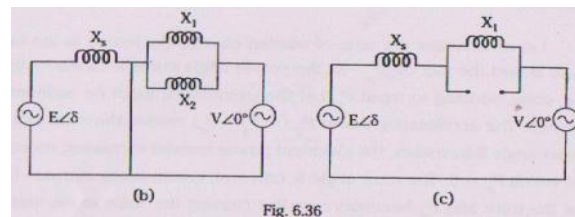
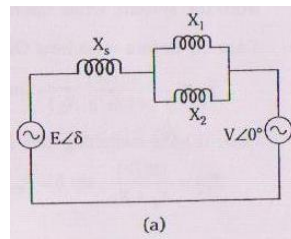


Fig. 6.36

Before the occurrence of a fault, the power angle curve is given by,

$$P_{e1} = [(|E| \cdot |V|) / \{X_s + (X_1 || X_2)\}] \cdot \sin \delta = P_{m1} \cdot \sin \delta$$

Upon occurrence of a three phase fault at the end of line-2, there is no power flow as seen from the fig i.e $P_{e2} = 0$.

The circuit breakers at the two ends of the faulted line open at the t_1 (corresponding to angle δ_1), called the clearing time, dis-connecting the faulted line. The power flow is now restored via the healthy line-1. With power angle curve is given as,

$$P_{e3} = [(|E| \cdot |V|) / (X_s + X_1)] \cdot \sin \delta = P_{m3} \cdot \sin \delta$$

obviously, $P_{m3} < P_{m1}$. The rotor now starts to decelerate as shown in fig 6.37. The system, will be stable if a decelerating area A_2 can be found equal to the accelerating area A_1 before δ reaches the maximum allowable value δ_m .

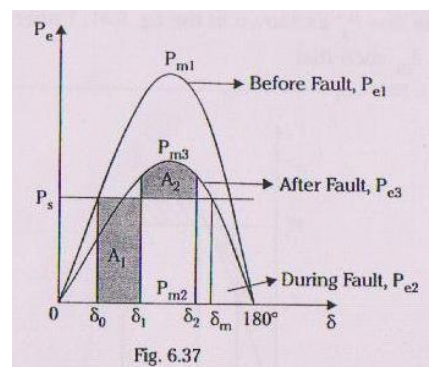


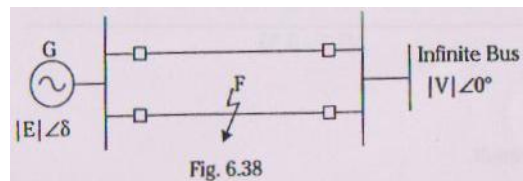
Fig. 6.37



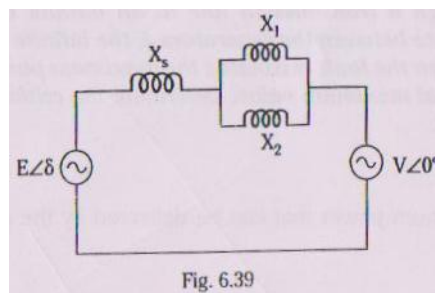
It easily follows that larger initial loading (P_s) increases A_1 for a given clearing angle δ_1 and therefore, quicker fault clearing would be needed to maintain stable operation.

Case ii) Short circuit away from line ends:

When a three phase fault occurs away from line ends (say in the middle of a line). There is some impedance between the paralleling buses and the fault. Therefore, some power is transmitted while the fault is still on the system. The one line diagram of the system is shown in fig 6.38.



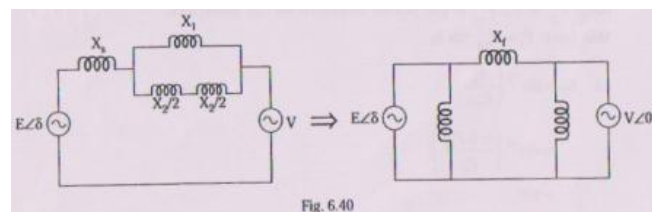
The equivalent circuit before occurrence of fault is shown in fig 6.39.



The power angle curve is given by,

$$P_{e1} = [(|E| \cdot |V|) / \{X_s + (X_1 || X_2)\}] \cdot \sin \delta = P_{m1} \cdot \sin \delta$$

circuit model of the system during fault is shown in fig 6.40.

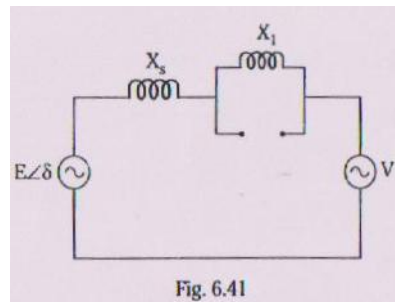


Here, X_f = transfer reactance of the system. The power angle curve during fault is therefore given by

$$P_{e2} = [(|E| \cdot |V|) / X_f] \cdot \sin \delta = P_{m2} \cdot \sin \delta$$



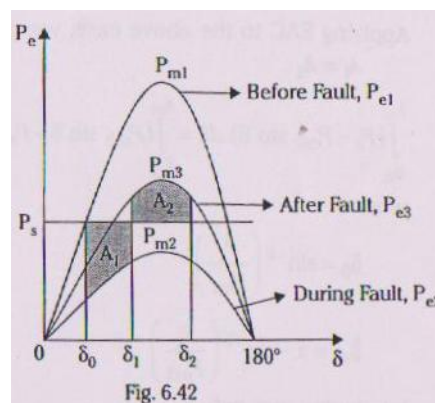
After the clearing of the fault by opening of the circuit breakers, the equivalent circuit is as shown in fig 6.41.



The Power angle curve is given by,

$$P_{e3} = [(|E| \cdot |V|) / (X_s + X_1)] \cdot \sin\delta = P_{m3} \cdot \sin\delta$$

The power angle curves corresponds to P_{e1} , P_{e2} and P_{e3} are shown in fig 6.42.



The system is stable only if it is possible to find an area A_2 equal A_1 .

7.4.6 Critical clearing angle and critical clearing time:

In the previous case if, P_s is increased, then δ_1 increase, area A_1 increases and to find $A_2=A_1$, δ_2 is increased till it has a value δ_m , the maximum allowable limit for stability. Then the system is said to be critically stable. The angle δ_1 is then called as the critical clearing angle (δ_{cc}). The time corresponding to this is called the critical clearing time (t_{cc}). The critical clearing angle can be determined from the EAC. However, the critical clearing time cannot be obtained from EAC. It is possible to estimate critical clearing time using the swing curve. This time is very much essential in designing the protective circuit breakers for the system.

The case of critical stability of a system is shown in fig 6.43.



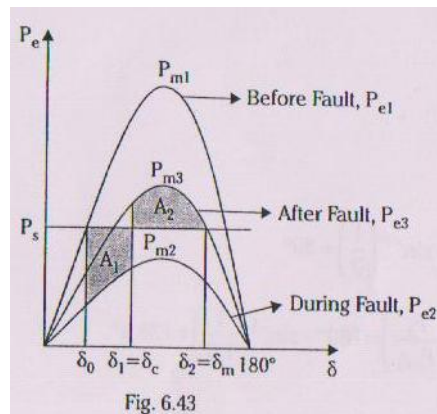


Fig. 6.43

Applying EAC to the above case, we get

$$A_1 = A_2$$

$$\int_{\delta_0}^{\delta_{cc}} (P_s - P_{m2} \sin \delta) \cdot d\delta = \int_{\delta_{cc}}^{\delta_m} (P_{m3} \sin \delta - P_s) \cdot d\delta$$

where,

$$\delta_0 = \sin^{-1}(P_s / P_{m1})$$

$$\delta_m = \pi - \sin^{-1}(P_s / P_{m3})$$

Integrating, we get

$$(P_s \cdot \delta + P_{m2} \cdot \cos \delta) \Big|_{\delta_0}^{\delta_{cc}} = (-P_{m3} \cdot \cos \delta - P_s \delta) \Big|_{\delta_{cc}}^{\delta_m}$$

or

$$P_s(\delta_{cc} - \delta_0) + P_{m2}(\cos \delta_{cc} - \cos \delta_0) + P_s(\delta_m - \delta_{cc}) + P_{m3}(\cos \delta_m - \cos \delta_{cc}) = 0$$

or

$$\cos \delta_{cc} = [P_s \cdot (\delta_m - \delta_0) - P_{m2} \cdot \cos \delta_0 + P_{m3} \cdot \cos \delta_m] / (P_{m3} - P_{m2}) \quad \dots\dots\dots 6.32$$

The angles in the above equation are in radians. If the angles are in degrees, the equation modifies as below. (π over 180°)

$$\cos \delta_{cc} = \frac{[\frac{\pi}{(180^\circ)} P_s \cdot (\delta_m - \delta_0) - P_{m2} \cdot \cos \delta_0 + P_{m3} \cdot \cos \delta_m]}{(P_{m3} - P_{m2})} \quad \dots\dots\dots 6.33$$

7.4.7 Methods of improving transient stability:

From the swing equation we have $(d^2\delta/dt^2) = (P_a/M)$ i.e the acceleration of the rotor $(d^2\delta/dt^2)$ is inversely proportional to the angular momentum of the machine when accelerating power is a constant. This means that higher the value of M, slower will be the change in the rotor angle of the machine and thus allows a



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longer time for the circuit breakers to isolate the fault before the machine passes through the critical clearing angle. However, to achieve higher value of M , a heavier rotor is required which in turn increases the cost of the machine. Therefore, this method cannot be employed in practice because of economic reasons.

The methods often employed in practice to improve system stability are:

- 1) Increase of system voltages.
- 2) Reduction of transfer reactance.
- 3) Use of high speed circuit breakers and auto-reclosing breakers.

It is observed from eq. 6.12 that $P_m = |E_g| \cdot |E_m| / X$. Thus, by increasing the system voltages or by reducing the system transfer reactance, the maximum power transfer (stability) can be increased. The system voltages can be increased by the use of high speed excitation systems (AVRs). The reactance of a transmission line can be decreased.

- i) by reducing the conductor spacing.
- ii) by increasing conductor diameter.
- iii) by the use of bundled conductors.
- iv) by increasing the number of parallel lines or
- v) by using series capacitors in the transmission lines.

The quicker a breaker operates, the faster the fault is removed from the system and better is the tendency of the system to store to normal operations.

Recent trends:

A brief account of some of the recent methods of maintaining stability is given below.

a) HVDC links:

A d.c transmission line does not have any stability problem in itself because, d.c operation is an asynchronous operation of the machines. Hence, the use of HVDC links to connect separate a.c systems improves the stability of the systems.

b) Braking Resistors:

For improving the stability of a system when a large load is suddenly lost, a resistive load called a braking resistor is connected at or near the generator terminals. This load compensates for at least some of the reduction of load on the generators and so reduces the acceleration ($d^2\delta/dt^2$) of the machines.



c)Fast valving:

In this method, the stability of a unit is improved by decreasing the mechanical input power to the turbine. When a fault occurs in the system, a control scheme detects the difference between a mechanical input and reduced electrical output of the generator, initiates the closing of a turbine valve to reduce the power input.

d)Full load rejection technique:

Sometimes, to maintain stability it becomes inevitable to take a faulty unit out of service. However, the loss of a major unit for a long time can be seriously hazardous for the remaining system. To avoid this, a full load rejection scheme could be utilized after the unit is separated from the system. To do this, the unit has to be equipped with a large bypass system. After the system has recovered from the shock caused by the fault, the unit could be synchronized and reloaded. The main disadvantage of this method is the extra cost of a large bypass system.

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8.1 Introduction:

The operation of three phase induction motor supplied with balanced voltages has been dealt in the previous semesters. In this chapter, we analyse the performance of a three phase induction motor under unbalanced conditions. The method of symmetrical components will be suitably exploited in study.

8.2 Performance of a three phase induction motor under unbalanced supply voltages:

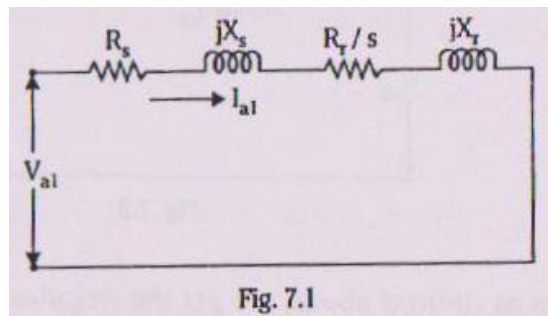
Let us assume that unbalanced voltages are applied across the stator of a 3-phase symmetrically wound induction motor. It is known that the unbalanced voltages can be resolved into positive, negative and zero sequence components of voltages. The effect of applying unbalanced voltages can be obtained by superposing the effects due to positive, negative and zero sequence components of voltages.

The positive sequence voltages constitute a balanced three phase system having the same phase sequence as that of the original unbalanced phasors. They induce positive sequence rotor currents which circulate a s times the supply frequency ($f_r = s.f$) and produce a positive torque. The negative sequence voltages also constitute a balanced three phase system, but having opposite phase sequence as that of the original unbalanced phasors. They induce negative sequence rotor currents at $(2-s)$ times the supply frequency in the reverse direction ($f_r' = (2-s)f$) and hence produce a negative torque. The zero sequence voltages do not constitute a three phase system. Hence they do not produce any net torque.

Now let us find expressions for positive and negative torques, the net torque and net power in an induction motor under unbalanced supply voltages. The analysis applies to a star connected (or equivalent star connected) induction motor. A delta connected stator winding of an induction motor, can however be transformed to an equivalent star connected stator winding for the purpose of analysis.

When positive sequence voltages are applied to a symmetrical star (or equivalent star) connected stator of a three phase induction motor, as already stated, the induced currents in the rotor circulate at slip s times the supply frequency. The approximate equivalent circuit of an induction motor on a single phase basis omitting the exciting circuit and assuming the voltage transformation ratio as unity is shown in fig 7.1





Here,

V_{a1} = positive-sequence component of phase voltage

R_s = stator resistance per phase

X_s = stator reactance per phase

R_r = rotor resistance per phase as referred to stator.

X_r = rotor reactance per phase as referred to stator.

Note:

In this case, the rotor impedance is same as the rotor impedance referred to stator as we have assumed voltage transformation ratio as unity.

The equivalent positive sequence current is given as

$$I_{a1} = V_{a1} / [(R_s + (R_r/s))^2 + (X_s + X_r)^2]^{1/2} \quad \dots\dots\dots 7.1$$

Therefore,

$$\text{Rotor copper loss/ph} = I_{a1}^2 \times R_r$$

In the case of an induction motor, we know that

$$\begin{aligned} \text{torque in synchronous} &= \text{rotor input} \\ &= \text{rotor copper loss} / s \\ &= \text{power} / (1-s) \end{aligned}$$

Using the above relations, we get the positive torque produced per phase due to positive sequence currents as

$$T_1 = I_{a1}^2 \times R_r / s \quad \text{syn. Watts} \quad \dots\dots\dots 7.2$$

and positive shaft power is,

$$\begin{aligned} P_1 &= T_1(1-s) \\ &= I_{a1}^2 \times R_r / s (1-s) \text{ watts} \quad \dots\dots\dots 7.3 \end{aligned}$$

Where, I_{a1} is defined through equation 7.1

When negative sequence voltages are applied to the stator, the negative sequence currents produced in the rotor circulate at $(2-s)$ times the supply frequency but in



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opposite direction. The approximate negative sequence equivalent circuit is shown in fig 7.2.

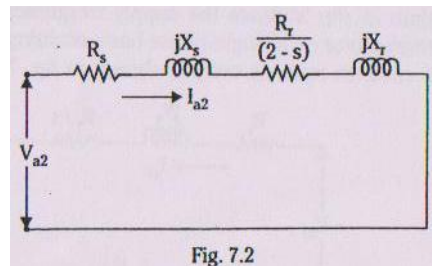


Fig. 7.2

On similar lines as derived above, we get the negative sequence current as:

$$I_{a2} = V_{a2} / [(R_s + (R_r/(2-s)))^2 + (X_s + X_r)^2]^{1/2} \dots\dots\dots 7.1$$

The negative torque produced per phase is

$$T_2 = - I_{a2}^2 \cdot (R_r/(2-s)) \text{ syn. Watts} \dots\dots\dots 7.5$$

and negative shaft power is

$$P_2 = T_2(1-s) = -I_{a2}^2 \cdot (R_r/(2-s)) \cdot (1-s) \text{ watts} \dots\dots\dots 7.6$$

The net three phase torque produced is thrice the sum of positive and negative i.e

$$T = 3(T_1 + T_2) = 3 \cdot [\{ I_{a1}^2 \cdot (R_r/s) \} - \{ I_{a2}^2 \cdot (R_r/(2-s)) \}] \text{ syn. Watts} \dots\dots\dots 7.7$$

similarly, the net three phase power produced is given by

$$P = 3(P_1 + P_2) = 3 \cdot [\{ I_{a1}^2 \cdot (R_r/s) \cdot (1-s) \} - \{ I_{a2}^2 \cdot (R_r/(2-s)) \cdot (1-s) \}] = 3 \cdot [\{ I_{a1}^2 \cdot (R_r/s) \} - \{ I_{a2}^2 \cdot (R_r/(2-s)) \}] \cdot (1-s) \dots\dots\dots 7.8$$

$$= T(1-s) \dots\dots\dots 7.9$$

Thus it can be observed that due to the negative torque produced by the negative sequence components of the unbalanced voltages, the net torque and hence the net power output is reduced. In addition to this, the negative sequence currents circulating in the rotor at (2-s) times the supply frequency, produce a large amount of coreless than their counter part of positive sequence currents which circulates at s times the supply frequency. This may some times result in overheating of rotor core. Hence, the performance of a three phase induction motor deteriorates when supplied with unbalanced voltages.

8.3 Single phasing of an induction motor:

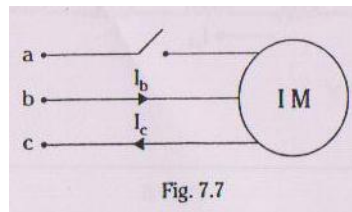
Single phasing means the opening of one wire (or leg) of a three phase circuit whereupon the remaining legs at once becomes single phase. When a three phase circuit is functioning normally, there are three distinct currents flowing in the



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circuit. As is known, one of the three phases act as a return path for the other two. Obviously, an open circuit in one wire kills two phases and there will be only one current or phase working, even though two wires are left intact. The usual cause of single phasing is due to the blowing out of a fuse in the switch gear because of over loading of the machine.

Consider a star (or equivalent star) connected stator of a three phase induction motor. Let us assume that an open circuit occurs in phase a. This is diagrammatically shown in fig 7.7.



The terminal conditions are,

$$I_a=0 \text{ (open)}$$

$$I_b=-I_c=I \text{ (say)}$$

In terms of symmetrical components, we can write

$$\begin{aligned} I_{a1} &= (1/3)(I_a + a \cdot I_b + a^2 \cdot I_c) \\ &= (1/3)(0 + a \cdot I - a^2 \cdot I) \\ &= ((a - a^2)/3) \cdot I \\ &= (j/\sqrt{3}) \cdot I \end{aligned}$$

similarly,

$$\begin{aligned} I_{a2} &= (1/3)(I_a + a^2 \cdot I_b + a \cdot I_c) \\ &= (1/3)(0 + a^2 \cdot I - a \cdot I) \\ &= ((a^2 - a)/3) \cdot I \\ &= (-j/\sqrt{3}) \cdot I \end{aligned}$$

It can be observed that,

$$I_{a1} = -I_{a2} = (j/\sqrt{3}) \cdot I \quad \dots\dots\dots 7.10$$

The only voltages that is known to us in this case is the line to line voltage between the lines b and c, that are not open.

Hence,

$$\begin{aligned} V_{bc} &= V_A \\ &= V_c - V_b \\ &= (V_{a0} + a \cdot V_{a1} + a^2 \cdot V_{a2}) - (V_{a0} + a^2 \cdot V_{a1} + a \cdot V_{a2}) \\ &= (a - a^2)(V_{a1} - V_{a2}) \end{aligned}$$

$$\text{or } V_{a1} - V_{a2} = V_A / (j/\sqrt{3})$$

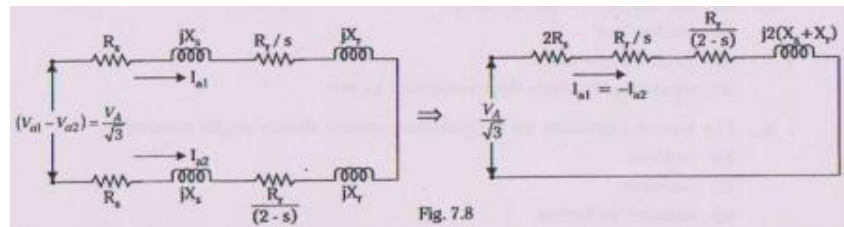


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$$V_{a1} - V_{a2} = V_A / \sqrt{3} \dots\dots\dots 7.11$$

= Magnitude of the phase voltage.

An equivalent circuit connecting the two sequence networks and satisfying equations 7.10 and 7.11 is as shown in fig 7.8.



From the circuit, it can be observed that,

$$I_{a1} = -I_{a2} = [V_A / \sqrt{3}] / [\{2R_s + (R_r/s) + (R_r/(2-s))\} + \{2(X_s + X_r)\}^2]^{1/2} \dots\dots\dots 7.12$$

The net three phase torque is given by,

$$T = 3 \cdot I_{a1}^2 [(R_r/s) - (R_r/(2-s))] = 6 \cdot I_{a1}^2 \cdot R_r \cdot [(1-s) / \{s \cdot (2-s)\}] \text{ syn. Watts} \dots\dots\dots 7.13$$

The net three phase shaft power output is (1-s) times the torque in syn. Watts.

Therefore,

$$P = 6 \cdot I_{a1}^2 \cdot R_r \cdot [(1-s)^2 / \{s \cdot (2-s)\}] \text{ watts} \dots\dots\dots 7.14$$

Equations 7.13 and 7.17 indicates that, for s=1(at standstill position), the net torque and the net power output are zero. This indicates that a stationary motor will not start with one line open. In fact, due to heavy standstill current, it is likely to burn out quickly unless immediately disconnected.

For 0 < s < 1, i.e during the running condition of the motor, the net torque and net power output is positive. This implies that a running motor will continue running with one-line open. However, if the motor is very heavily loaded, then it will stop under single phasing. Since it can neither restart not blow out the remaining fuses, the burn out of the machine is very prompt.

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