Module 3: TRANSISTOR and FET FREQUENCY RESPONSE

The frequency response of an amplifier is the plot of the magnitude of voltage gain as a function of frequency. In transistor amplifier the low frequency response is governed by coupling and bypass capacitors. The high frequency response is affected by the transistor parasitic capacitances and stray wiring capacitances. The mid frequency response is unaffected by these capacitances.

General frequency considerations:

The response of a single stage or multistage amplifier depends on the frequency of the applied signal. The coupling and bypass capacitors affect the low frequency response since the reactance of these capacitors decreases with increase in frequency. The internal capacitances of the active devices and the stray wiring capacitances will limit the high frequency response of the system. An increase in the number of stages of a cascaded system will also limit the low and high frequency responses.

Frequency response of RC Coupled amplifier:

Fig. (1) shows the frequency response of RC coupled amplifier. The horizontal scale is a logarithmic scale to permit a plot extending from low to high frequency regions. The frequency range is divided into 3 regions.

(i) Low frequency region.
(ii) Mid frequency region.
(iii) High frequency region.

The drop in the gain at low frequencies is due to the coupling capacitors \( (C_C, C_S) \) and bypass capacitors \( (C_E) \). At high frequencies the drop in gain is due to the internal device capacitances and the stray wiring capacitances. In the mid frequency range the gain is almost independent of the frequency. This is due to the fact that at mid frequencies the coupling and bypass capacitors act as short circuits and the device and stray wiring capacitances act as open circuits due to their low capacitance. The mid band gain is denoted as \( A_{v_{\text{mid}}} \).
Fig. (1) Frequency response of RC coupled amplifier.

**Frequency response of transformer coupled amplifier:**

Fig. (2) shows frequency response of transformer coupled amplifier. The magnetizing inductive reactance of the transformer winding is $X_L=2\pi fL$. At low frequencies the gain drops due to small value of $X_L$. At $f=0$ (DC) there is no change in flux in the core. As a result the secondary induced voltage or output voltage is zero and hence the gain. At high frequencies the gain drops due to stray capacitance between the turns of primary and secondary windings.

Fig. (2) Frequency response of transformer coupled amplifier.

**Frequency response of direct coupled amplifier:**

Fig. (3) shows frequency response of direct coupled amplifier. Since there are no coupling or bypass capacitors, there is no drop in gain at low frequencies. It has a flat response to the upper cut-off frequency. Gain drops at high frequencies due to device internal capacitances and the stray wiring capacitances.
Fig. (3) Frequency response of direct coupled amplifier.

**Half power frequencies and bandwidth:**

The frequencies \( f_1 \) and \( f_2 \) at which the gain is 0.707 \( A_{\text{mid}} \) are called cut-off frequencies or corner frequencies or break frequencies. \( f_1 \) is called the lower cut-off frequency and \( f_2 \) is called the upper cut-off frequency.

Bandwidth or pass band of the amplifier is

\[
\text{BW} = f_2 - f_1 \tag{1}
\]

The output voltage in the mid band is \( |V_O| = |A_{\text{mid}}| |V_i| \)

Output power in the mid band is

\[
P_{O(\text{mid})} = \frac{|V_O|^2}{R_O} = \frac{|A_{\text{mid}}|^2|V_i|^2}{R_O} \tag{2}
\]

The output voltage at cut-off frequencies is

\[
|V_O| = 0.707 A_{\text{mid}} |V_i| \]

The output power at cut-off frequencies is

\[
P_{O(\text{cut-off})} = \frac{0.707 A_{\text{mid}}^2 |V_i|^2}{R_O}
\]
Thus, the output power at cut-off frequencies is half the mid band power output. \( f_1 \) is called the lower half power frequency and \( f_2 \) is called the upper half power frequency.

**Normalized gain V/s Frequency plot:**

The normalized gain is obtained by dividing the gain \( A_V \) at each frequency by the mid band gain \( A_{V_{mid}} \).

Therefore normalized gain \( = \frac{A_V}{A_{V_{mid}}} \) (4)

Fig. (4) shows the normalized gain V/s frequency plot for an RC coupled amplifier.

The normalized mid band gain is \( \frac{A_{V_{mid}}}{A_{V_{mid}}} = 1 \)

The normalized gain at cut-off frequencies is \( \frac{0.707A_{V_{mid}}}{A_{V_{mid}}} = 0.707 \)

![Normalized gain V/s frequency plot](image)

**Normalized decibel gain is** \( \frac{A_V}{A_{V_{mid}}} \) \( \text{dB} = 20 \log_{10} \left[ \frac{A_V}{A_{V_{mid}}} \right] \) (5)

Normalized decibel voltage gain in mid band is
20 \log_{10} \left[ \frac{A_v}{A_{v_{mid}}} \right] = 0

Normalized decibel voltage gain at cut-off frequencies is

20 \log_{10} \left[ \frac{0.707A_v}{A_{v_{mid}}} \right] = -3\text{dB}

Since normalized decibel voltage gain at cut-off frequencies is 3dB less than the normalized decibel mid band voltage gain. \( f_1 \) and \( f_2 \) are also called 3dB frequencies.

\( f_1 \rightarrow \text{lower 3dB frequency} \)

\( f_2 \rightarrow \text{upper 3dB frequency} \)

Fig. (5) shows the plot of normalized dB voltage gain V/s frequency for an RC coupled amplifier.

Fig. (5) Plot of normalized decibel voltage gain V/s frequency.

**Phase angle plot:**

A single stage RC coupled amplifier introduces a 180° phase shift between input and output signals in the mid band region. At low frequencies the output voltage \( V_O \) lags \( V_i \) by an additional angle \( \theta_1 \). Therefore, the total phase phase shift between \( V_O \) and \( V_i \) is more than 180°. At high frequencies, \( V_O \) leads \( V_i \) by an additional angle \( \theta_2 \). As a result, the total phase shift drops below 180°. Fig. (6) shows the phase plot for a single stage RC coupled amplifier.
Fig. (6) Phase plot of single stage RC coupled amplifier.

**Low frequency analysis:**

In the low frequency region of a single stage BJT amplifier, the amplifier gain increases with frequency. Hence it can be modelled as a high pass RC circuit as shown in Fig. (7).

![Amplifier modelled as high pass RC circuit.](image)

The capacitor C represents the combined effect of coupling and bypass capacitors and the resistance R represents the combined effect of resistive elements of the amplifier network.

The capacitive reactance is

\[
X_C = \frac{1}{2\pi f_C} \quad (6)
\]

At \( f=0 \), \( X_C = \infty \Omega \). i.e. at low frequencies, the capacitor acts as a open circuit as shown in Fig. (8)

From Fig. (8), \( V_O=0 \).
At high frequencies, $X_C \approx 0 \Omega$ i.e. at high frequencies, capacitor acts as a short circuit as shown in Fig. (9).

From Fig. (9), $V_o \approx V_L$

Hence as the input signal frequency increases from zero to mid band value, the output voltage rises from zero to $V_i$ and hence the gain from zero to 1.

**Mathematical analysis:**
Apply Voltage division rule to circuit in Fig. (7),

\[ V_O = \frac{V_i R}{R - jX_C} \]

Voltage gain is given by

\[ A_v = \frac{V_o}{V_i} = \frac{R}{R - jX_C} = \frac{R}{R[1 - j\frac{X_C}{R}]} \]

\[ A_v = \frac{1}{1 - j[\frac{X_C}{R}]} \quad \text{(7)} \]

The magnitude of voltage gain is

\[ |A_v| = \frac{1}{\sqrt{1 + [\frac{X_C}{R}]^2}} \quad \text{(8)} \]

(i) At \( f=0 \), \( X_C = \frac{1}{2\pi f_C} = \infty \Omega \) therefore \( |A_v|=0 \)

(ii) At high frequencies, \( f \to \infty \), therefore \( X_C \to 0 \). Hence \( |A_v| \to 1 = |A_v|_{\text{mid}} \)

\[ |A_v|_{\text{mid (dB)}} = 20 \log_{10}(1) = 0\text{dB} \]

(iii) When \( X_C = R \) \quad \text{(9)}

\[ |A_v| = \frac{1}{\sqrt{2}} \Rightarrow \frac{V_o}{V_i} = \frac{1}{\sqrt{2}} \text{ or } V_o = 0.707V_i \]

The corresponding decibel gain is

\[ 20 \log_{10} \frac{1}{\sqrt{2}} = -3\text{dB} \]

From Eq. (9), \( \frac{1}{2\pi f_C} = R \)

\[ f = \frac{1}{2\pi f_C} \]

The frequency given by the above Eq. is the lower cut-off frequency or lower 3dB cut-off frequency denoted by \( f_1 \).

Therefore \( f_1 = \frac{1}{2\pi f_C} \quad \text{(10)} \)
Using in Eq. (7) and (8)

\[
A_V = \frac{1}{1 - j \left( \frac{f_1}{f} \right)} 
\tag{11}
\]

\[
|A_V| = \frac{1}{\sqrt{1 + \left( \frac{f_1}{f} \right)^2}} 
\tag{12}
\]

From Eq. (11), phase angle of \( A_V \) is

\[
\theta_1 = \tan^{-1} \left( \frac{f_1}{f} \right) 
\tag{13}
\]

Since \( \theta_1 \) is +Ve, \( V_o \) leads \( V_i \) by an angle \( \theta_1 \)

In magnitude and phase form, Eq. (11) can be written as

\[
A_V = |A_V| \Delta \theta_1 
= \frac{1}{\sqrt{1 + \left( \frac{f_1}{f} \right)^2}} \Delta \tan^{-1} \left( \frac{f_1}{f} \right) 
\tag{14}
\]

Fig. (10) shows the plot of \( |A_V| \) V/s frequency.

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**Fig. (10) Low frequency response of high pass RC circuit.**
Bode plot of low frequency response:

From Eq. (12), we have

\[ |A_V| = \frac{1}{\sqrt{1 + \left(\frac{f_1}{f}\right)^2}} \]

Voltage gain in dB is

\[ |A_V|_{dB} = 20 \log_{10} \left[ \frac{1}{\sqrt{1 + \left(\frac{f_1}{f}\right)^2}} \right] \]

\[ |A_V|_{dB} = -20 \log_{10} \sqrt{1 + \left(\frac{f_1}{f}\right)^2} \quad ---- (15) \]

To construct the plot of \( |A_V|_{dB} \) V/s frequency using straight line segments, we consider the following frequency ranges.

(a) For frequencies \( f << f_1 \) or \( \frac{f_1}{f} >> 1 \)

Eq. (15) can be approximated as

\[ |A_V|_{dB} = -20 \log_{10} \sqrt{\left(\frac{f_1}{f}\right)^2} \]

\[ |A_V|_{dB} = -20 \log_{10} \left(\frac{f_1}{f}\right) \quad ---- (16) \]

\( |A_V|_{dB} \) is calculated at different values of \( \frac{f_1}{f} \) and tabulated in table (1).

<p>| ( f )     | ( \frac{f_1}{f} ) | ( |A_V|<em>{dB} = -20 \log</em>{10} \left(\frac{f_1}{f}\right) ) |
|------------|----------------------|--------------------------------------------------|
| ( \frac{f_1}{10} ) | 10                   | -20dB                                            |
| ( \frac{f_1}{4} )  | 4                    | -12dB                                            |</p>
<table>
<thead>
<tr>
<th>f_1 \hspace{0.5cm}</th>
<th>2 \hspace{1cm}</th>
<th>-6dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>f_1/2 \hspace{0.5cm}</td>
<td>1 \hspace{1cm}</td>
<td>0dB</td>
</tr>
</tbody>
</table>

Table (1): |A_V|_dB at different frequencies.

Following conclusions can be drawn from table (1).

(i) A change in frequency by a factor of 2 is equal to one octave. When the frequency changes from \(\frac{f_1}{4}\) to \(\frac{f_1}{2}\) or \(\frac{f_1}{2}\) to \(f_1\) (one octave), the gain increases by 6dB.

(ii) A change in frequency by a factor of ten is equal to one decade. When the frequency changes from \(\frac{f_1}{10}\) to \(f_1\) (one decade), the gain increases by 20dB.

(iii) If a plot of |A_V|_dB against log scale in the frequency range \(\frac{f_1}{10} < f < f_1\) is plotted, a straight line with slope 6dB/octave or 20dB/decade is obtained as shown in Fig. (11).

Fig. 11: Bode plot for low frequency region.

(b) For frequencies, \(f >> f_1\) or \(\frac{f_1}{f} << 1\)

Eq. (15) can be approximated by

\[ |A_V|_dB \approx -20 \log_{10} 1=0dB. \]
The plot of $|A_J|_{dB}$ against log scale for the frequency range $f >> f_1$, is a straight line on the frequency axis as shown in Fig. (11). The slope of this line is zero, since the gain is constant at 0dB. The plot in Fig. (11) is made of 2 straight line segments called asymptotes with a break point at $f_1$. Hence $f_1$ is called break frequency or corner frequency. This piecewise linear plot is also called bode magnitude plot or simply bode plot.

From bode plot, at $f = f_1$, $|A_J|_{dB} = 0$.

From bode plot, at $f = f_1$, $|A_J|_{dB} = -3$.

Thus, at $f = f_1$, the gain read from bode plot differs from the actual gain by 3dB.

**Phase Plot:**

At low frequencies, $V_O$ leads $V_i$ by an angle $\theta_1$ given by

$$\theta_1 = \tan^{-1}\left[\frac{f_1}{f}\right] \quad \quad (17)$$

The value of $\theta_1$ is calculated at different values of $\frac{f_1}{f}$ as shown in table (2).

**Table (2): Phase angle between $V_O$ and $V_i$**

<table>
<thead>
<tr>
<th>$f$</th>
<th>$\frac{f_1}{f}$</th>
<th>$\theta_1 = \tan^{-1}\left[\frac{f_1}{f}\right]$</th>
<th>Total phase shift $\theta = 180 + \theta_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\infty$</td>
<td>$90^0$</td>
<td>$270^0$</td>
</tr>
<tr>
<td>$\frac{f_1}{100}$</td>
<td>100</td>
<td>$89.4^0$</td>
<td>$269.4^0$</td>
</tr>
<tr>
<td>$f_1$</td>
<td>1</td>
<td>$45^0$</td>
<td>$225^0$</td>
</tr>
<tr>
<td>$100f_1$</td>
<td>0.01</td>
<td>$0.572^0$</td>
<td>$180.572^0$</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0</td>
<td>$0^0$</td>
<td>$180^0$</td>
</tr>
</tbody>
</table>
The total phase shift $\theta_1$ between $V_O$ and $V_i$ is the sum of the phase shift of RC network and inherent phase shift ($180^0$) introduced by the amplifier.

From table (2),

(i) The phase shift $\theta_1$ due to RC network decreases from $90^0$ to $0^0$. The plot of $\theta_1$ V/s frequency is shown in Fig. (12).

(ii) The total phase $\theta$, decreases from $270^0$ to $180^0$.

![Phase response of RC network](image)

**Fig. (12) Phase response of RC network.**

**Low frequency Response of BJT amplifier:**

Fig. (13) shows the circuit of single stage BJT amplifier. The coupling capacitors $C_S$ and $C_C$ and bypass capacitor $C_E$ determines the low frequency response.

**Effect of $C_S$ on low frequency response:**

The input coupling capacitor $C_S$ couples the source signal to BJT. First, we will neglect the effects of $C_C$ and $C_E$ i.e. they are treated as short circuits.

The AC equivalent circuit is obtained by reducing VCC to zero and $C_C$ and $C_E$ by their short circuit equivalent as shown in Fig. (14).
The resistance of the transistor between base-emitter is $h_{ie}$. The input AC equivalent circuit is shown in Fig. (15).

Let $R_i = R_1 \parallel R_2 \parallel h_{ie}$ ------ (18)

Where $h_{ie} = \beta r_e$ ------ (19)

Using voltage division rule in the circuit of Fig. (15) the voltage applied to the amplifier is

\[ V_i = \frac{V_S R_i}{(R_S + R_i) - f X C_S} \] ------ (20)
Where \( X_{CS} = \frac{1}{2\pi f_C S} \) ----- (21)

\[
V_i = \frac{V_S}{1 - j \frac{X_{CS}}{R_i + R_S}}
\]

\[
V_i = \frac{|V_S| \frac{R_i}{R_S + R_i} \sqrt{1 + \left( \frac{X_{CS}}{R_i + R_S} \right)^2}} {\sqrt{1 + \left( \frac{X_{CS}}{R_i + R_S} \right)^2}}
\] ----- (22)

In the mid frequency band, \( f \) is large. As a result, \( X_{CS} \rightarrow 0 \).

Therefore from Eq. (22),

\[
|V_i|_{mid} = \frac{|V_S|R_i}{(R_S + R_i)}
\] ----- (23)

Therefore Eq. (22) becomes,

\[
|V_i| = \frac{|V_i|_{mid}} {\sqrt{1 + \left( \frac{X_{CS}}{R_i + R_S} \right)^2}}
\] ----- (24)

The lower 3dB cut-off occurs when \( |V_i| = \frac{|V_i|_{mid}} {\sqrt{2}} = 0.707 |V_i|_{mid} \)

Therefore Eq. (24) becomes,

\[
0.707 |V_i|_{mid} = \frac{|V_i|_{mid}} {\sqrt{1 + \left( \frac{X_{CS}}{R_i + R_S} \right)^2}}
\]

This condition is satisfie, if \( \frac{X_{CS}}{R_i + R_S} = 1 \) or \( X_{CS} = R_i + R_S \)

\[
\frac{1}{2\pi f_{CS}} = R_i + R_S
\]

Therefore \( f = \frac{1}{2\pi (R_S + R_i) C_S} \) ----- (25)

Eq. (25) gives the lower 3dB cut-off frequency due to \( C_S \).
Therefore \( f_{ls} = \frac{1}{2\pi(R_s + R_l)c_s} \) ----- (26)

**Effect of output coupling capacitor \( C_C \) on low frequency response :**

The output coupling capacitor \( C_C \) couples the output of the BJT to the load. The equivalent circuit on the output side by neglecting the effect of \( C_S \) and \( C_E \) by treating them as short circuits is as shown in Fig. (16).

![Fig. (16) (a) AC equivalent circuit of output side (b) Simplified AC equivalent circuit](image)

Let \( R_O = r_O \parallel R_C \) ----- (27)

\( V_C \) = output voltage of BJT

\( V_O \) = load voltage

Using voltage division rule in circuit of Fig. (16) (a), the load voltage is,

\[
V_O = \frac{V_C R_L}{(R_O + R_L) - jXc_C} \quad \text{----- (28)}
\]

Where \( Xc_C = \frac{1}{2\pi f c_C} \) ----- (29)

\[
V_O = \frac{V_c \left( \frac{R_L}{R_O + R_L} \right)}{1 - j \left( \frac{Xc_C}{R_O + R_L} \right)}
\]

\[
|V_O| = \frac{|V_c| \left( \frac{R_L}{R_O + R_L} \right)}{\sqrt{1 + j \left( \frac{Xc_C}{R_O + R_L} \right)^2}} \quad \text{----- (30)}
\]

In the mid frequency band, \( Xc_C \rightarrow 0 \)
Therefore \(|V_o|_{\text{mid}} = \frac{|V_c|R_L}{(R_O+R_L)} \) ----- (31)

Substitute Eq. (31) in (30)

\(|V_o| = \sqrt{\frac{|V_o|_{\text{mid}}}{1+\left[\frac{X_{C_C}}{R_O+R_L}\right]^2}} \) ----- (32)

The lower 3dB cut-off occurs when \(|V_o| = \frac{|V_o|_{\text{mid}}}{\sqrt{2}} = 0.707 \ |V_o|_{\text{mid}}\)

This is possible iff,

\(\frac{X_{C_C}}{R_O+R_L} = 1 \) or \(X_{C_C} = R_O + R_L\)

Therefore \(f = \frac{1}{2\pi(R_O+R_L)C_C} \) ----- (33)

Eq. (33) gives the lower 3dB cut-off frequency due to \(C_C\).

Therefore \(f_{lc} = \frac{1}{2\pi(R_O+R_L)C_C} \) ----- (34)

**Effect of Emitter bypass capacitor \(C_E\) on low frequency response :**

The equivalent circuit (13) considering the effect of \(C_E\) is as shown in Fig. (17). Hence the effect of \(C_S\) and \(C_C\) are neglected.

![Fig. (17) AC equivalent circuit](image)

Replacing the transistor by its low frequency small signal hybrid model the AC equivalent circuit is as shown in Fig. (18).
Fig. (18) AC equivalent circuit using hybrid model

\( R_e \) is the AC equivalent resistance seen by \( C_e \). To find \( R_e \), \( V_S \) is reduced to 0 as shown in Fig. (18).

Let \( \hat{I}_i = R_S \parallel R_1 \parallel R_2 \) ---- (35)

The AC equivalent circuit is redrawn as shown in Fig. (20)

Fig. (18) AC equivalent circuit to find \( R_e \)

\( R_S = \beta r_e \) is in base circuit. When it is transformed to emitter circuit, it is divided by \( \beta \). Therefore \( I_E \approx I_C = \beta I_B \).

The resulting circuit is shown in Fig. (21).

\( R_e = R_E \parallel \frac{R_S}{\beta} + r_e \) ---- (36)

From Fig. (18), the lower cut-off frequency due to \( C_E \) is

\[ f_{LE} = \frac{1}{2\pi R_e C_E} \] ---- (37)

**Effect of \( C_E \) on voltage gain:**

The mid band voltage gain of amplifier of Fig. (13) without \( C_E \) is given by,

\[ A_{V_{mid}} = -\frac{R_O \parallel R_E}{r_e + R_E} \] ---- (38)

Where \( R_O = R_C \parallel r_0 \)

If \( C_E \) is connected in parallel with \( R_E \), then voltage gain becomes a function of frequency. The voltage gain at any frequency is
\[ A_V = -\frac{R_O R_L}{r_e + R_E X_{CE}} \] ---- (39)

Where \( X_{CE} = \frac{1}{2\pi f_{CE}} \) ---- (40)

As the frequency increases:

(i) \( X_{CE} \) decreases.
(ii) \( R_E \parallel X_{CE} \) decreases.
(iii) \( A_V \) increases in magnitude.

As the frequency approaches the mid band value

(i) \( X_{CE} \) approaches zero.
(ii) \( R_E \parallel X_{CE} \) approaches zero. (i.e. \( R_E \) is shorted out)
(iii) \( A_V \) approaches maximum value or mid band value.

\[ A_{V_{mid}} = -\frac{R_O R_L}{r_e} \] ---- (41)

**Overall lower cutoff frequency:**

The low frequency response of the amplifier is influenced by the capacitors \( C_S, C_C \) and \( C_E \). The lower cut-off frequencies due to \( C_S, C_C \) and \( C_E \) are \( f_{LS}, f_{LC} \) and \( f_{LE} \) respectively. If these cut-off frequencies are relatively apart (i.e. one is greater than the other by 4 times or more) the higher of the 3 is approximately the lower cut-off frequency for the amplifier stage.

Ex: \( f_{LS} = 6\text{Hz}, f_{LC} = 25\text{Hz} \) and \( f_{LE} = 320\text{Hz} \)

Then the lower cut-off frequency of amplifier is \( f_{LE} = 320\text{Hz} \) Because, \( f_{LE} > 4f_{LS} \)

\( f_{LE} > 4f_{LC} \).

**Miller Effect Capacitance:**

Fig. (19) shows an inverting amplifier with a capacitance \( C_f \) between the input and output nodes. WKT, \( A_V \) is \(-Ve\) for inverting amplifier since \( V_O \) and \( V_i \) are
180° out of phase. Using Millers theorem we can find the loading effect of $C_f$ on the input and output circuits of the amplifier.

![Inverting amplifier with capacitance between input and output nodes.](image)

**Fig. (19) Inverting amplifier with capacitance between input and output nodes.**

**To find Miller-Input Capacitance ($C_{mi}$) :**

From Fig. (19),

$$R_i = \frac{V_i}{I_1} \Rightarrow I_1 = \frac{V_i}{R_i}$$

$$Z_i = \frac{V_i}{I_i} \Rightarrow I_i = \frac{V_i}{Z_i}$$

Apply KCL at input node A,

$$I_i = I_1 + I_2 \quad \text{---- (42)}$$

From Fig. (23),

$$I_2 = \frac{V_i - V_o}{X_{C_f}}$$

But $V_0 = A_V V_i$

Therefore $I_2 = \frac{V_i - A_V V_i}{X_{C_f}} = \frac{V_i [1 - A_V]}{X_{C_f}}$

Substitute for $I_i$, $I_1$ and $I_2$ in Eq. (42), we get
\[ \frac{V_i}{Z_i} = \frac{V_i}{R_i} + \frac{V_i[1-A_V]}{X_{C_f}} \]

Eliminating \( V_i \) through,

\[ \frac{1}{Z_i} = \frac{1}{R_i} + \frac{[1-A_V]}{X_{C_f}} = \frac{1}{R_i} + \frac{1}{X_{C_f}} \frac{1}{1-A_V} \]

Let \( X_{C_{mi}} = \frac{X_{C_f}}{1-A_V} \) ----- (43)

\[ \frac{1}{Z_i} = \frac{1}{R_i} + \frac{1}{X_{C_{mi}}} \] ----- (44)

But \( X_{C_f} = \frac{1}{2\pi f C_f} \)

\[ X_{C_{mi}} = \frac{1}{[1-A_V]2\pi f C_f} \] ----- (45)

Where \( C_{mi} = [1 - A_V]C_f \) = miller input capacitance ----- (46)

From Eq. (44), \( Z_i \) can be interpreted as the impedance resulting from the parallel combination of \( R_i \) and \( C_{mi} \) as shown in Fig. (24).

**To find Miller output capacitance (\( C_{mo} \))**:

From Fig. (19),

\[ R_O = \frac{V_o}{V_i} \Rightarrow I_1 = \frac{V_o}{R_O} \]

\[ Z_O = \frac{V_o}{I_o} \Rightarrow I_0 = \frac{V_o}{Z_O} \]

Apply KCL at node B,

\[ I_0 = I_1 + I_2 \] ----- (47)

From Fig. (19)
\( I_2 = \frac{V_o - V_i}{X_{cf}} \)

But \( V_0 = A_V + V_i \) or \( V_i = \frac{V_o}{A_V} \)

Therefore \( I_2 = \frac{V_o - \frac{V_o}{A_V}}{X_{cf}} = \frac{V_o \left[1 - \frac{1}{A_V}\right]}{X_{cf}} \)

Usually, \( R_O \) is large therefore \( \frac{V_o}{R_o} \) can be neglected

Eq. (47), \( \frac{V_o}{Z_0} = \frac{V_o}{R_o} + \frac{V_o \left[1 - \frac{1}{A_V}\right]}{X_{cf}} \)

\[ I_0 \approx \frac{V_o \left[1 - \frac{1}{A_V}\right]}{X_{cf}} \]

\[ Z_0 = \frac{V_o}{I_0} = \frac{X_{cf}}{1 - \frac{1}{A_V}} = \frac{1}{\left[1 - \frac{1}{A_V}\right]2\pi fC_f} \]

\[ Z_0 = \frac{1}{2\pi fC_m_o} \quad \text{(48)} \]

Where, \( C_{m_o} = \left[1 - \frac{1}{A_V}\right]C_f \quad \text{(49)} \)

\( C_{m_o} \) is called Miller output Capacitance.

**Statement of Millar’s theorem:**

A capacitance \( C_f \) connected between the input and output nodes of an inverting amplifier can be replaced by

(i) Miller input capacitance, \( C_{m_i} = [1 - A_V]C_f \) connected between input node and ground.
(ii) Miller output capacitance, \( C_{m_o} = \left[ 1 - \frac{1}{A_V} \right] C_f \) connected between output node and ground.

For an non-inverting amplifier \( A_V \) is positive. In order to obtain positive values for \( C_{m_i} \) and \( C_{m_o} \), Eq. (46) and Eq. (49) should be modified as follows

\[
C_{m_i} = [1 - A_V] C_f \quad (50)
\]
\[
C_{m_o} = \left[ 1 + \frac{1}{A_V} \right] C_f \quad (51)
\]

Applications of Miller’s theorem to the amplifier of Fig. (19) results in network shown in Fig. (20).

Fig. (20) Amplifier with \( C_f \) replaced by Miller’s capacitances.

**High Frequency Response of BJT amplifier:**

In the high frequency response of BJT amplifier, the upper 3dB cut-off point is defined by the following factors.

(i) The network capacitance which includes the parasitic capacitances of the transistor and the wiring capacitances.

(ii) The frequency dependence of short circuit \( C_E \) current gain \( h_{fe} \) or \( \beta \).

**Network Parameters:**

Fig. (21) shows the RC coupled amplifier with parasitic and wiring capacitances. \( C_{be}, C_{bc} \) and \( C_{ce} \) are the parasitic capacitances of the transistor. \( C_{w_i} \) and \( C_{w_o} \) are
input and output wiring capacitances which are introduced during the construction of the amplifier circuit.

Fig. (21) RC-coupled amplifier with parasitic and wiring capacitances.

Fig. (22) shows the high frequency AC equivalent circuit of RC-coupled amplifier.

Fig. (23) High frequency AC equivalent circuit of amplifier of Fig. (22).

(i) Using Miller’s theorem, the transit capacitance, $C_{bc}$ can be replaced by two capacitances; $C_{mi}$ at the input and $C_{mo}$ at output.
(ii) The total capacitance \( C_i \) is the sum of \( C_{mi} \), \( C_{be} \) and \( C_{wi} \).

i.e. \( C_i = C_{mi} + C_{be} + C_{wi} \) ----- (52)
where \( C_{mi} = [1 - A_V] C_{bc} \) ----- (53)

(iii) The total output capacitance is the sum of \( C_{mo} \), \( C_{ce} \) and \( C_{wo} \).

i.e. \( C_0 = C_{wo} + C_{ce} + C_{mo} \) ----- (54)
where \( C_{mo} = \left[ 1 + \frac{1}{A_V} \right] C_f \)

**Upper cut-off frequency due to \( C_i \):**

Apply voltage division rule to circuit of Fig. (22),

\[
E_{Thi} = V_s \left[ \frac{R_1 || R_2 \ || \ \hat{R}_i}{R_S + R_1 \ || \ R_2 \ || \ \hat{R}_i} \right] \quad ----- (56)
\]

From circuit in Fig. (21);

\[
R_{Thi} = R_S + R_1 \ || \ R_2 \ || \ \hat{R}_i \quad ----- (57)
\]

Where \( \hat{R}_i = \beta r_e \)

From Fig. (29) (b), Apply \( V_g \) division rule,

\[
|V_i| = |E_{Thi}| \left[ \frac{X_{Cl}}{\sqrt{(R_{Thi})^2 + (X_{Cl})^2}} \right]
\]

\[
|V_i| = |E_{Thi}| \left[ \frac{|E_{Thi}|}{\sqrt{1 + \left( \frac{R_{Thi}}{X_{Cl}} \right)^2}} \right] \quad ----- (58)
\]

Where \( X_{Cl} = \frac{1}{2\pi f C_i} \) ----- (59)

In the mid band, the effect of \( C_i \) is negligible. As a result, \( X_{Cl} \) can be treated as open circuit i.e. \( X_{Cl} = \infty \).

Therefore \( |V_i|_{mid} \approx |E_{Thi}| \)

At high frequencies, \( C_i \) cannot be neglected with increase in \( f \), \( X_{Cl} \) decreases, \( \frac{R_{Thi}}{X_{Cl}} \) increases, \( |V_i| \) decreases and hence the voltage gain decreases.
3dB cut-off occurs at a frequency at which

\[ |V_i| = \frac{|V_i|_{mid}}{\sqrt{2}} = \frac{|E_{Thi}|}{\sqrt{2}} \]

From (58), this condition occurs, when

\[ R_{Thi} = X_{Cl} \]

\[ R_{Thi} = \frac{1}{2\pi f_C i} \]

Or \( f = f_{Hi} = \frac{1}{2\pi R_{Thi} C_i} \)

\[
\begin{bmatrix}
  f \\
  f_{Hi}
\end{bmatrix}
\]

Therefore Eq. (58) becomes,

\[ |V_i| = \frac{|E_{Thi}|}{\sqrt{1 + \left(\frac{f}{f_{Hi}}\right)^2}} \quad (62) \]

Thus, due to \( C_i \), \( V_g \) gain decreases at the rate of 20dB/decade.

**Upper cut-off frequency due to output capacitance \( C_o \):**

Consider the output circuit of Fig.(22) which is shown in Fig. (23).

\( \beta I_b, r_o \) and \( R_C || R_L \) is connected to voltage source as shown in Fig. (22).

\[ R_{Tho} = r_o || R_C || R_L \quad (63) \]

\[ E_{Tho} = [-\beta I_b] [r_o || R_C || R_L] \quad (64) \]

Using the same procedure as listed above, we have

\[ |V_o| = \frac{|E_{Tho}|}{\sqrt{1 + \left(\frac{R_{Tho}}{X_{Co}}\right)^2}} \quad (65) \]

Where \( X_{Co} = \frac{1}{2\pi f_C} \quad (66) \)
The output voltage in mid band is

\[ |V_{o}\mid_{\text{mid}} \approx |E_{THo}| \quad (67) \]

The Upper 3dB cutoff frequency due to \( C_o \) is

\[ f_{HO} = \frac{1}{2\pi R_{THo}C_o} \quad (68) \]

and magnitude of voltage gain is

\[ |A_V| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{HO}}\right)^2}} \quad (69) \]

Thus due to \( C_o \), \( Vg \) gain decreases at the rate of 20dB/ Decade.

**Combined effect of \( C_i \) and \( C_O \) on high frequency response:**

(i) The input capacitance \( C_i \), defines upper cut-off frequency \( f_{Hi} \).

(ii) The output capacitance \( C_o \), defines another upper cut-off frequency \( f_{Ho} \).

(iii) The lowest of these 2 frequencies will be taken as overall upper cut-off frequency.

(iv) If the variation of \( h_{fe} \) with frequency is considered then the actual cut-off frequency may be lower than \( f_{Hi} \) or \( f_{Ho} \).

**Variation of \( h_{fe} \) with frequency:**

Fig. (24) shows hybrid-\( \pi \) high frequency small signal model of BJT.

![Hybrid-\( \pi \) high frequency small signal model of BJT](image)

Fig. (24) Hybrid-\( \pi \) high frequency small signal model of BJT.
The B-E input capacitance $C_\pi$ ($C_{be}$) and B-C depletion capacitance $C_u$ ($C_{bc}$) makes the short circuit current gain $h_{fe}$ to vary with frequency in high frequency region.

**Expression for $h_{fe}$ as a function of frequency:**

Following assumptions are made:

(i) $r_b$ is few tens of $\Omega$. Hence it is treated as short circuit.
(ii) $r_u$ is few tens of $\text{M}\Omega$. Hence it is treated as open circuit.

To find $h_{fe}$, short the output terminals. The resulting circuit is shown in Fig. (24).

Fig. (24) circuit to find $h_{fe}$.

WKT, $h_{fe} = \frac{I_C}{I_b} \left| V_{ce} = 0 \right.$ ----- (70)

The current $g_m V_\pi$ flows into short circuit.

Therefore $I_C = g_m V_\pi$ ----- (71)

to find $I_b$ FROM Fig 24

Let $Z = r_\pi \parallel \frac{1}{j\omega(C_\pi + C_u)}$ ----- (72)

$$Z = \frac{r_\pi}{1 + j\omega(C_\pi + C_u) r_\pi}$$

Now, $V_\pi = I_b Z$

$$V_\pi = \frac{l_b r_\pi}{1 + j\omega(C_\pi + C_u) r_\pi}$$ ----- (73)

Substitute Eq. (73) in Eq. (71), we have

$$I_c = \frac{g_m r_\pi}{1 + j2\pi f(C_\pi + C_u) r_\pi}$$ ----- (74)

Let $f_\beta = \frac{1}{2\pi(C_\pi + C_u) r_\pi}$ ----- (75)

Substitute Eq. (75) in Eq. (74), we have
\[ h_{fe} = \frac{g_m r_n}{1 + j \left( \frac{f}{f_\beta} \right)} \] ----- (76)

\[ |h_{fe}| = \frac{g_m r_n}{\sqrt{1 + j \left( \frac{f}{f_\beta} \right)^2}} \] ----- (77)

In the mid band, \( f << f_\beta \), As a result \( \left( \frac{f}{f_\beta} \right)^2 << 1 \)

From Eq. (77), we have

\[ |h_{fe}|_{\text{mid}} = g_m r_n = h_{fe_{\text{mid}}} \] ----- (78)

\( h_{fe_{\text{mid}}} \) is also denoted by \( \text{In } \beta_{\text{mid}} \).

Substitute Eq. (78) in Eq. (77),

\[ |h_{fe}| = \frac{|h_{fe}|_{\text{mid}}}{\sqrt{1 + j \left( \frac{f}{f_\beta} \right)^2}} \] ----- (79)

Eq. (79) gives the variation of \( |h_{fe}| \) with frequency.

(i) As \( f \) increases, \( \left( \frac{f}{f_\beta} \right)^2 \) increases and hence \( |h_{fe}| \) decreases.

(ii) When \( f = f_\beta \), \( |h_{fe}| = \frac{h_{fe_{\text{mid}}}}{\sqrt{2}} = \frac{\beta_{\text{mid}}}{\sqrt{2}} \)

\( f_\beta \) defines the upper 3dB cut-off point for short circuit current gain \( h_{fe} \). \( f_\beta \) is also denoted by \( f h_{fe} \). Eq. (75) can be written as

\[ f_\beta = f h_{fe} = \frac{1}{2\pi (C_n + C_u) r_n} \] ----- (80)

But, \( r_n = \beta_re = \beta_{\text{mid}} r_e \)

\[ f_\beta = f h_{fe} = \frac{1}{2\pi} \frac{1}{\sqrt{\beta_{\text{mid}} r_e (C_n + C_u)}} \] ----- (80)
$f_\beta$ is called $\beta$ cut-off frequency. $f_\beta$ is also the bandwidth for the short circuit current gain $h_{fe}$. Fig. (38) shows the variation of $|h_{fe}|$ with frequency. $|h_{fe}|$ decreases from its mid band value $h_{fe,mid}$ with a slope of 20dB/decade.

**Expression for gain bandwidth product $f_T$:**

$f_T$ is the frequency at which $|h_{fe}| = 1$ or $|h_{fe}|_{dB} = 0$dB.

Using this in Eq. (79),

$$\frac{h_{fe,mid}}{\sqrt{1 + j \left(\frac{f}{f_\beta}\right)^2}} \bigg|_{f = f_T} = 1$$

$$\frac{h_{fe,mid}}{\sqrt{1 + j \left(\frac{f_T}{f_\beta}\right)^2}} ----- (82)$$

Since $f_T \gg f_\beta$, $\left(\frac{f_T}{f_\beta}\right)^2 \gg 1$

Therefore

$$\sqrt{1 + j \left(\frac{f_T}{f_\beta}\right)^2} \approx \frac{f_T}{f_\beta}$$

Therefore Eq. (82) becomes

$$\frac{h_{fe,mid}}{\frac{f_T}{f_\beta}} = 1$$

Or $f_T = f_\beta h_{fe,mid} = \beta_{mid} f_\beta ----- (83)$

Since $h_{fe,mid}$ is the mid band short circuit current gain and $f_\beta$ is the bandwidth, $f_T$ is called gain-bandwidth product.

Eq. (81) in Eq. (83),

$$f_T = \beta_{mid} \times \frac{1}{2\pi \beta_{mid} r_e(C_n + C_u)} = \frac{1}{2\pi(C_n + C_u) r_e} ----- (84)$$
Low frequency response of FET amplifier:

Consider a common source amplifier as shown in Fig. 25

![Fig 25 Capacitive elements that affect the low-frequency response of a JFET amplifier.](image)

**Effect of $C_G$ on Low frequency response:**

The lower cut-off frequency of this network is shown in fig 26

![Fig 26 Determining the effect of $CG$ on the low-frequency response.](image)

\[ f_{CG} = \frac{1}{2\pi (R_{sig} + R_i) C_G} \]

Where $R_i = R_G \parallel R_{in(gate)}$

\[ R_{in(gate)} = \left| \frac{V_{GS}}{I_{GSS}} \right| \]

$I_{GSS}$ = gate reverse current

$R_G >> R_{sig}$ and $R_{in(gate)} >> R_G$
\[ R_i \approx R_G \]

Therefore \[ f_{CG} = \frac{1}{2\pi R_G C_G} \]

**Effect of C\textsubscript{C} on Low frequency response:**

Consider output part of equivalent circuit as shown in Fig 27

![Fig 27 Determining the effect of Cc on the low-frequency response.](image)

\[ R_o = r_d \parallel R_D \]

Therefore \[ f_{LC} = \frac{1}{2\pi (R_o+R_L)C_G} \]

**Effect of C\textsubscript{S} on Low frequency response:**

Consider the RC network shown in Fig 28 formed by C\textsubscript{S} and let \( R_{eq} \) be the resistance looking in at the source.

![Fig 28 Determining the effect of CS on the low-frequency response.](image)

\[ f_{LS} = \frac{1}{2\pi R_{eq} C_S} \]

\[ R_{eq} = \frac{R_S}{1+R_S(1+g_m r_d) (r_d+R_D||R_L)} \]
Therefore \( r_d \gg R_D \)

**High Frequency Response FET amplifier:**

Fig. 29 shows CS JFET amplifier with inter electrode capacitances and wiring capacitances. The capacitors \( C_{gs} \) and \( C_{gd} \) vary from 1pF, whereas the capacitance \( C_{ds} \) is smaller ranging from 0.1pF to 1pF.

![Capacitive elements](image)

**Fig 29 Capacitive elements that affect the high frequency response of a JFET amplifier**

Fig. 30 shows high-frequency AC equivalent circuit. At high frequencies, \( C_i \) will approach a short circuit equivalent and \( V_{gs} \) will drop in value and reduce the overall gain. At frequencies where \( C_o \) approaches its short circuit equivalent, the parallel output voltage \( V_o \) will drop in magnitude.

![High-frequency AC equivalent circuit](image)

**Fig 30 High-frequency ac equivalent circuit for Fig. 29.**
Effect of $C_i$ on high frequency response:

![Thévenin equivalent circuits for the (a) input circuit and (b) output circuit.](image)

$$f_{Hi} = \frac{1}{2\pi (R_{Th_i})C_i}$$

Where $R_{Thi} = R_{sig} \parallel R_G$

$$C_i = C_{wi} + C_{gs} + (1 - A_V)C_{gd}$$

Effect of $C_o$ on high frequency response:

$$f_{Ho} = \frac{1}{2\pi (R_{Tho})C_o}$$

Where $R_{Tho} = r_d \parallel R_D \parallel R_L$

$$C_o = C_{wo} + C_{ds} + (1 - 1/A_V)C_{gd}$$

Multistage Frequency Effects:

For a second transistor stage connected directly to the output of a first stage, there will be a significant change in the overall frequency response. In the high frequency region, the output capacitance $C_o$ must include the wiring capacitance ($C_{w1}$), parasitic capacitance ($C_{be}$) and miller capacitance ($C_{mi}$) of the following stage. There will be additional low frequency cut-off levels due to the 2nd stage, which will further reduce the overall gain of the system in the region. For each additional stage, the upper cutoff frequency will be determined by the stage having the lowest cutoff frequency. The low frequency cutoff is determined by the stage
having the highest low-frequency cutoff frequency. The multistage amplifier frequency response is shown in Fig 32.

![Fig 32 Effect of an increased number of stages on the cutoff frequencies and the bandwidth.](image)

Assuming identical stage, for low frequency region

\[ A_{V \text{ low (overall)}} = A_{V_{1 \text{ low}}} \quad A_{V_{2 \text{ low}}} \quad \ldots \quad A_{V_{n \text{ low}}} \]

Since all stages are identical, \( A_{V_{1 \text{ low}}} = A_{V_{2 \text{ low}}} = \ldots \)

Therefore \( A_{V \text{ low (overall)}} = (A_{V_{1 \text{ low}}})^n \)

Or \( \frac{A_{V \text{ low (overall)}}}{A_{V_{\text{mid (overall)}}}} = \left(\frac{A_{V_{\text{low}}}}{A_{V_{\text{mid}}}}\right)^n = \frac{1}{(1 - j\frac{f_1}{f})^n} \)

At 3dB frequency

\[ \frac{1}{\sqrt{(1 + j\left(\frac{f_1}{f}\right)^2)^n}} = \frac{1}{\sqrt{2}} \]

\[ \left(1 + \left(\frac{f_1}{f}\right)^{1/2}\right)^n = 2^{1/2} \]

\[ f_1' = \frac{f_1}{\sqrt[2^{1/n - 1}]} \]

Similarly \( f_2' = \sqrt{2^{1/n} - 1} \ f_2 \)