Module – 1

Design of rectangular slab type combined footing.

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1.1.1 Introduction
Whenever two or more columns in a straight line are carried on a single spread footing, it is called a combined footing. Isolated footings for each column are generally the economical. Combined footings are provided only when it is absolutely necessary, as
i) When two columns are close together, causing overlap of adjacent isolated footings
ii) Where soil bearing capacity is low, causing overlap of adjacent isolated footings
iii) Proximity of building line or existing building or sewer, adjacent to a building column.

The combined footing may be rectangular, trapezoidal or Tee-shaped in plan. The geometric proportions and shape are so fixed that the centroid of the footing area coincides with the resultant of the column loads. This results in uniform pressure below the entire area of footing.

Trapezoidal footing is provided when one column load is much more than the other. As a result, the both projections of footing beyond the faces of the columns will be restricted. Rectangular footing is provided when one of the projections of the footing is restricted or the width of the footing is restricted.

**Rectangular combined footing**

Longitudinally, the footing acts as an upward loaded beam spanning between columns and cantilevering beyond. Using statics, the shear force and bending moment diagrams in the longitudinal direction are drawn. Moment is checked at the faces of the column. Shear force is critical at distance ‘d’ from the faces of columns or at the point of contra flexure. Two-way shear is checked under the heavier column.

The footing is also subjected to transverse bending and this bending is spread over a transverse strip near the column.

Combined footing may be of slab type or slab and beam type or slab and strap beam type

**Design:**
1. Locate the point of application of the column loads on the footing.
2. Proportion the footing such that the resultant of loads passes through the centre of footing
3. Compute the area of footing such that the allowable soil pressure is not exceeded.
4. Calculate the shear forces and bending moments at the salient points and hence draw SFD and BMD.
5. Fix the depth of footing from the maximum bending moment.
6. Calculate the transverse bending moment and design the transverse section for depth and reinforcement. Check for anchorage and shear.
7. Check the footing for longitudinal shear and hence design the longitudinal steel
8. Design the reinforcement for the longitudinal moment and place them in the appropriate positions.
9. Check the development length for longitudinal steel
10. Curtail the longitudinal bars for economy
11. Draw and detail the reinforcement
12. Prepare the bar bending schedule

Section 1-1, 2-2, 5-5, and 6-6 are sections for critical moments
Section 3-3, 4-4 are sections for critical shear (one

CRITICAL SECTIONS FOR MOMENTS AND
1.1.2 Objective

1. To provide basic knowledge in the areas of limit state method and concept of design of RC and Steel structures
2. To identify, formulate and solve engineering problems in RC and Steel Structures

1.1.3 Design example

Design of combined footing – Slab and Beam type

Two interior columns A and B carry 700 kN and 1000 kN loads respectively. Column A is 350 mm x 350 mm and column B is 400 mm X 400 mm in section. The centre to centre spacing between columns is 4.6 m. The soil on which the footing rests is capable of providing resistance of 130 kN/m². Design a combined footing by providing a central beam joining the two columns. Use concrete grade M25 and mild steel reinforcement.

Solution: Data

\( f_{ck} = 25 \text{ N/mm}^2, \)
\( f_y = 250 \text{ N/mm}^2, \)
\( f_b \text{ (SBC)} = 130 \text{ kN/m}^2, \)
Column A = 350 mm x 350 mm,
Column B = 400 mm x 400 mm,
c/c spacing of columns = 4.6 m,
\( P_A = 700 \text{ kN} \) and \( P_B = 1000 \text{ kN} \)

Required: To design combined footing with central beam joining the two columns.

Ultimate loads
\[
P_{uA} = 1.5 \times 700 = 1050 \text{ kN}, \quad P_{uB} = 1.5 \times 1000 = 1500 \text{ kN}
\]

**Proportioning of base size**

Working load carried by column A = \( P_A \)
\[
= 700 \text{ kN}
\]

Working load carried by column B = \( P_B \)
\[
= 1000 \text{ kN}
\]

Self weight of footing 10 % \( x (P_A + P_B) \)
\[
= 170 \text{ kN}
\]

Total working load = 1870 kN

Required area of footing = \( A_f = \text{Total load} / \text{SBC} = 1870 / 130 = 14.38 \text{ m}^2 \) Let the width of the footing = \( B_f = 2 \text{ m} \)

Required length of footing = \( L_f = A_f / B_f = 14.38 / 2 = 7.19 \text{ m} \)

Provide footing of size 7.2 m X 2 m, \( A_f = 7.2 \times 2 \)
\[
= 14.4 \text{ m}^2
\]

For uniform pressure distribution the C.G. of the footing should coincide with the C.G. of column loads. Let \( x \) be the distance of C.G. from the centre line of column A

\[
\begin{align*}
\text{Combined footing with loads} \\
\text{Combined footing with loads}
\end{align*}
\]

Then \( x = (P_B \times 4.6) / (P_A + P_B) = (1000 \times 4.6) / (1000 + 700) = 2.7 \text{ m} \) from column A.

If the cantilever projection of footing beyond column A is ‘a’
then, \(a + 2.7 = \frac{L_f}{2} = 7.2/2\), Therefore \(a = 0.9\) m
Similarly if the cantilever projection of footing beyond B is ‘b’
then, \(b + (4.6-2.7) = \frac{L_f}{2} = 3.6\) m, Therefore \(b = 3.6 - 1.9 = 1.7\) m
The details are shown in Figure

Total ultimate load from columns = \(P_u = 1.5(700 + 1000) = 2550\) kN.
Upward intensity of soil pressure \(w_u = \frac{P}{A_f} = \frac{2550}{14.4} = 177\) kN/m\(^2\) 1.5 SBC or UBC

**Design of slab:**
Intensity of upward pressure = \(p_u = 177\) kN/m\(^2\)
Consider one meter width of the slab (b=1m)
Load per m run of slab at ultimate = \(177 \times 1 = 177\) kN/m

**Rectangular Footing with Central Beam:- Design of Bottom slab.**
Cantilever projection of the slab (For smaller column) = \(1000 - 350/2 = 825\) mm Maximum ultimate moment = \(177 \times 0.825^2/2 = 60.2\) kN-m

For M25 and Fe 250, \(Q_{u,max} = 3.71\) N/mm\(^2\)
Required effective depth = \(\sqrt{60.2 \times 10^6/(3.71 \times 1000)} = 128\) mm
Since the slab is in contact with the soil, clear cover of 50 mm is assumed.
Using 20 mm diameter bars
Required total depth = \(128 + 20/2 + 50 = 188\) mm say 200 mm
Provided effective depth = \(d = 200-50-20/2 = 140\) mm

**To find steel**
\(M_u/ bd^2 = 3.07\) 3.73, URS
\(P_t = 1.7\%
\(A_d = 2380\) mm\(^2\)
Use \(\Phi 20\) mm diameter bar at spacing = \(1000 \times 314 / 23 84\) say 130 mm Area provided =\(1000 \times 314 / 130 = 2415\) mm\(^2\)

**Check the depth for one - way shear considerations**
Design shear force=\(V_u = 177x(0.825-0.140) = 121\) kN
Nominal shear stress:
\[ \tau_v = \frac{V_u}{bd} = \frac{121000}{1000 \times 140} = 0.866 \text{ MPa} \]

Permissible shear stress:
\[ P_t = \frac{100 \times 2415}{1000 \times 140} = 1.7 \text{ %}, \quad \tau_{uc} = 0.772 \text{ N/mm}^2 \]

Value of \( k \) for 200 mm thick slab = 1.2

Permissible shear stress = 1.2 \times 0.772 = 0.926

\( \tau_{uc} > \tau_v \) and hence safe.

The depth may be reduced uniformly to 150 mm at the edges.

**Check for development length**
\[ L_{dl} = \left[ \frac{0.87 \times 250}{(4 \times 1.4)} \right] = 39 \times 20 = 780 \text{ mm} \]

Available length of bar = 825 - 25 = 800 mm > 780 mm and hence safe.

**Transverse reinforcement**

Required \( A_{st} = 0.15 \times bD / 100 = 0.15 \times 1000 \times 200 / 100 = 300 \text{ mm}^2 \)

Using 8 mm bars, spacing = 1000 \times 50 / 300 = 160 mm

Provide distribution steel of 8 mm at 160 mm c/c

(c) **Design of Longitudinal Beam:**

Two columns are joined by means of a beam monolithic with the footing slab. The load from the slab will be transferred to the beam. As the width of the footing is 2 m, the net upward soil pressure per meter length of the beam

\[ w_u = 177 \times 2 = 354 \text{ kN/m} \]

**Shear Force and Bending Moment**

\[ V_{AC} = 354 \times 0.9 = 318.6 \text{ kN}, \quad V_{AB} = 1050-318.6 = 731.4 \text{ kN} \]

\[ V_{BD} = 354 \times 1.7 = 601.8 \text{ kN}, \quad V_{BA} = 1500-601.8 = 898.2 \text{ kN} \]

Point of zero shear from left end C

\[ X_1 = \frac{1050}{354} = 2.97 \text{ m from C or } X_2 = 7.2-2.97 = 4.23 \text{ m} \]

from D Maximum B.M. occurs at a distance of 4.23 m from D

\[ M_{oE} = 354 \times 4.23^2 / 2 - 1500 (4.23 - 1.7) = -628 \text{ kN.m} \]
Bending moment under column A = \( M_{uA} = 354 \times 0.9^2 / 2 = 143.37 \) kN.m Bending moment under column B = \( M_{uB} = 354 \times 1.7^2 = 511.5 \) kN-m Let the point of contra flexure be at a distance \( x \) from the centre of column A Then, \( M_x = 1050x - 354(x + 0.9)^2 / 2 = 0 \)

Therefore \( x = 0.206 \) m and 3.92 m from column A i.e. 0.68 m from B.

Depth of beam from B.M. Considerations

The width of beam is kept equal to the maximum width of the column i.e. 400 mm. Determine the depth of the beam where T-beam action is not available. The beam acts as a rectangular section in the cantilever portion, where the maximum positive moment = 511.5 kN/m.

\[
d = \sqrt{\frac{511.5 \times 10^6}{3.73 \times 400}} = 586 \text{ mm}
\]
Provide total depth of 750 mm. Assuming two rows of bars at an effective cover of 70 mm. Effective depth provided = d = 750 - 70 = 680 mm (Less than 750mm and hence no side face steel is needed).

**Check the depth for Two-way Shear**

The column B can punch through the footing only if it shears against the depth of the beam along its two opposite edges, and along the depth of the slab on the remaining two edges. The critical section for two-way shear is taken at distance d/2 (i.e. 680/2 mm) from the face of the column. Therefore, the critical section will be taken at a distance half the effective depth of the slab (d_s/2) on the other side as shown in Fig.

![Diagram](image.png)

In this case b = D = 400 mm, d_b = 680 mm, d_s = 140 mm Area resisting two - way shear

\[ = 2(b \times d_b + d_s \times d_s) + 2(D + d_b)d_s \]

\[ = 2(400 \times 680 + 140 \times 140) + 2(400 + 680) 140 \]

885600 mm²

Design shear = \( P_{ud} = \) column load – \( W_u \times \) area at critical section

\[ = 1500 - 177 \times (0.400 + 0.140) \times (0.400 + 0.680) \]
Shear stress resisted by concrete = \( \tau_{uc} = \frac{P_{ud}}{b_o d} \)

\[ \tau_{uc} = \frac{1377.65 \times 1000}{885600} = 1.56 \text{ MPa} \]

Shear stress resisted by concrete = \( \tau_{uc} = \tau_{uc} \times K_s \)

where, \( \tau_{uc} = 0.25 \sqrt{f_{ck}} = 0.25 \sqrt{25} = 1.25 \text{ N/mm}^2 \)

\( K_s = 0.5 + \frac{d}{D} = 0.5 + \frac{400}{400} = 1.5 \leq 1 \)

Hence \( K_s = 1 \)

\( \tau_{uc} = 1 \times 1.25 = 1.25 \text{ N/mm}^2 \)

Therefore unsafe and the depth of slab need to be increased. However the same depth is taken.

**Area of Reinforcement**

**Cantilever portion BD**

Length of cantilever from the face of column = 1.7 - 0.4 / 2 = 1.5 m. Ultimate moment at the face of column = 354 x 1.5^2 / 2 = 398.25 kN-m

\( M_{umax} = 3.71 \times 400 \times 680^2 \times 10^6 = 686 \text{ kN.m} > 398.25 \text{ kN-m} \)

Therefore Section is singly reinforced.

\( M_{ud}/bd = 398.25 \times 10 / (400 \times 680) = 2.15 \geq 3.73, URS \)

\( A_{st} = 3030 \text{ mm}^2 \)

Provide 3 - \( \Phi \) 32 mm + 4 - \( \Phi \) 16 mm at bottom face, Area provided = 3217 mm^2

\( L_{ad} = 39 \times 32 = 1248 \text{ mm} \)

**Curtailment**

All bottom bars will be continued up to the end of cantilever. The bottom bars of 3 - 32 will be curtailed at a distance \( d (= 680 \text{ mm}) \) from the point of contra flexure (\( \lambda = 680 \text{ mm} \)) in the portion BE with its distance from the centre of support equal to 1360 mm from B.

**Cantilever portion AC**

Length of cantilever from the face of column = 900 - 350 / 2 = 725 mm

Ultimate moment = 354 x 0.725^2 / 2 = 93 kN-m

\( M_{ud}/bd = 93 \times 10 / (400 \times 680) = 0.52 \geq 3.73, URS \)

\( A_{st} = 660 \text{ mm}^2 \)

Provide 4 - 16 mm at bottom face, Area provided = 804 mm^2

Continue all 4 bars of 16 mm diameter through out at bottom.

**Region AB between points of contra flexures**
The beam acts as an isolated T- beam.

\[ b_t = \left[ \frac{L_o}{L_o/b + 4} \right] + b_w, \text{ where,} \]

\[ L_o = 4.6 - 0.206 - 0.68 = 3.714 \text{ m} = 3714 \text{ mm} \]

\[ b = \text{actual width of flange} = 2000 \text{ mm}, \quad b_w = 400 \text{ mm} \]

\[ b_f = \left[ \frac{3714}{(3714 / 2000 + 4)} + 400 \right] = 1034 \text{ mm} < 2000 \text{ mm} \]

\[ D_f = 200 \text{ mm}, \quad M_u = 628 \text{ kN-m} \]

Moment of resistance \( M_{uf} \) of a beam for \( x_u = D_f \) is:

\[ (M_{uf}) = [0.36 \times 25 \times 1034 \times 200 (680 - 0.42 \times 200)] \times 10^{-6} = 1109 \text{ kN.m} > M_u (= 628 \text{ kN-m}) \]

Therefore \( X_u < D_f \)

\[ M_u = 0.87f_y A_{st}(d-f_y A_{st}/f_{ck} b_f)^2 \]

Provide 5 bars of 32 mm and 3 bars of 16 mm,

Area provided = 4021 + 603 = 4624 mm\(^2\) > 4542 mm\(^2\)

\[ p_t = 100 \times 4624/(400 \times 680) = 1.7 \% \]

Curtailment

Consider that 2 - 32 mm are to be curtailed

No. of bars to be continued = 3 - 16 + 3 - 32 giving area = \( A_{st} = 3016 \) mm\(^2\)

Moment of resistance of continuing bars

\[ M_{ur} = (0.87 \times 250 \times 3016 (680 - ((250 \times 3016) / (25 \times 400)) \times 10^{-6} = 396.6 \text{ kN-m} \]

Let the theoretical point of curtailment be at a distance \( X \) from the free end C, then \( M_{uc} = M_{ur} \)

Therefore -354 \( x^2 \) / 2 + 1050 (x - 0.9) = 396.6

\[ x^2 - 5.93x + 7.58 = 0, \]

Therefore \( x = 4.06 \) m or 1.86 m from C

Actual point of curtailment = 4.06 + 0.68 = 4.74 m from C or 1.86 - 0.68 = 1.18 m from C

Terminate 2 - \( \Phi 32 \) mm bars at a distance of 280 mm (= 1180 - 900) from the column A and 760 mm (= 5500 - 4740) from column B. Remaining bars 3 - \( \Phi 32 \) shall be continued beyond the point of inflection for a distance of 680 mm i.e. 460 mm from column A and up to the outer face of column B. Remaining bars of 3 - \( \Phi 16 \) continued in the cantilever portion.

**Design of shear reinforcement**

**Portion between column i.e. AB**

In this case the crack due to diagonal tension will occur at the point of contra flexure because the distance of the point of contra flexure from the column is less than the effective depth \( d (= 680 \text{ mm}) \)
(i) Maximum shear force at B = \( V_{\text{umax}} \) = 898.2 kN
Shear at the point of contra flexure = \( V_{\text{uD}} \) - 898.2 - 354 x 0.68 = 657.48 kN
\( \tau_v = 657000/(400 \times 680) = 2.42 \text{ MPa} \quad \tau_{\text{c,max.}} \)
Area of steel available = \( 3 \times \Phi 16 + 3 \times \Phi 32 \), \( A_{\text{st}} = 3016 \) mm\(^2\)
\( \rho_t = 100 \times 3016 / (400 \times 680) = 1.1\% \)
\( \tau_c = 0.664 \text{ MPa} \)
\( \tau_v > \tau_c \)
Design shear reinforcement is required.
Using 12 mm diameter - legged stirrups,
Spacing = \( 0.87 \times 250 \times (4 \times 113) / (2.42-0.664)\times 400 = 139 \text{ mm say} 120 \text{ mm c/c} \)
Zone of shear reinforcements between \( \tau_v \) to \( \tau_c \)
\( = \) \( m \) from support B towards A

(ii) Maximum shear force at A = \( V_{\text{umax}} \) = 731.4 kN.
Shear at the point contra flexure = \( V_{\text{uD}} \) = 731.4 - 0.206 x 354 = 658.5 kN
\( \tau_v = 658500/(400 \times 680) = 2.42 \text{ MPa} \quad \tau_{\text{c,max.}} \)
Area of steel available = 4624 mm\(^2\), \( \rho_t = 100 \times 4624 / (400 \times 680) = 1.7\% \)
\( \tau_v > \tau_c \)
Design shear reinforcement is required.
Using 12 mm diameter - legged stirrups,
Spacing = \( 0.87 \times 250 \times (4 \times 113) / (2.42-0.774)\times 400 = 149 \text{ mm say} 140 \text{ mm c/c} \)
Zone of shear reinforcement.
From A to B for a distance as shown in figure
For the remaining central portion of 1.88 m provide minimum shear reinforcement using 12 mm diameter 2 - legged stirrups at
Spacing , \( s = 0.87 \times 250 \times (2 \times 113) / (0.4 \times 400) = 300 \text{ mm c/c} < 0.75d \)

**Cantilever portion BD**
\( V_{\text{umax}} = 601.8 \text{ kN}, \quad V_{\text{uD}} = 601.8 - 354(0.400 / 2 + 0.680) = 290.28 \text{ kN} \)
\( \tau_v = 290280/(400 \times 680) = 1.067 \text{ MPa} < \tau_{\text{c,max.}} \)
\( A_{\text{st}} = 3217 \text{ mm}^2 \)
Therefore \( \rho_t = 100 \times 3217/(400 \times 680) = 1.18\% \)
\( \tau_c = 0.683 \text{ N/mm}^2 \quad (\text{Table IS:456-2000}) \)
τ_v > τ_c and τ_v - τ_c < 0.4. Provide minimum steel.

Using 12 mm diameter 2-legged stirrups,

Spacing = \(0.87 \times 250 \times (2 \times 113) / (0.4 \times 400) = 307.2 \text{ mm say 300 mm c/c}\)

Cantilever portion AC

Minimum shear reinforcement of \(\Phi 12\) mm diameters 2-legged stirrups at 300mm c/c will be sufficient in the cantilever portions of the beam as the shear is very less.
1.1.4 Outcome

Able to design and detailing the combined footing as per IS code provisions

1.1.5 Future Study

https://nptel.ac.in/courses/105108069/3