UNIT – IV

ASTRONOMICAL SURVEYING

Celestial sphere - Astronomical terms and definitions - Motion of sun and stars - Apparent altitude and corrections - Celestial co-ordinate systems - Different time systems –Use of Nautical almanac - Star constellations - calculations for azimuth of a line.

Celestial Sphere.

The millions of stars that we see in the sky on a clear cloudless night are all at varying distances from us. Since we are concerned with their relative distance rather than their actual distance from the observer. It is exceedingly convenient to picture the stars as distributed over the surface of an imaginary spherical sky having its center at the position of the observer. This imaginary sphere on which the star appear to lie or to be studded is known as the celestial sphere. The radius of the celestial sphere may be of any value – from a few thousand metres to a few thousand kilometers. Since the stars are very distant from us, the center of the earth may be taken as the center of the celestial sphere.

Zenith, Nadir and Celestial Horizon.

The Zenith (Z) is the point on the upper portion of the celestial sphere marked by plumb line above the observer. It is thus the point on the celestial sphere immediately above the observer's station.

The Nadir (Z') is the point on the lower portion of the celestial sphere marked by the plum line below the observer. It is thus the point on the celestial sphere vertically below the observer's station.

Celestial Horizon. (True or Rational horizon or geocentric horizon): It is the great circle traced upon the celestial sphere by that plane which is perpendicular to the Zenith-Nadir line, and which passes through the center of the earth. (Great circle is a section of a sphere when the cutting plane passes through the center of the sphere).

Terrestrial Poles and Equator, Celestial Poles and Equator.

The terrestrial poles are the two points in which the earth's axis of rotation meets the earth's sphere. The terrestrial equator is the great circle of the earth, the plane of which is at right angles to the axis of rotation. The two poles are equidistant from it.

If the earth's axis of rotation is produced indefinitely, it will meet the celestial sphere in two points called the north and south celestial poles (P and P'). The celestial equator is the great circle of the celestial sphere in which it is intersected by the plane of terrestrial equator.

Sensible Horizon and Visible Horizon.

It is a circle in which a plane passing through the point of observation and tangential to the earth's surface (or perpendicular to the Zenith-Nadir line) intersects with celestial sphere. The line of sight of an accurately leveled telescope lies in this plane.

It is the circle of contract, with the earth, of the cone of visual rays passing through the point of observation. The circle of contact is a small circle of the earth and its radius depends on the altitude of the point of observation.

Vertical Circle, Observer's Meridian and Prime Vertical?

A vertical circle of the celestial sphere is great circle passing through the Zenith and Nadir. They all cut the celestial horizon at right angles.

The Meridian of any particular point is that circle which passes through the Zenith and Nadir of the point as well as through the poles. It is thus a vertical circle.

It is that particular vertical circle which is at right angles to the meridian, and which, therefore passes through the east and west points of the horizon.

Latitude (θ) and Co-latitude (c).

Latitude (θ): It is angular distance of any place on the earth's surface north or south of the equator, and is measured on the meridian of the place. It is marked + or – (or N or S) according as the place is north or south of the equator. The latitude may also be defined as the angle between the zenith and the celestial equator.

The Co-latitude of a place is the angular distance from the zenith to the pole. It is the complement of the latitude and equal to (90°-θ).

longitude (ϕ) and altitude (α).

The longitude of a place is the angle between a fixed reference meridian called the prime of first meridian and the meridian of the place. The prime meridian universally adopted is that of Greenwich. Te longitude of any place varies between 0° and 180°, and is reckoned as Φ° east or west of Greenwich.

The altitude of celestial or heavenly body (i.e, the sun or a star) is its angular distance above the horizon, measured on the vertical circle passing through the body.

Co-altitude or Zenith Distance (z) and azimuth (A).

It is the angular distance of heavenly body from the zenith. It is the complement or the altitude, i.e $z = (90° - \alpha)$.

The azimuth of a heavenly body is the angle between the observer's meridian and the vertical circle passing through the body.

Declination (δ) and Co-declination or Polar Distance (p).

The declination of a celestial body is angular distance from the plane of the equator, measured along the star's meridian generally called the declination circle, (i.e., great circle passing through the heavenly body and the celestial pole). Declination varies from 0° to 90°, and is marked + or – according as the body is north or south of the equator.

It is the angular distance of the heavenly body from the near pole. It is the complement of the declination. i.e., $p = 90° - \delta$.

Hour Circle, Hour Angle and Right ascension (R.A).

Hour circles are great circles passing though the north and south celestial poles. The declination circle of a heavenly body is thus its hour circle.

The hour angle of a heavenly body is the angle between the observer's meridian and the declination circle passing through the body. The hour angle is always measured westwards.

Right ascension (R.A): It is the equatorial angular distance measured eastward from the First Point of Aries to the hour circle through the heavenly body.

Equinoctial Points.

The points of the intersection of the ecliptic with the equator are called the equinoctial points. The declination of the sun is zero at the equinoctial points. The Vernal Equinox or the First point of Aries (Y) is the sun's declination changes from south to north, and marks the commencement of spring. It is a fixed point of the celestial sphere. The Autumnal Equinox or the First Point of Libra (Ω) is the point in which sun's declination changes from north to south, and marks the commencement of autumn. Both the equinoctial points are six months apart in time.

ecliptic and Solstices?

Ecliptic is the great circle of the heavens which the sun appears to describe on the celestial sphere with the earth as a centre in the course of a year. The plan of the ecliptic is inclined to the plan of the equator at an angle (called the obliquity) of about 23° 27', but is subjected to a diminution of about 5" in a century.

Solstices are the points at which the north and south declination of the sun is a maximum. The point C at which the north declination of the sun is maximum is called the summer solstice; while the point C at which south declination of the sun is maximum is know as the winter solstice. The case is just the reverse in the southern hemisphere.

North, South, East and West Direction.

The north and south points correspond to the projection of the north and south poles on the horizon. The meridian line is the line in which the observer's meridian plane meets horizon place, and the north and south points are the points on the extremities of it. The direction ZP (in plan on the plane of horizon) is the direction of north, while the direction PZ is the direction of south. The east-west line is the line in which the prime vertical meets the horizon, and east and west points are the extremities of it. Since the meridian place is perpendicular to both the equatorial plan as well as horizontal place, the intersections of the equator and horizon determine the east and west points.

spherical excess and spherical Triangle?

The spherical excess of a spherical triangle is the value by which the sum of three angles of the triangle exceeds 180°.

Thus, spherical excess E = (A + B + C - 180°)

A spherical triangle is that triangle which is formed upon the surface of the sphere by intersection of three arcs of great circles and the angles formed by the arcs at the vertices of the triangle are called the spherical angles of the triangle.

Properties of a spherical triangle.

The following are the properties of a spherical triangle:

1. Any angle is less than two right angles or π.
2. The sum of the three angles is less than six right angles or 3π and greater than two right angles or π.
3. The sum of any two sides is greater than the third.
4. If the sum of any two sides is equal to two right angles or π, the sum of the angles opposite them is equal to two right angles or π.
5. The smaller angle is opposite the smaller side, and vice versa.

formulae involved in Spherical Trigonometry?

The six quantities involved in a spherical triangle are three angles A, B and C and the three sides a, b and c. Out of these, if three quantities are known, the other three can very easily be computed by the use of the following formulae in spherical trigonometry:

1. Sine formula: $\dfrac{\sin a}{\sin A} = \dfrac{\sin b}{\sin B} = \dfrac{\sin c}{\sin C}$

2. Cosine formula: $\cos A = \dfrac{\cos a - \cos b \cos c}{\sin b \sin c}$

Or $\cos a = \cos b \cos c + \sin b \sin c \cos A$

Also, $\cos A = - \cos B \cos C + \sin B \sin C \cos a$

systems used for measuring time?

There are the following systems used for measuring time:

1. Sidereal Time
2. Solar Apparent Time
3. Mean Solar Time
4. Standard Time

terrestrial latitude and longitude.

In order to mark the position of a point on the earth's surface, it is necessary to use a system of co-ordinates. The terrestrial latitudes and longitudes are used for this purpose.

The terrestrial meridian is any great circle whose plane passes through the axis of the earth (i.e., through the north and south poles). Terrestrial equator is great circle whose plane is perpendicular to the earth's axis. The latitude θ of a place is the angle subtended at the centre of the earth north by the are of meridian intercepted between the place and the equator.

The latitude is north or positive when measured above the equator, and is south or negative when measured below the equator. The latitude of a point upon the equator is thus 0°, while at the North and South Poles, it is 90° N and 90° S latitude respectively. The co-latitude is the complement of the latitude, and is the distance between the point and pole measured along the meridian.

The longitude (ϕ) of a place is the angle made by its meridian plane with some fixed meridian plane arbitrarily chosen, and is measured by the arc of equator intercepted between these two meridians. The prime meridian universally adopted is that of Greenwich. The longitude of any place varies between 0° to 180°, and is reckoned as ϕ° east or west of Greenwich. All the points on meridian have the same longitude.

Spherical Triangle? & its properties.

A spherical triangle is that triangle which is formed upon the surface of the sphere by intersection of three arcs of great circles and the angles formed by the arcs at the vertices of the triangle are called the spherical angles of the triangle.

AB, BC and CA are the three arcs of great circles and intersect each other at A, B and C. It is usual to denote the angles by A, B and C and the sides respectively opposite to them, as a, b and c. The sides of spherical triangle are proportional to the angle subtended by them at the centre of the sphere and are, therefore, expressed in angular measure. Thus, by sin b we mean the sine of the angle subtended at the centre by the arc AC. A spherical angle is an angle between two great circles, and is defined by the plane angle between the tangents to the circles at their point of intersection. Thus, the spherical angle at A is measured by the plane angle A1AA2 between the tangents AA1 and AA2 to the great circles AB and AC.

Properties of a spherical triangle

The following are the properties of a spherical triangle:

1. Any angle is less than two right angles or π.
2. The sum of the three angles is less than six right angles or 3π and greater than two right angles or π.
3. The sum of any two sides is greater than the third.
4. If the sum of any two sides is equal to two right angles or π, the sum of the angles opposite them is equal to two right angles or π.
5. The smaller angle is opposite the smaller side, and vice versa.

Formulae in Spherical Trigonometry

The six quantities involved in a spherical triangle are three angles A, B and C and the three sides a, b and c. Out of these, if three quantities are known, the other three can very easily be computed by the use of the following formulae in spherical trigonometry:

1. since formula : $\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$

2. Cosine formula :cos A = $\frac{\cos a - \cos b \cos c}{\sin b \sin c}$

or cos a = cos b cos c + sin b sin c cos A

Also, $\cos A = -\cos B \cos C + \sin B \sin C \cos a$

The Spherical Excess

The spherical excess of a spherical triangle is the value by which the sum of three angles of the triangle exceeds 180°.

Thus, spherical excess E = (A + B + C - 180°)

Also, $$\tan^2 \frac{1}{2} E = \tan \frac{1}{2} s \tan \frac{1}{2}(s-a) \tan \frac{1}{2}(s-b) \tan \frac{1}{2}(s-c)$$

In geodetic work the spherical triangles on the earth's surface are comparatively small and the spherical excess seldom exceeds more than a few seconds of arc. The spherical excess, in such case, can be expressed by the approximate formula

$$E = \frac{\Delta}{R^2 \sin 1''} \text{ seconds}$$

where R is the radius of the earth and Δ is the area of triangle expressed in the same linear units as R.

the relationship between co-ordinates?

1. **The Relation between Altitude of the Pole and Latitude of the Observer.**

In the sketch, H-H is the horizon plane and E-E is the equatorial plane. O is the centre of the earth. ZO is perpendicular to HH while OP is perpendicular to EE.

Now latitude of place = $\theta = \angle EOZ$

And altitude of pole = $\alpha = \angle HOP$

$$\angle EOP = 90° = \angle EOZ + \angle ZOP$$
$$= \theta + \angle ZOP \quad \text{.... (i)}$$
$$\angle HOZ = 90° = \angle HOP + \angle POZ$$
$$= \alpha + \angle POZ \quad \text{.... (ii)}$$

Equating the two, we get

$\theta + \angle ZOP = \alpha + \angle POZ$ or $\theta = \alpha$

Hence the altitude of the pole is always equal to the latitude of the observer.

2. The Relation between Latitude of Observer and the Declination and Altitude of a Point on the Meridian.

For star M1, EM1 = δ = declination.

SM1 = α = meridian altitude of star.

M1Z = z = meridian zenith distance of star.

EZ = θ = latitude of the observer.

Evidently, EZ = EM1 + M1Z

Or $\theta = \delta + z$ …. (1)

The above equation covers all cases. If the star is below the equator, negative sign should be given to δ. If the star is to the north of zenith, negative sign should be given to z.

If the star is north of the zenith but above the pole, as at M2, we have

ZP = Z M2 + M2 P

or $(90° - \theta) = (90° - \alpha) + p$, where p = polar distance = M2 P

or $\theta = \alpha - p$ …. (2)

Similarly, if the star is north of the zenith but below the pole, as at M3, we have

ZM3 = ZP + PM3

$(90° - \alpha) = (90° - \theta) + p$, where p = polar distance = M3 P

$\theta = \alpha + p$ …. (3)

The above relations form the basis for the usual observation for latitude.

3. The Relation between Right Ascension and Hour Angle.

Fig 1.22. shows the plan of the stellar sphere on the plane of the equator. M is the position of the star and ∠SPM is its westerly hour angle. HM. Y is the position of the First Point of Aries and angle SPY is its westerly hour angle. ∠YPM is the rit ascension of the star. Evidently, we have

∴ Hour angle of Equinox = Hour angle of star + R.A. of star.

Find the difference of longitude between two places A and B from their following longitudes :]

(1) Longitude of A = 40° W

Longitude of B = 73° W

(2) Long. Of A = 20° E

Long. Of B = 150° E

(3) Longitude of A = 20° W

Longitude of B = 50° W

Solution.

(1) The difference of longitude between A and B = 73° - 40° = **33°**

(2) The difference of longitude between A and B = 150° - 20° = **130°**

(3) The difference of longitude between A and B = 20° - (- 50°) = **70°**

(4) The difference of longitude between A and B = 40° - (- 150°) = **190°**

Since it is greater than 180°, it represents the obtuse angular difference. The acute angular difference of longitude between A and B, therefore, is equal to

360° - 190° = **170°.**

Calculate the distance in kilometers between two points A and B along the parallel of latitude, given that

(1) Lat. Of A, 28° 42' N : longitude of A, 31° 12' W

Lat. Of B, 28° 42' N : longitude of B, 47° 24' W

(2) Lat. Of A, 12° 36' S : longitude of A, 115° 6' W

Lat. Of B, 12° 36' S : longitude of B, 150° 24' E.

Solution.

The distance in nautical miles between A and B along the parallel of latitude = difference of longitude in minutes x cos latitude.

(1) Difference of longitude between A and B = 47° 24' – 31° 12' =

16° 12' = 972 minutes

∴ Distance = 972 cos 28° 42' = 851.72 nautical miles

= 851.72 x 1.852 = **1577.34 km.**

(2) Difference of longitude between A and B

= 360° - { 115° 6' – (- 150° 24') } = 94° 30' = 5670 min.

∴ Distance = 5670 cos 12° 36’ = 5533.45 nautical miles

= 5533.45 x 1.852 = **10,247.2 km.**

Find the shortest distance between two places A and B, given that the longitudes of A and B are 15° 0’ N and 12° 6’ N and their longitudes are 50° 12’ E and 54° 0’ E respectively. Find also the direction of B on the great circle route.

Radius of earth = 6370 km.

Solution.

The positions of A and B have been shown.

In the spherical triangle ABP,

B = AP = 90° - latitude of A

= 90° - 15° 0’ = 75°

A = BP = 90° - latitude of B

= 90° - 12° 6’ = 77° 54’

P = ∠A P B = difference of longitude

= 54° 0’ – 50° 12’ = 3° 48’.

The shortest distance between two points is the distance along the great circle passing through the two points.

Knowing the two sides one angle, the third side AB (=p) can be easily computed by the cosine rule.

Thus cos P = $\dfrac{\cos p - \cos a \cos b}{\sin a \sin b}$

or cos p = cos P sin a sin b + cos a cos b

= cos 3° 48’ sin 77° 54’ sin 75° + cos 77° 54’ cos 75°

= 0.94236 + 0.05425 = 0.99661

∴ p = AB = 4° 40’ = 4°.7

Now, arc ≈ radius x central angle = $\dfrac{6370x4^{\circ}.7x\pi}{180^{\circ}} = 522.54km.$

Hence distance AB = **522.54 km.**

Direction of A from B :

The direction of A from B is the angle B, and the direction of B from A is the angle A.

Angles A and B can be found by the tangent semi-sum and semi-difference formulae

Thus $$\tan \tfrac{1}{2}(A+B) = \frac{\cos\frac{1}{2}(a-b)}{\cos\frac{1}{2}(a+b)}\cot\frac{1}{2}p$$

And $$\tan \tfrac{1}{2}(A-B) = \frac{\sin\frac{1}{2}(a-b)}{\sin\frac{1}{2}(a+b)}\cot\frac{1}{2}p$$

Here $$\frac{(a-b)}{2} = \frac{77°54'-75°}{2} = \frac{2°54'}{2} = 1°27'$$

$$\frac{(a+b)}{2} = \frac{77°54'+75°}{2} = \frac{152°54'}{2} = 76°27'; \frac{p}{2} = \frac{3°48'}{2} = 1°54'$$

$\therefore$ $$\tan\frac{1}{2}(A+B) = \frac{\cos 1°27'}{\cos 76°27'}\cot 1°54$$

From which, $$\frac{A+B}{2} = 89°35' \qquad \text{.... (i)}$$

and $$\tan\frac{1}{2}(A-B) = \frac{\sin 1°27'}{\sin 76°27'}\cot 1°54'$$

From which, $$\frac{A-B}{2} = 38°6' \qquad \text{.... (ii)}$$

$\therefore$ Direction of B from A = angle A = 89°35’ + 38°6’ = 127°41’ = **S 52°19’ E**

$\therefore$ Direction of A from B = angle B = 89°35’ + 38°6’ = 51°29’ = **N 51°29’ W.**

Determine the hour angle and declination of a star from the following data :

(i)	**Altitude of the star**	**= 22° 36’**
(ii)	**Azimuth of the star**	**= 42° W**
(iii)	**Latitude of the place of observation**	**= 40° N.**

Solution.

Since the azimuth of the star is 42° W, the star is in the western hemi-sphere.

In the astronomical Δ PZM, we have

PZ = co-latitude = 90° - 40° = 50° ;

ZM = co-altitude = 90° - 22° 36° = 67 24’ ;

angle A = 42°

Knowing the two sides and the included angle, the third side can be calculated from the cosine formula

Thus, cos PM = cos PZ . cos ZM + sin PZ. Sin ZM. cos A

= cos 50°. cos 67° 24’ + sin 50°. sin 67° 24’. cos 42°

= 0.24702 + 0.52556 = 0.77258

∴ PM = 39° 25’

∴ Declination of the star = δ = 90° - PM = 90° - 39° 25’ = **50° 35’ N.**

Similarly, knowing all the three sides, the hour angle H can be calculated from Eq. 1.2

$$\cos H = \frac{\cos ZM - \cos PZ.\cos PM}{\sin PZ.\sin PM} = \frac{\cos 67°24' - \cos 50°.\cos 39°25'}{\sin 50°.\sin 39°25'}$$

$$= \quad = \frac{0.38430 - 0.49659}{0.48640} = -0.23086$$

∴ cos (180° - H) = 0.23086 ∴ 180° - H = 76° 39’

H = 103° 21’.

astronomical parameters of the earth and the sun.

The Earth:

The Earth is considered approximately spherical in shape. But actually it is very approximately an oblate spheroid. Oblate spheroid is the figure formed by revolving an ellipse about its minor axis. The earth is flattened at poles – its diameter along the polar axis being lesser than its diameter at the equator. The equatorial radius a of the earth, according to Hayford's spheroid is 6378.388 km and the polar radius b of the earth is 6356.912 km. The ellipticity is expressed by the ratio $\frac{a-b}{a}$, which reduces to $\frac{1}{297}$. For the Survey of India; Everest's first constants were used as follows:

a = 20,922,932 ft and b = 20,853,642 ft, the ellipticity being $\frac{1}{311.04}$.

The earth revolves about its minor or shorter axis (i.e. polar axis), on an average, once in twenty-four hours, from West to East. If the earth is considered stationary, the whole celestial sphere along with its celestial bodies like the stars, sum, moon etc. appear to revolve round the earth from East to West. The axis of rotation of earth is known as the polar axis, and the points at which it intersects the surface of earth are termed the North and South Geographical or Terrestrial Poles. In addition to the motion of rotation about its own polar axis, the earth has a motion of rotation relative to the sun, in a plane inclined at an angle of 23° 27' to the plane of the equator. The time of a complete revolution round the sun is one year. The apparent path of the sun in the heavens is the result of both the diurnal and annual real motions of the earth.

The earth has been divided into certain zones depending upon the parallels of latitude of certain value above and below the equator. The zone between the parallels of latitude 23° 27 ½ ' N and 23° 27 ½ ' S is known as the torrid zone (see Fig. 1.12). This is the hottest portion of the earth's surface. The belt included between 23° 27 ½ ' N and 66° 32 ½ ' N of equator is called the north temperate zone. Similarly, the belt included between 23° 27 ½ ' S and 66° 32 ½ ' S is called south temperate zone. The belt between 66° 32 ½ ' N and the north pole is called the north frigid zone and the belt between 66° 32 ½ ' S and the south pole is called south frigid zone.

The sun:

The sun is at a distance of 93,005,000 miles from the earth. The distance is only about $\frac{1}{250,000}$ of that of the nearest star. The diameter of the sun is about 109 times the diameter of the earth, and subtends and angle of 31' 59" at the centre of the earth. The mass of the sun is about 332,000 times that of the earth. The temperature at the centre of the sun is computed to be about 20 million degrees.

The sun has twp apparent motions, one with respect to the earth from east to west, and the other with respect to the fixed stars in the celestial sphere. The former apparent path of the sun is in the plane which passes through the centre of the celestial sphere and intersects it in a great circle called the ecliptic. The apparent motion of the sun is along this great circle. The angle between the plane of equator and the ecliptic is called the Obliquity of Ecliptic, its value being 23° 27'. The obliquity of ecliptic changes with a mean annual diminution of 0'.47.

The points of the intersection of the ecliptic with the equator are called the equinoctial points, the declination of the sun being zero at these points. The Vernal Equinox or the First point of Aries (Y) is the point in which the sun's declination changes from south to north. Autumnal Equinox or the First point of Libra ($\underline{\Omega}$) is the point in which the sun's declination changes from north to south. The points at which sun's declinations are a maximum are called solstices. The point at which the north declination of sun is maximum is called the summer solstice, while the point at which the south declination of the sun is maximum is known as the winter solstice.

The earth moves eastward around the sun once in a year in a pat that is very nearly a huge circle with a radius of about 93 millions of miles. More accurately, the path is described as an ellipse, one focus of the ellipse being occupied by the sun.

Various measurements of time.

Due to the intimate relationship with hour angle, right ascension and longitude, the knowledge of measurement of time is most essential. The measurement of time is based upon the apparent motion of heavenly bodies caused by earth's rotation on its axis. Time is the interval which lapses, between any two instants. In the subsequent pages, we shall use the following abbreviations.

G.M.T.	… Greenwhich Mean Time	G.M.M.	… Greenwhich Mean Midnight
G.A.T.	… Greenwich Apparent Time	L.A.N.	… Local Apparent Noon
G.S.T.	… Greenwich Sidereal Time	L.M.M.	… Local Mean Midnight
L.M.T.	… Local Mean Time	L.Std.T.	… Local Standard Time
L.A.T.	… Local Apparent Time	N.A.	… Nautical Almanac
L.S.T.	… Local Sidereal Time	S.A.	… Star Almanac
G.M.N.	… Greenwich Mean Noon		

The units of time.

There are the following systems used for measuring time :

1.	Sidereal Time	2.	Solar Apparent Time
2.	Mean Solar Time	4.	Standard Time

Sidereal Time:

Since the earth rotates on its axis from west to east, all heavenly bodies (i.e. the sun and the fixed stars) appear to revolve from east to west (i.e. in clock-wise direction) around the earth. Such motion of the heavenly bodies is known as apparent motion. We may consider the earth to turn on it axis with absolute regular speed. Due to this, the stars appear to complete one revolution round the celestial pole as centre in constant interval of time, and they cross the observer's meridian twice each day. For astronomical purposes the sidereal day is one of the principal units of time. The sidereal day is the interval of time between two successive upper transits of the first point of Aries (Y). It begins at the instant when the first point of Aries records 0h, 0m, 0s. At any other instant, the sidereal time will be the hour angle of Y reckoned westward from 0h to 24h. The sidereal day is divided into 24 hours, each hour subdivided into 60 minutes and each minute into 60 seconds. However, the position of the Vernal Equinox is not fixed. It has slow (and variable) westward motion caused by the precessional movement of the axis, the actual interval between two transits of the equinox differs about 0.01 second of time from the true time of one rotation.

Local Sidereal Time (L.S.T.):

The local sidereal time is the time interval which has elapsed since the transit of the first point of Aries over the meridian of the place. It is, therefore, a measure of the angle through which the earth has rotated since the equinox was on the meridian. The local sidereal time is, thus, equal to the right ascension of the observer's meridian.

Since the sidereal time is the hour angle of the first point of Aries, the hour angle of a star is the sidereal time that has elapsed since its transit. M1 is the position of a star having SPM1 (= H) as its hour angle measured westward and YPM1 is its right ascension (R.A.) measured eastward. SPY is the hour angle of Y and hence the local sidereal time.

Hence, we have SPM1 + M1PY = SPY

or **star's hour angle + star's right ascension = local sidereal time** … (1)

If this sum is greater than 24 hours, deduct 24 hours, while if it is negative add, 24 hours.

The star M2 is in the other position. Y PM2 is its Right Ascension (eastward) and ZPM2 is its hour angle (westward). Evidently,

ZPM2 (exterior) + YPM2 – 24h = SPY = L.S.T.

or **star's hour angle + star's right ascension – 24h = L.S.T**

This supports the preposition proved with reference to Fig. 1.30 (a). The relationship is true for all positions of the star.

When the star is on the meridian, its hour angle is zero. Hence equation 1 reduces to

Star's right ascension = local sidereal time at its transit.

A sidereal clock, therefore, records the right ascension of stars as they make their upper transits.

The hour angle and the right ascension are generally measured in time in preference to angular units. Since one complete rotation of celestial sphere through 360° occupies 24 hours, we have

$$24 \text{ hours} = 360° \quad ; \quad 1 \text{ hour} = 15°$$

The difference between the local sidereal times of two places is evidently equal to the difference in their longitudes.

Solar Apparent Time:

Since a man regulates his time with the recurrence of light and darkness due to rising and setting of the sun, the sidereal division of time is not suited to the needs of every day life, for the purposes of which the sun is the most convenient time measurer. A solar day is the interval of time that elapes between two successive lower transits of the sun's centers over the meridian of the place. The lower transit is chosen in order that the date may change at midnight. The solar time at any instant is the hour angle of the sun's centre reckoned westward from 0h to 24h. This is called the apparent solar time, and is the time indicated by a sun-dial. Unfortunately, the apparent solar day is not of constant length throughout the year but changes. Hence our modern clocks and chronometers cannot be used to give us the apparent solar time. The non-uniform length of the day is due to two reasons :

(1) The orbit of the earth round the sun is not circular but elliptical with sun at one of its foci. The distance of the earth from the sun is thus variable. In accordance with the law of gravitation, the apparent angular motion of the sun is not uniform – it moves faster when is nearer to the earth and slower when away. Due to this, the sun reaches the meridian sometimes earlier and sometimes later with the result that the days are of different lengths at different seasons.

(2) The apparent diurnal path of the sun lies in the ecliptic. Due to this, even though the eastward progress of the sun in the ecliptic were uniform, the time elapsing between the departure of a meridian from the sun and its return thereto would vary because of the obliquity of the ecliptic.

The sun changes its right ascension from 0h to 24h in one year, advancing eastward among the stars at the rate of about 1° a day. Due to this, the earth will have to turn nearly 361° about its axis to complete one solar day, which will consequently be about minutes longer than a sidereal day. Both the obliquity of the ecliptic and the sun's unequal motion cause a variable rate of increase of the sun's right ascension. If the rate of change of the sun's right ascension were uniform, the solar day would be of constant length throughout the year.

Mean Solar Time :

Since our modern clocks and chronometers cannot record the variable apparent solar time, a fictitious sun called the mean sun is imagined to move at a uniform rate along the equator. The motion of the mean sun is the average of that of the true sun in its right ascension. It is supposed to start from the vernal equinox at the same time as the true sun and to return the vernal equinox with the true sun. The mean solar day may be defied as the interval between successive transit of the mean sun. The mean solar day is the average of all the apparent solar days of the year. The mean sun has the constant rate of increase of right ascension which is the average rate of increase of the true sun's right ascension.

The local mean noon (L.M.N.) is the instant when the mean sun is on the meridian. The mean time at any other instant is given by the hour angle of the mean sun reckoned westward from 0 to 24 hours. For civil purposes, however, it is found more convenient to begin the day at midnight and complete it at the next midnight, dividing it into two periods of 12 hours each. Thus, the zero hour of the mean day is at the local mean midnight (L.M.M.). The local mean time (L.M.T.) is that reckoned from the local mean midnight. The difference between the local mean time between two places is evidently equal to the difference in the longitudes.

From Fig. 1.30 (a) if M1 is the position of the sun, we have

Local sidereal time = R.A. of the sun + hour angle of the sun … (1)

Similarly,

Local sidereal time = R.A. of the mean sun + hour angle of the mean sun … (2)

The hour angle of the sun is zero at its upper transit. Hence

Local sidereal time of apparent noon = R.A. of the sun … (3)

Local sidereal time of mean noon = R.A. of the mean sun … (4)

Again, since the our angle of the sun (true or mean) is zero at its upper transit while the solar time (apparent or mean) is zero as its lower transit, we have

The apparent solar time = the hour angle of the sun + 12h … (5)

The mean solar time = the hour angle of mean sun + 12h … (6)

Thus, if the hour angle of the mean sun is 15° (1 hour) the mean time is 12 + 1 = 13 hours, which is the same thing as 1 o’clock mean time in the afternoon; if the hour angle of the mean sun is 195° (13 hours), the mean time is 12 + 13 =25 hours, which is the same as 1 o’clock mean time after the midnight (i.e., next. Day).

The Equation of Time

The difference between the mean and the apparent solar time at any instant is known as the equation of time. Since the mean sun is entirely a fictitious body, there is no means to directly observe its progress. Formerly, the apparent time was determined by solar observations and was reduced to mean time by equation of time. Now-a-days, however, mean time is obtained more easily by first determining the sidereal time by steller observations and then converting it to mean time through the medium of wireless signals. Due to this reason it is more convenient to regard the equation of time as the correction that must be applied to mean time to obtain apparent time. The nautical almanac tabulates the value of the equation of time for every day in the year, in this sense (i.e. apparent – mean). Thus, we have

Equation of time = Apparent solar time – Mean solar time

The equation of time is positive when the apparent solar time is more than the mean solar time ; to get the apparent solar time, the equation of time should then be added to mean solar time. For example, at 0h G.M.T. on 15 October 1949, the equation of the time is + 13m 12s. This means that the apparent time at 0h mean time is 0h 13m 12s. In other words, the true sun is 13m 12s ahead of the mean sun. Similarly, the equation of time is negative when the apparent time is less than the mean time. For example, at 0h G.M.T. on 18 Jan., 1949, the equation of time is – 10m 47s. This means that the apparent time at 0h mean time will be 23h 49m 13s on January 17. In other words, the true sun is behind the mean sun at that time.

The value of the equation of time varies in magnitude throughout the year and its value is given in the Nautical Almanac at the instant of apparent midnight for the places on the meridian of Greenwich for each day of the year. For any other time it must be found by adding or subtracting the amount by which the equation has increased or diminished since midnight.

It is obvious that the equation of time is the value expressed in time, of the difference at any instant between the respective hour angles or right ascensions of the true and mean suns.

The amount of equation of the time and its variations are due to two reasons :

(1) obliquity of the ecliptic, and (2) elasticity of the orbit. We shall discuss both the effects separately and then combine them to get the equation of time.

Explain the conversion of local time to standard time and vice versa.

The difference between the standard time and the local mean time at a place is equal to the difference of longitudes between the place and the standard meridian.

If the meridian of the place is situated east of the standard meridian, the sun, while moving apparently from east to west, will transit the meridian of the place earlier than the standard meridian. Hence the local time will be greater than the standard time. Similarly, if the meridian of the place is to the west of the standard meridian, the sun will transit the standard meridian earlier than the meridian of the place and hence the local time will be lesser than the standard time. Thus, we have

L.M.T = Standard M.T ± Difference in the longitudes $\left(\frac{E}{W}\right)$

L.A.T = Standard A.T ± Difference in the longitudes $\left(\frac{E}{W}\right)$

L.S.T = Standard S.T ± Difference in the longitudes $\left(\frac{E}{W}\right)$

Use (+) sign if the meridian of place is to the east of the standard meridian, and (-) Sign if it to the west of the standard meridian.

If the local time is to be found from the given Greenwich time, we have

L.M.T = Standard M.T ± Difference in the longitudes $\left(\frac{E}{W}\right)$

The standard time meridian in India is 82° 30' E. If the standard time at any instant is 20 hours 24 minutes 6 seconds, find the local mean time for two places having longtitudes (a) 20° E, (b) 20° W.

Solution:

(a) The longitude of the place = 20° E

Longtitude of the standard meridian = 82° 30'E

∴ Difference in the longitudes = 82° 30' – 20° = 62° 30', E. the place being to the west of the standard meridian.

Now 62° of longitude = $\frac{62}{15}$ h = $4^h\, 8^m\, 0^s$

Now 30' of longitude = $\frac{30}{15}$ m = $0^h\, 2^m\, 0^s$

Total = $4^h\, 10^m\, 0^s$

Now L.M.T = Standard time – Difference in longitude (W)

= $20^h\, 24^m\, 6^{s}$ - -