Analysis of Determinate Structures

Module 1

Introduction and analysis of Structures

Trusses

Forms of structure

Structure refers to a system of connected parts that can support loads while performing its primary functions.

A structural system is composed of structural members joined together by structural connections. Each structural system may be composed of one or more of the following four basic types of structures:

1. Frame Structure
2. Surface Structure
3. Trusses
4. Cable or Arches

1. Frame Structure:

* Frames are commonly used in building structures
* Frames are composed of beams and columns that are connected together.
* Steel frames and concrete frames are
the most commonly used structures in buildings

* There are two types of frames
  1. planar frame or plane frame -
     All members are in one plane.
  2. space frame: All the members exist in more than one plane.

2. Surface structures:
* membranes, plates or shell type of structures with less thickness as compared to it's other dimension form the surface structure.
* This kind of structure exists in a single plane.
* surface structures may be made of rigid materials such as reinforced concrete.

3. Trusses:
* Trusses consists of slender members arranged in a triangular pattern.
* All the members are connected together by pins which are free to rotate.
* Loads that cause the entire truss to bend are converted into axial, tensile and
compressive forces in the members. There are two types of trusses:

1. plane truss
   - Examples: Bridges and roofs
2. space truss
   - Examples: Stadium, transmission line tower
3. cables and Arches:
   - Cables and Arch type of structures are used to span long distances.
   - Cables are usually flexible and carry their loads in tension.
   - The external load is usually applied vertically (not along the axis of the cable).
   - As a result, the cable deforms with a sag.
4. Cables are commonly used to support bridges and building roofs.
5. Cables have an advantage over beams and trusses, especially for span greater than 150 feet.
6. An Arch has a reverse curvature of cable and it achieves its strength in compression.
7. An arch must be rigid in order to maintain its shape.
The degree of freedom of a hinged support is equal to one since it is only allowed to or free to rotate in any direction. \( \text{DOF} = 1 \)

2. Roller and Simply Supported:

The degree of freedom for roller and simply supported is equal to two since it is free to move in one direction.

4. Free end - At this point in the beam the degree of freedom is equal to free since it is allowed to move or rotate in any direction.
Degree of indeterminacy:

The number of equations required over an above the equations of static equilibrium for the analysis of a structure is known as the degree of indeterminacy.

It is also termed as degree of redundancy.

For example:

For a beam as above, the number of reactions is equal to three. Therefore, the degree of indeterminacy is equal to number of reactions minus the equations of equilibrium that is equal to number of reactions is equal to zero.

\[3 - 3 = 0\]

Number of reactions = 6
Number of equations = 3

\[\therefore \text{degree of indeterminacy} = 3\]
Determinate and indeterminate structures *
* Structures are grouped into statically determinate and statically indeterminate structures.

**Statically determinate structure**: The structure which can be analysed that is the support reaction, bending moment and shear forces can be determined with the equations of static equilibrium only are known as statically determinate beams.

For ex: - Simply supported beam, cantilever beam, one-end hinged and the other end roller supported beam.

**Statically indeterminate structure**: The structure which cannot be analysed that is if the support reactions, bending moment and shear forces cannot be determined with the equation of static equilibrium are known as statically indeterminate structure.
To analyse statically indeterminate structure, one has to make use of compactibility conditions and has to determine various deformations.

**Linear and Non-Linear Structures**

**Linear Structure:** If the material has linear stress-strain relationship in a structure [only small deformation is allowed], then it is called a linear structure.

**Non-Linear Structure:** If the material does not linear stress-strain relationships or if the deformation is so large that the change of geometry cannot be neglected in the analysis of the structure, then that kind of structure is known as a non-linear structure or system.

There are two types of non-linear systems:

1. Material non-linearity:

   Due to stress-strain relationships of the material.

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2 Geometric Non-linearity: Due to considerable change in geometry, kinematically indeterminate structures.

The kinematic indeterminacy is the minimum number of independent displacement quantities that you need to define to be able to get the complete displaced geometry of a structure. It is also referred to as the degree of freedom of the system.

In the Simply Supported beam as shown above, the static or the degree of static indeterminacy is determined using the number of reactions. Therefore degree of static indeterminacy is equal to number of reactions minus number of equilibrium equations.

\[ \text{Degree of static indeterminacy} = 3 - 3 \]
But to determine the degree of kinematic indeterminacy, the degree of freedom of the system has to be determined.

At Joint A,

\[ \text{DOF} = 1 \quad \Rightarrow \quad \text{A} \]

At Joint B,

\[ \text{DOF} = 2 \quad \Rightarrow \quad \text{B} \]

\[ \therefore \text{Total DOF} = 1 + 2 = 3 \]

And in line with the definition, the degree of kinematic indeterminacy is:

\[ \text{DOF} = 3 \]

\[ \therefore \text{Even though the beam is statically determinate, it has the kinematic indeterminacy to the degree of 3.} \]

Analysis of trusses

A truss is a very common structure used in constructing bridges, building roofs, towers, and lateral bracings of high rise buildings.
Basic assumptions of truss analysis:

* Members are connected at their joints by smooth frictionless pin.

* All members function has two force members i.e. they are subjected to either tension or compression.

* The centroidal axis of each member is straight and coincides with a line connecting the joints centres at the end of the member.

* Loads and reactions are applied to the truss at joints only.

Before analysing a truss we need to find out the statical determinancy of the structure both internally and externally.

To find the static determinancy of the truss externally, we first find the number of reactions at the supports we are to be found out and use the equations of equilibrium to find out the degree of indeterminacy. To find out whether the
Structure is internally determinate, the number of members, joints and equations of equilibrium are taken into consideration.

The degree of static indeterminacy of the truss can be found out using the formula \( m - 2j + r \) where

'\( m \)' is the number of members

'\( j \)' is the number of joints

'\( r \)' is the number of equilibrium equations

For example

\[
\begin{align*}
\text{Number of reactions} &= 3 \\
\text{Number of equations} &= 3 \\
\text{of equilibrium} &= 3 - 3 = 0
\end{align*}
\]

is the d
To find static internal indeterminacy of the structure.

Number of members = 7
Number of joints = 5
Number of reactions = 3

\[ m - 2j + r \]
\[ 7 - 2(5) + 3 \]
\[ = 0 \]

Method of Joints:

In the method of joints, each joint is isolated as a free body and the two equations of equilibrium:

\[ \Sigma H = 0 \quad \text{and} \quad \Sigma V = 0 \]

are applied at each joint sequentially.

At each joint there should not be more than two unknown forces. In the method of joints, the forces in the members are initially assumed to be in tension. If the force is later found as a negative value, then the member is compression.
Problem:

1. Find the forces in all the members of the warren truss as shown in the figure.

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Step 1 - Degree of Indeterminacy

(i) External
Reactions = 3
No. of equations = 3

Degree of Static External Indeterminacy
= 3 - 3 = 0

The given truss is statically externally determinate.

(ii) Internal
m = 11, \( j = 7 \), \( r = 3 \)

\[ m - 2j + r = 0 \]
\[ 11 - 2(7) + 3 = 11 - 14 + 3 = 0 \]

\[ \therefore \text{ The structure is internally determinate.} \]
```
Step (2): Find the reactions at the supports.

$$\Sigma H = 0$$

$$H_A = 0$$

$$\Sigma V = 0$$

$$\Rightarrow V_A + V_B - 30 - 60 = 0$$

$$V_A + V_B = 90$$

$$\Rightarrow \Sigma m_A = 0$$

$$(- V_B \times 12) + (60 \times 8) + (30 \times 4) = 0$$

$$-12V_B + 480 + 120 = 0$$

$$-12V_B = -600$$

$$V_B = 50 \text{ KN}$$

$$V_A + V_B = 90$$

$$V_A + 50 = 90$$

$$V_A = 40 \text{ KN}$$
Step (3): Find the forces in the members

Joint 1

- \( H_A = 0 \)
- \( V_A = 40 \text{ kN} \)

From the figure:

\[
\cos \theta = \frac{2m}{4m} = 0.5
\]

\( \theta = 60^\circ \)

\[\Sigma H = 0 \]

\[\Rightarrow H_A + F_{13} + F_{12} \cos 60^\circ = 0 \]

\[\Rightarrow F_{13} + 0.5 F_{12} = 0 \quad \text{(1)} \]

\[\Sigma V = 0 \]

\[\Rightarrow V_A + F_{12} \sin 60^\circ = 0 \]

\[\Rightarrow 40 + 0.866 F_{12} = 0 \]

\[\Rightarrow F_{12} = \frac{-40}{0.866} \]

\[F_{12} = -46.189 \text{ kN} \]

\( F_{12} \) is a compression member.

From eqn (1):

\[F_{13} + 0.5 F_{12} = 0 \]

\[F_{13} + 0.5(-46.189) = 0 \]

\[F_{13} - 23.094 = 0 \]

\[F_{13} = 23.09 \text{ kN} \]
Joint 2

\[ F_{24} = \frac{F_{12}}{\sin 60^\circ} = \frac{F_{23}}{\sin 60^\circ} = \frac{F_3}{\sin \beta} = \frac{F_3}{\sin \gamma} = k \]

\[ \frac{F_{24}}{\sin 60^\circ} = \frac{F_{23}}{\sin (180+60)} = \frac{F_{12}}{8 \cdot \sin 60} \]

\[ \frac{F_{24}}{0.8660} = \frac{F_{23}}{-0.8660} = \frac{F_{12}}{6.8660} \]

\[ F_{24} = \frac{-46.189}{0.8660} = -53.033 KN \]

\[ F_{23} = \frac{-46.189}{0.8660} = 53.033 KN \]
At joint 3

\[ \Sigma H = 0 \]

\[ F_{35} + F_{34} \cos 60^\circ - F_{32} \cos 60^\circ = 0 \]

\[ F_{35} + 0.5 F_{34} = 46.189 \]

\[ \Sigma V = 0 \]

\[ 46.1189 \sin 60^\circ + F_{34} \sin 60^\circ - 30 = 0 \]

\[ F_{34} = -11.548 \text{ kN} \]

\[ F_{35} + 0.5(-11.548) = 46.189 \]

\[ F_{35} = 51.963 \text{ kN} \]

Joint 4
\[ F_{42} = -46.189 \text{ kN} \]
\[ F_{43} = -11.548 \text{ kN} \]

\[ F_{46} + F_{45} \cos 60^\circ = (-11.548 \cos 60^\circ) - (-46.189) = 0 \]

\[ F_{46} + F_{45} \cos 60^\circ = -51.963 \]

\[ \sum V = 0 \]

\[ - (-11.548 \sin 60^\circ) - (F_{45} \sin 60^\circ) = 0 \]

\[ F_{45} = 11.548 \text{ kN} \]

\[ F_{46} + 11.548 \cos 60^\circ = -51.963 \]

\[ F_{46} = -57.73 \text{ kN} \]

\[ \text{Joint-5} \]

\[ \sum H = 0 \]

\[ F_{57} - 51.963 - 11.548 \cos 60^\circ + F_{56} \cos 60^\circ = 0 \]

\[ F_{57} + F_{56} \cos 60^\circ = 57.737 \]
\[ \sum V = 0 \]
\[ -60 + 11.548 \sin 60^\circ + F_{56} \sin 60^\circ = 0 \]
\[ F_{56} = 57.734 \text{ kN} \]
\[ F_{57} + 57.734 \cos 60^\circ = 57.737 \]
\[ F_{57} = 28.868 \text{ kN} \]

**Joint 6**

\[
\begin{align*}
F_{64} & \quad 60^\circ \\
F_{65} & \quad 60^\circ \\
\frac{-57.737}{\sin 60^\circ} & = \frac{57.734}{\sin (180+60^\circ)} = \frac{F_{67}}{\sin 60^\circ} \\
F_{67} & = -57.737 \\
\frac{-57.737}{\sin 60^\circ} & = \frac{57.737}{\sin 60^\circ} \\
F_{67} & = 57.737 \text{ kN} 
\end{align*}
\]

**Joint 7**

\[ V_B = 50 \text{ kN} \]
\[ F_{76} = -57.737 \text{ kN} \]
\[ F_{76} = 28.868 \text{ kN} \]
Find the forces in all the members of k - Truss as shown in the figure.

Step-2

\[ \Sigma H = 0 \]

\[ H_A = 0 \]

\[ \Sigma V = 0 \]

\[ V_A + V_G - (50 + 100) = 0 \]

\[ V_A + V_G = 150 \text{ KN} \]

\[ \Sigma M_A = 0 \]

\[ 50 \times 8 + 100 \times 12 - V_G \times 24 = 0 \]

\[ V_G = 66.667 \text{ KN} \]

\[ V_A + V_G = 150 \]

\[ V_A + 66.667 = 150 \]

\[ V_A = 83.333 \text{ KN} \]
step-1 - degree of indeterminacy

(i) External

No. of reactions = 3
No. of equations = 3

\[ \text{Degree of Static External indeterminacy} = 3 - 3 = 0 \]

\[ \therefore \text{The given truss is statically externally determinate} \]

(ii) Internal

\[ m = (\text{No. of member}) = 29 \]
\[ j = 16 \]
\[ r = 3 \]

\[ m = 2j + r \]
\[ 29 = 2(16) + 3 \]
\[ \therefore \text{The structure is internally determinate} \]
Step-3  Find the forces in the members

Joint (A)

\[ \tan \theta = \frac{6}{4} \]
\[ \theta = \tan^{-1}\left(\frac{6}{4}\right) \]
\[ \theta = 56.30 \]

\[ \Sigma H = 0 \Rightarrow F_{AL} + F_{AB} \cos 56.30 + H_A = 0 \]
\[ F_{AL} + F_{AB} \cos 56.30 = 0 \quad \rightarrow (3) \]

\[ \Sigma V = 0 \Rightarrow V_A + F_{AB} \sin 56.30 = 0 \]
\[ 83.33 + F_{AB} \sin(56.30) = 0 \]
\[ F_{AB} = -100.180 \text{ kN} \]

\[ F_{AL} + F_{AB} \cos 56.30 = 0 \]
\[ F_{AL} + (-100.180) \cos 56.30 = 0 \]
\[ F_{AL} = 55.57 \text{ kN} \]

Joint (B)

\[ \frac{F_{BC}}{\sin(90-56.3)} = \frac{F_{BM}}{\sin(180+56.3)} = \frac{F_{AB}}{\sin 90} \]

\[ F_{BM} = -100.180 \frac{\sin 90}{\sin 90} \]
\[ F_{BM} = 83.34 \text{ kN} \]
\[
\frac{F_{EC}}{\sin(90-56.3)} = \frac{-100.180}{\sin 90^\circ}
\]

\[F_{EC} = -55.58 \text{ kN}\]

At joint L

\[
\frac{F_{AL}}{\sin 90^\circ} = \frac{F_{LK}}{\sin 90^\circ} = \frac{F_{LM}}{\sin(180^\circ)}
\]

\[
\frac{F_{AL}}{\sin 90^\circ} = \frac{F_{LK}}{\sin 90^\circ} = \frac{F_{LM}}{\sin(180^\circ)}
\]

\[
\frac{55.57}{\sin 90^\circ} = \frac{F_{LK}}{\sin 90^\circ}
\]

\[F_{LK} = 55.57 \text{ kN}\]

\[F_{LM} = 0\]

At joint M

\[\theta_1 = \tan^{-1}\left(\frac{3}{4}\right) = 36.86^\circ\]

\[\theta_1 = 36.86^\circ\]
\[ \Sigma H = 0 \implies F_{mc} \cos 36.86 + F_{mk} \cos 36.86 = 0 \quad (3) \]

\[ \Sigma V = 0 \implies F_{Bm} - F_{ml} + F_{mc} \sin 36.86 = -F_{mk} \sin 36.86 = 0 \]

Using eqn (2)

\[ 83.34 + F_{mc} \sin 36.86 + F_{mk} \sin 36.86 = 0 \]

\[ \Rightarrow F_{mc} \cos 36.86 = -F_{mk} \cos 36.86 \]

\[ F_{mc} = -F_{mk} \quad (4) \]

Using eqn (4) in the above equation

\[ 83.34 - F_{mk} \sin 36.86 - F_{mk} \sin 36.86 = 0 \]

\[ 83.34 - 2 F_{mk} \sin 36.86 = 0 \]

\[ F_{mk} = \frac{83.34}{2 \times \sin 36.86} \]

\[ F_{mk} = 69.46 \text{ KN} \]

\[ F_{mc} = -F_{mk} \]

\[ F_{mc} = -69.46 \text{ KN} \]
At joint C

\[ F_{cc} - F_{cd} - F_{cm} = 0 \]

\[ 55.58 - F_{cm} \cos 36.86^\circ = 0 \]

\[ F_{cm} = 41.68 \text{ kN} \]

At joint KL

\[ F_{km} + F_{kn} = 50 \text{ kN} \]
\[ \Sigma H = 0 \]
\[ F_{kj} - F_{kl} - F_{km} \cos 36.86 = 0 \]
\[ F_{kj} = (55.48) - 69.46 \cos(36.86) = 0 \]
\[ F_{kj} = 110.68 \text{ KN} \]

\[ \Sigma V = 0 \]
\[ \Rightarrow F_{kn} + F_{km} \sin 36.86 - 50 = 0 \]
\[ F_{kn} + 69.46 \sin 36.86 - 50 = 0 \]
\[ F_{kn} = 8.32 \text{ KN} \]

At Joint (N)

\[ \Sigma H = 0 \]
\[ F_{no} \sin 53.13 + F_{nj} \sin 53.13 = 0 \rightarrow (1) \]

\[ \Sigma V = 0 \]
\[ 41.68 + F_{no} \cos 53.13 - F_{nj} \cos 53.13 - 8.32 = 0 \]
\[ F_{no} \cos 53.13 - F_{nj} \cos 53.13 = 33.46 \rightarrow (2) \]

Solving (1) & (2)
\[ F_{no} = -27.799 \text{ KN} \]
\[ F_{nj} = 27.799 \text{ KN} \]
Joint (G):

\[ \frac{66.667}{\sin(56.30)} = \frac{F_{GF}}{\sin 90^\circ} = \frac{F_{GH}}{\sin(180 + 33.7)} \]

\[ F_{GF} = 80.13 \text{ KN} \]

\[ F_{GH} = -64.46 \text{ KN} \]

Joint (F):

\[ \frac{F_{FE}}{\sin(33.7)} = \frac{80.13}{\sin(90^\circ)} = \frac{F_{FP}}{\sin(180 + 56.3)} \]

\[ F_{FE} = 44.45 \text{ KN} \]

\[ F_{FP} = -66.66 \text{ KN} \]
\[ \text{At Joint H:} \]

\[ F_{HG} = -44.46 \text{ kN} \]

\[ \frac{F_{HP}}{\sin(180)} = \frac{F_{HI}}{\sin(90)} \]

\[ F_{HP} = 0 \]

\[ F_{HI} = -44.46 \text{ kN} \]

\[ \sum H = 0 \]

\[ -F_{PE} \sin 53.13 - F_P \sin 53.13 = 0 \]

\[ \sum V = 0 \]

\[ F_{FP} - F_{PH} = F_{PE} \cos 53.13 - F_{PR} \cos 53.13 = 0 \]

\[ -66.66 + F_{PE} \cos 53.13 - F_{PR} \cos 53.13 = 0 \]
\[ F_{PE} \cos 53.13 - F_{PR} \cos 53.13 = 66.66 \]

\[ F_{PE} = 55.54 \text{ kN} \]
\[ F_{PR} = -55.54 \text{ kN} \]

At Joint I:

\[ \Sigma H = 0 \]
\[ -44.46 - F_{JH} - 55.54 \sin 53.13 = 0 \]
\[ F_{JH} = -86.89 \text{ kN} \]

\[ \Sigma V = 0 \]
\[ F_{J0} + (-55.54 \cos 53.13) = 0 \]
\[ F_{J0} = 33.32 \text{ kN} \]
\[ \Sigma H = 0 \]

\[ 44.45 + 55.54 \cos 36.86 - F_{DE} = 0, \]

\[ F_{DE} = 88.88 \text{ kN} \]

\[ \Sigma V = 0 \]

\[ -F_{EO} - 55.54 \sin 36.86 = 0 \]

\[ F_{EO} = -33.31 \text{ kN} \]

**Joint - I**

\[ \Sigma V = 0 \]

\[ F_{I0} - 55.55 \sin (36.869) = 0 \]

\[ F_{I0} = 33.329 \text{ kN} \]

\[ \Sigma H = 0 \]

\[ -F_{IJ} - 55.55 \cos (36.869) - 44.439 = 0 \]

\[ F_{IJ} = -88.879 \text{ kN} \]
Joint - O

\[ F_{OE} = -33.339 \text{ kN} \]
\[ F_{OI} = 33.329 \text{ kN} \]

\[ \sum H = 0 \]
\[ -F_{OD} \cos(36.869) - F_{OJ} \cos(36.869) = 0 \]
\[ F_{OD} + F_{OJ} = 0 \rightarrow 0 \]

\[ \sum V = 0 \]
\[ -33.329 - 33.329 + F_{OD} \sin(36.869) - F_{OJ} \sin(36.869) = 0 \]
\[ F_{OD} - F_{OJ} = +11.09 \rightarrow 0 \]

\[ F_{OD} = 55.545 \text{ kN} \]
\[ F_{OJ} = 55.545 \text{ kN} \]

Joint - O

\[ F_{DC} = 111.30 \text{ kN} \]
\[ F_{DE} = 88.849 \text{ kN} \]
\[ F_{DN} = -27.148 \text{ kN} \]
\[ F_{DO} = 555.45 \text{ kN} \]

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\[ \Sigma V = 0 \]
\[ -F_{DJ} + 27.148 \sin \left(36.869\right) - 55.545 \sin(36.869) = 0 \]
\[ F_{DJ} = -17.037 \text{ kN} \]

**Method of Sections**

Unlike the method of joints, the method of section is based on the concept that each section of the truss will be in equilibrium when cut into two different sections. This method is specifically helpful in problems where two or three forces are to be found out in the entire truss.

First an optimum section is taken to solve the problem and then the equilibrium equations are used to solve the problem. Here all three equations of equilibrium, \( \Sigma H = 0, \Sigma V = 0, \Sigma M = 0 \) are used and member forces are found out.
problem

0 Find the forces in the member BD, BE, CE of the truss as shown in the figure.

\[ \begin{align*}
\text{Step 1: Degree of indeterminacy} \\
0 \text{ External} \\
\text{Reactions} &= 3 \\
\text{Equations} &= 3 \\
3 - 3 &= 0
\end{align*} \]

0 Internal

\[ \begin{align*}
m &= 13 \\
j &= 8 \\
r &= 3
\end{align*} \]

\[ \begin{align*}
m - 2j + r &= 13 - 2(8) + 3 \\
&= 0
\end{align*} \]

\[ \begin{align*}
\text{Step 2: Find the reactions} \\
\sum H = 0 &\Rightarrow H_A^{+3} = 0 \\
H_A &= -3 \text{ KN}
\end{align*} \]
\( \Sigma V = 0 \Rightarrow V_A + V_H - 4 = 0 \)

\( V_A + V_H = 4 \quad (1) \)

\( \Sigma m_A = 0 \Rightarrow (-V_H \times 20) + 4(15) + 3(5) = 0 \)

\[ V_H = 3.75 \text{ kN} \]

\( V_A + V_H = 4 \)

\( V_A + 3.75 = 4 \)

\[ V_A = 0.25 \text{ kN} \]

\[ \tan \theta = \frac{5}{5} \]

\[ \theta = 45^\circ \]

\( \Sigma H = 0 \Rightarrow H_A + 3 + F_{CE} + F_{BD} + F'_{BE} \cos 45^\circ = 0 \)

\[ F_{CE} + F_{BD} + F'_{BE} \cos 45^\circ = 0 \quad (2) \]

\( \Sigma V = 0 \)

\[ V_A - F'_{BE} \sin 45 = 0 \]

\[ 0.25 - F'_{BE} \sin 45 = 0 \]

\[ F'_{BE} = 0.35 \text{ kN} \]
\[ M_b = 0 \]

\[ \Rightarrow -(-3 \times 5) + (0.25 \times 5) - (F_{CE} \times 5) = 0 \]

\[ F_{CE} = 3.75 \text{ KN} \]

Sub in \( \varnothing \)

\[ F_{BD} = -3.49 \text{ KN} \]

2) Find the forces in the members CE, DF and GI.

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Step-1 Degree of indeterminacy

Reactions = 3

Equations = 3

3 - 3 = 0

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Internal

\[ m = 15 \]
\[ \theta = 9 \]
\[ r = 3 \]

**Step 2**

Find the reactions

\[ \Sigma H = 0 \]
\[ \Rightarrow H_H + 3 = 0 \]
\[ H_H = -3 \text{ kN} \]

\[ \Sigma V = 0 \]
\[ \Rightarrow V_H + V_I - 5 - 4 = 0 \]
\[ V_H + V_I = 9 \text{ kN} \]

\[ \Sigma m_H = 0 \]
\[ (-V_I \times 2) + (4 \times 2) + (3 \times 6) = 0 \]
\[ -V_I = -13 \]
\[ V_I = 13 \text{ kN} \]

\[ V_H + V_I = 9 \]
\[ V_H + 13 = 9 \]
\[ V_H = -4 \text{ kN} \]
\[ H = 0 \]
\[ \Rightarrow 3 - F_{CD} \cos 45 = 0 \]
\[ F_{CD} = 4.24 \text{ kN} \]

\[ V = 0 \]
\[ -5 - 4 - F_{CD} \sin 45 - F_{BD} - F_{CE} = 0 \]
\[ -5 - 4 - 4.24 \sin 45 - F_{BD} - F_{CE} = 0 \]
\[ - F_{BD} - F_{CE} = 11.99 = 0 \]
\[ F_{BD} + F_{CE} = 11.99 \text{ kN} \]

\[ m_c = 0 \]
\[ -F_{BD} \times 2 - 5 \times 2 = 0 \]
\[ F_{BD} = -5 \text{ kN} \]
\[ -5 + F_{CE} = 11.99 \]
\[ F_{CE} = -6.99 \text{ kN} \]
\[ \sum H = 0 \]
\[ 3 - F_{DG_1} \cos 45 = 0 \]
\[ F_{DG_1} = 4.24 \text{ KN} \]

\[ \sum V = 0 \]
\[-5 - 4 - F_{DG_1} \sin 45 - F_{DF} - F_{EG_1} = 0 \]
\[-5 - 4 - 4.24 \sin 45 - F_{DF} - F_{EG_1} = 0 \]
\[-F_{DF} - F_{EG_1} = G \]
\[ F_{DF} + F_{EG_1} = -G \]

\[ \sum M = 0 \]
\[ 3 \times 2 + 4 \times 2 + F_{EG_1} \times 2 = 0 \]
\[ F_{EG_1} = -7 \text{ KN} \]
\[ F_{DF} - T = G \]

\[ F_{DF} = 13 \text{ kN} \]

**Section 3-3**

\[ \sum V = 0 \]

\[ -4 + 13 + F_{HF} + F_{GJ} + F_{HG} \sin 45^\circ = 0 \]

\[ -4 + 13 + F_{HF} + F_{GJ} + 4.84 \sin 45^\circ = 0 \]

\[ F_{HF} + F_{GJ} = -11.99 \]

\[ \sum M_H = 0 \]

\[ -13 \times 2 - F_{GJ} \times 2 = 0 \]

\[ F_{GJ} \times 2 = -26 \]

\[ F_{GJ} = -13 \text{ kN} \]
3. A pin joint truss is loaded and supported as shown in the figure. Determine forces in members BC, GF, and CG along with the nature of the forces. Use the method of sections.

\[ F_{HF} = 13 = -11.99 \]
\[ F_{HF} = 1.01 \text{ kN} \]

Step-1

Degree of indeterminacy

Reactions = 3
Equations = 3

\[ 3 - 3 = 0 \]

Internal

\[ m - (2j + r) = 11 \]
\[ j = 7 \]
\[ r = 3 \]

So it is internally determined.
Step 2: Find the reactions

\[ \Sigma H = 0 \]
\[ V_D + V_E = 0 \]

\[ \Sigma V = 0 \]
\[ -H_D - 4 - G = 0 \]
\[ -H_D - 10 = 0 \]
\[ H_D = -10 \text{ KN} \]

\[ \Sigma M = 0 \]
\[ 4 \times 8 + 6 \times 19 - V_E \times 9 = 0 \]
\[ V_E = 11.55 \text{ KN} \]

\[ V_D + V_E = 0 \]
\[ V_D + 11.55 = 0 \]
\[ V_D = -11.55 \text{ KN} \]
Step-3

Section-1

\[
\begin{align*}
\tan \theta &= \frac{3}{4} \\
\theta_1 &= 36.86^\circ \\
\theta_2 &= 36.86^\circ \\
\sum H &= 0 \\
- F_{CG} \cos 36.86 - F_{FG} \cos 36.86 - F_{BC} &= 0.3 \text{ KN} \\
\sum V &= 0 \\
- (6 \times 4) + F_{CG} \sin 36.86 - F_{FG} \sin 36.86 &= 10 \\
F_{CG} \sin 36.86 - F_{FG} \sin 36.86 &= 10 \rightarrow 0 \\
\sum M_{G_0} &= 0 \\
(6 \times 4) - F_{BC} \times 3 &= 0 \\
F_{BC} &= 8 \text{ KN} \\
- F_{CG} \cos 36.86 - F_{FG} \cos 36.86 &= 8 \rightarrow 0 \\
\text{solving } 0 \text{ and } 0 \\
F_{CG} &= 3.33 \text{ KN} \\
F_{FG} &= -13.33 \text{ KN} \\
\end{align*}
\]
The forces in members BC, CG, and GF are 8, 3.33, -13.33 kN respectively.

2) Using the method of sections determine the bar forces indicated in the members of the truss as shown in the figure.

![Diagram of the truss with forces and reactions labeled.]

**Step-1**

Degree of indeterminacy

Reactions = 3

Equations = 3

\[3 - 3 = 0\]

Internal

\[m = 25\]

\[j = 10\]

\[r = 3\]

\[m - 2j + r\]

\[25 - (2 \times 10) + 3\]

\[= 0\]

\[\therefore \text{So it is internally determinate.}\]
Step-2  Find the reactions

\[ \Sigma H = 0 \]
\[ H_{10} = 0 \]

\[ \Sigma V = 0 \]
\[ V_{10} - 50 - 100 - 50 + V_{14} = 0 \]
\[ V_{10} + V_{14} = 200 \]

\[ \Sigma m_{10} = 0 \]
\[ (-V_{14} \times 14.4) + (50 \times 10.8) + (100 \times 7.2) + (50 \times 3.6) = 0 \]
\[ V_{14} = 100 \text{ kN} \]

\[ V_{10} + V_{14} = 200 \]
\[ V_{10} + 100 = 200 \]
\[ V_{10} = 100 \text{ kN} \]

Step-3:

\[ F_{2,11} = 3.6 \text{ kN} \]
\[ F_{2,2} = 2.7 \text{ kN} \]

source diginotes.in
\[ \Sigma H = 0 \]
\[ H_{10} + F_{11-12} + F_{9-3} = 0 \]
\[ F_{11-12} + F_{9-3} = 0 \rightarrow \]
\[ \Sigma V = 0 \]
\[ V_{10} - 50 - F_{9-3} + F_{11-12} = 0 \]
\[ 100 - 50 = F_{9-3} + F_{11-12} = 0 \]
\[ 50 - F_{9-3} + F_{11-12} = 0 \]
\[ - F_{9-3} + F_{11-12} = -50 \rightarrow \]
\[ \Sigma M_{11} = 0 \]
\[ F_{23} \times 5.4 + V_{10} \times 3.6 = 0 \]
\[ F_{23} \times 5.4 + 100 \times 3.6 = 0 \]
\[ F_{23} = -66.66 \text{ KN} \]
\[ \Sigma M_{12} = 0 \]
\[ - F_{11-12} \times 5.4 - 0 \times 5.4 \times 100 \times 3.6 = 0 \]
\[ F_{11-12} = 66.66 \text{ KN} \]
\[ \Sigma M_{10} = 0 \]
\[ + 50 \times 3.6 - 66.66 \times 5.4 - F_{11-4} \times 3.6 \]
\[ + F_{2-7} \times 3.6 = 0 \]
\[ 180 - 359.964 = F_{11-4} \times 3.6 + F_{2-7} \times 3.6 = 0 \]
\[ F_{2-7} \times 3.6 - F_{7-11} \times 3.6 = 179.964 \rightarrow \]

(source: diginotes.in)
\[-F_{2-3} + F_{7-11} = -50\]

Section B-B

\[
\begin{align*}
\sum H &= 0 \\
\Rightarrow F_{7-3} \cos 36.86 + F_{7-12} \cos 36.86 + 66.67 - 66.67 &= 0 \\
\Rightarrow F_{7-3} + F_{7-12} &= 0 \\
\Rightarrow F_{7-3} - F_{7-12} &= 0
\end{align*}
\]

\[
\sum V = 0
\]

\[
\begin{align*}
100 - 50 + F_{7-3} \sin 36.86 - F_{7-12} \sin 36.86 &= 0 \\
F_{7-3} \sin 36.86 - F_{7-12} \sin 36.86 &= -50
\end{align*}
\]

\[
F_{7-3} = -41.67 \text{ kN}
\]

\[
F_{7-12} = 41.67 \text{ kN}
\]
Considering Joint 3

Considering Joint 10

![Diagram showing forces and equilibrium equations]

\[ \Sigma H = 0 \Rightarrow F_{10-11} = 0 \]

\[ \Sigma V = 0 \Rightarrow F_{G-10} = -100 \]

Joint 11

\[ \Sigma H = 0 \]

\[ \Rightarrow -F_{G-11} \cos 36.86 + F_{11-12} = 0 \]

\[ \Rightarrow F_{G-11} = \frac{\cos 36.86}{\cos 36.86} (66.67) \]

\[ F_{6-11} = +83.34 \text{ kN} \]

\[ \Sigma V = 0 \]

\[ \Rightarrow -50 + F_{11-7} + F_{G-11} \sin 36.86 = 0 \]

\[ F_{11-7} = \frac{99}{99} \approx 100 \text{ kN} \]

\[ F_{G-9} = +50 \text{ kN} \]