BENDING MOMENT AND SHEAR FORCES

INTRODUCTION

Beam is a structural member which has negligible cross-section compared to its length. It carries load perpendicular to the axis in the plane of the beam. Due to the loading on the beam, the beam deforms and is called as deflection in the direction of loading. This deflection is due to bending moment and shear force generated as resistance to the bending. Bending Moment is defined as the internal resistance moment to counteract the external moment due to the loads and mathematically it is equal to algebraic sum of moments of the loads acting on one side of the section. It can also be defined as the unbalanced moment on the beam at that section.

Shear force is the internal resistance developed to counteract the shearing action due to external load and mathematically it is equal to algebraic sum of vertical loads on one side of the section and this act tangential to cross section. These two are shown in Fig 3.01 (a).

Unbalanced Moment = Bending Moment
(M) & Unbalanced Force = Shear Force

Fig. 3.01 (a)
For shear force Left side Upward force to the section is Positive (LUP) and Right side Upward force to the section is Negative (RUN) as shown in Fig. 3.01 (b).

For Bending Moment, Moment producing sagging action to the beam or clockwise moment to the left of the section and anti-clockwise moment to the right of the section is treated as positive and Moment producing hogging action to the beam or anti-clockwise moment to the left of the section and clockwise moment to the right of the section is treated as Negative as shown in Fig. 3.01(b).

**Sign Convention**

![Sign Convention Diagram]

**Elastic Curve**

Generally the beam is represented by a line and the beam bends after the loading. The depiction of the bent portion of the beam is known as elastic curve.

The shape of the elastic curve is the best way to find the sign of the Bending Moment as shown in the Fig. 3.02

**Support Reactions:**

The various structural members are connected to the surroundings by various types of supports. The structural members exert forces on supports known as action. Similarly, supports exert forces on structural members known as reaction.

A beam is a horizontal member, which is generally placed on supports.
The beam is subjected to the vertical forces known as action. Supports exert forces on beam known as reaction.

**Types of supports:**
1) Simple supports
2) Roller supports
3) Hinged or pinned supports
4) Fixed supports

1) **Simple supports:**

![Simple supports diagram](image1)

Fig. 3.03

Simple supports are those supports, which exert reactions perpendicular to the plane of support. It restricts the translation of body in one direction only, but not rotation.

2) **Roller supports:**

![Roller supports diagram](image2)

Fig. 3.04

Roller supports are the supports consisting of rollers which exert reactions perpendicular to the plane of the support. They restrict translation along one direction and no rotation.

3) **Hinged or Pinned supports:**

![Hinged or Pinned supports diagram](image3)

Fig. 3.05

Hinged supports are the supports which exert reactions in any direction but for our convenient point of view it is resolved in to two components. Therefore hinged supports restrict translation in both directions. But rotation is possible.

4) **Fixed supports:**

Fixed supports are those supports which restricts both translation and rotation of the body. Fixed supports develop an internal moment known as restraint moment to prevent the rotation of the body.
Types of Beams:

1) Simply supported Beam:

   ![Fig. 3.07 Simply supported Beam]
   
   It is a beam which consists of simple supports. Such a beam can resist forces normal to the axis of the beam.

2) Continuous Beam:

   ![Fig. 3.08 Continuous Beam]
   
   It is a beam which consists of three or more supports.

3) Cantilever beam:

   ![Fig. 3.09 Cantilever beam]
   
   It is a beam whose one end is fixed and the other end is free.

3) Propped cantilever Beam:

   ![Fig. 3.10 Propped cantilever Beam]
   
   It is a beam whose one end is fixed and other end is simply supported.

4) Overhanging Beam:

   ![Fig. 3.11 Overhanging Beam]
   
   It is a beam whose one end is exceeded beyond the support.
Types of loads:

1) **Concentrated load**: A load which is concentrated at a point in a beam is known as concentrated load.

![Figure 3.12](image_url)

2) **Uniformly Distributed load**: A load which is distributed uniformly along the entire length of the beam is known as Uniformly Distributed Load.

![Figure 3.13](image_url)

Convert the U.D.L. into point load which is acting at the centre of particular span
Magnitude of point load = \( \frac{20\text{KN}}{m} \times 3\text{m} = 60\text{kN} \)

3) **Uniformly Varying load**: A load which varies with the length of the beam is known as Uniformly Varying load

![Figure 3.14](image_url)

Magnitude of point load = Area of triangle and which is acting at the C.G. of triangle.
Problems on Equilibrium of coplanar non concurrent force system.

Tips to find the support reactions:
1) In coplanar concurrent force system, three conditions of equilibrium can be applied namely
   $$\sum F_x = 0, \sum F_y = 0$$ and $$\Sigma M = 0$$
2) Draw the free body diagram of the given beam by showing all the forces and reactions acting on the beam
3) Apply the three conditions of equilibrium to calculate the unknown reactions at the supports. **Determinate structures** are those which can be solved with the fundamental equations of equilibrium. i.e. the 3 unknown reactions can be solved with the three equations of equilibrium.

Relationship between Uniformly distributed load (udl), Shear force and Bending Moment.

Consider a simply supported beam subjected to distributed load $$\omega$$ which is a function of $$x$$ as shown in Fig. 3.15(a). Consider section 11 at a distance $$x$$ from left support and another section 22 at a small distance $$dx$$ from section 11. The free body diagram of the element is as shown in Fig. 3.15(b). To the left of the section 11 the internal force $$V$$ and the moment $$M$$ acts in the +ve direction. To the right of the section 22 the internal force and the moment are assumed to increase by a small amount and are respectively $$V+dV$$ and $$M+dM$$ acting in the +ve direction.

For the equilibrium of the system, the algebraic sum of all the vertical forces must be zero.
\[ \rightarrow + \text{ve} \sum V = 0; \]
\[ V - \omega dx - (V + dV) = 0 \]
\[ -\omega dx - dV = 0 \]
\[ -\omega = \frac{dV}{dx} \]

...(01)

Eq. 01 the udl at any section is given by the negative slope of shear force with respect to distance \( x \) or negative udl is given by the rate of change of shear force with respect to distance \( x \).

Within a limit of distributed force \( \omega_1 \) and \( \omega_2 \) over a distance of \( a \), shear force is written as
\[ V = \int_{\omega_1}^{\omega_2} -\omega dx \]

For the equilibrium of the system, the algebraic sum Moments of all the forces must be zero. Taking moment about section 22
\[ \sum M = 0; \]
\[ M + V dx - (\omega dx) \left( \frac{dx}{2} \right) - (M + dM) = 0 \]

Ignoring the higher order derivatives, we get
\[ V dx - dM = 0 \]
\[ \text{or } V = \frac{dM}{dx} \]

02

Eq. 02 shows the shear force at any section is given by rate of change in bending moment with respect to distance \( x \).

Within a limit of distributed force \( \omega_1 \) and \( \omega_2 \) and shear force \( V_1 \) and \( V_2 \) over a distance of \( a \), we can write bending moment as
\[ M = \int_{V_1}^{V_2} V dx \]
**Point of contra flexure or point of inflection.**

These are the points where the sign of the bending moment changes, either from positive to negative or from negative to positive. The bending moment at these points will be zero.

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**Procedure to draw Shear Force and Bending Moment Diagram**

- Determine the reactions including reactive moments if any using the conditions of equilibrium viz. \( \Sigma H = 0; \Sigma V = 0; \Sigma M = 0 \)

**Shear Force Diagram (SFD)**

- Draw a horizontal line to represent the beam equal to the length of the beam to some scale as zero shear line.
- The shear line is vertical under vertical load, inclined under the portion of uniformly distributed load and parabolic under the portion of uniformly varying load. The shear line will be horizontal under no load portion. Remember that the shear force diagram is only concerned with vertical loads only and not with horizontal force or moments.
- Start from the left extreme edge of the horizontal line (For a cantilever from the fixed end), draw the shear line as per the above described method. Continue until all the loads are completed and the check is that the shear line should terminate at the horizontal line.
- The portion above the horizontal line is positive shear force and below the line is negative shear force.
- To join the shear line under the portion of uniformly varying load, which is a parabola, it is to be remembered that the parabola should be tangential to the horizontal if the
corresponding load at the loading diagram is lesser and will be tangential to vertical if the corresponding load at the loading diagram is greater.

Bending Moment Diagram (BMD)

- Draw a horizontal line to represent the beam equal to the length of zero shear line under the SFD.
- The Bending Moment line is vertical under the applied moment, inclined or horizontal under the no load portion, parabolic under the portion of uniformly distributed load and cubic parabola under the portion of uniformly varying load.
- Compute the Bending Moment values as per the procedure at the salient points.
- Bending Moment should be computed just to the left and just to the right under section where applied moment is acting, i.e. $M_{AL}$ and $M_{AR}$. Once the applied moment is to be ignored and next the moment is to be considered as per the sign convention.
- Draw these values as vertical ordinates above or below the horizontal line corresponding to positive or negative values.
- Start the Bending Moment line from the left extreme edge of the horizontal line, draw as per the above described method under prescribed loading conditions. Continue until the end of the beam and the check is that the line should terminate at the horizontal line.

- The portion above the horizontal line is positive Bending Moment and below the line is negative Bending Moment.

- Locate the point of Maximum Bending Moment. It occurs at the section where Shear Force is zero.

- Locate the Point of Contra flexure where the Bending Moment line crosses the horizontal line, i.e. the sign of Bending Moment line changes its sign.

To join the Bending Moment line under the portion of uniformly distributed load which is a parabola, it is to be remembered that the parabola should be tangential to the horizontal if the corresponding shear force value at the loading diagram is lesser and will be tangential to vertical if the corresponding shear force line at the shear force diagram is greater as shown in Fig. 3.17.

In case of the beam being a **Cantilever**, start the Shear force from the fixed end, i.e. arrange the cantilever such that the fixed end is towards left end.
Eccentric Concentrated Load

Consider a simply supported beam of span $l$ with an eccentric point load $W$ acting at a distance $a$ from support as shown in Fig. 3.20.

The reactions can be obtained from the equations of equilibrium.

(Write the Upward acting forces on one side and downward acting forces on the other side of the equation to avoid confusion among sign convention).

$$\sum V_A = 0; \quad R_A + R_B = W$$

Taking moments about $A$,

$$\sum M_A = 0;$$

(Write the clockwise moments on one side and anti-clockwise moments on the other side of the equation to avoid confusion among sign convention).

$$(R_B)(l) = (W)(a)$$

$$R_B = \frac{Wa}{l}$$

Similarly Taking moments about $B$,

$$\sum M_B = 0;$$

$$(R_A)(l) = (W)(l-a)$$

$$R_A = \frac{W(l-a)}{l}$$

Check

To check the computations, substitute in Eq. 01, we have

$$R_A + R_B = \frac{Wa}{l} + \frac{W(l-a)}{l} = W\left[\frac{a+l-a}{l}\right] = W$$

and hence OK.

Shear Force Values

$$V_A = 0 + R_A = \frac{W(l-a)}{l}$$

$$V_C = \frac{W(l-a)}{l}$$
\[ V_C = \frac{W(l-a)}{l} - W = -\frac{Wa}{l} \]

\[ V_B = \frac{Wa}{l} \]

\[ V_B = \frac{Wa}{l} - \frac{Wa}{l} = 0 \]

**Bending Moment Values**

*Note:* The Bending Moment will always be zero at the end of the beam unless there is an applied moment at the end.

\[ M_A = 0 \]

\[ M_B = 0 \]

\[ M_C = (R_A)a = \frac{W(l-a)}{l} \times a = W(l-a) \frac{a}{l} \] also

\[ M_C = (RB)(l-a) = \left(\frac{Wa}{l}\right) \times (l-a) = W(l-a) \frac{a}{l} \]

**Uniformly Distributed Load**

Consider a simply supported beam of span \( l \) with an uniformly distributed load \( \omega/\text{m} \) acting over the entire span as shown in Fig. 3.35

The reactions can be obtained from the conditions of equilibrium.

As the loading is symmetrical
\[ R_A = R_B \] and hence
\[ \sum V_A = 0; R_A + R_B = 2 R_A = 2 R_B = \omega x l \]

\[ R_A = R_B = \frac{\omega l}{2} \]

**Shear Force Values**

\[ V_A = R_A = \frac{\omega l}{2} \]

\[ V_B = \frac{\omega l}{2} - \omega l = -\frac{\omega l}{2} \]

Shear Force at Midsection will be

\[ V_C = \frac{\omega l}{2} - \frac{\omega l}{2} = 0 \]

**Bending Moment Values**

\[ M_A = 0 \]
\[ M_B = 0 \]
\[ M_C = (R_A) \frac{l}{2} \left[ \frac{\omega l}{2} \right] \times \left[ \frac{l}{2} \right] = \frac{\omega l^2}{4} \]

**Uniformly Varying Load**

Consider a simply supported beam of span \( l \) with an uniformly varying load \( \omega/m \) acting over the entire span as shown in Fig. 3.24

The reactions can be obtained from the conditions of equilibrium.

\[ \Sigma V_A = 0; \]
\[ R_A + R_B = \left( \frac{\omega l}{2} \right) \quad (01) \]

Taking moments about A,

\[ \Sigma M_A = 0; \]
\[ R_B \times l = \left( \frac{\omega l}{2} \right) \left( \frac{l}{3} \right) = \frac{\omega l^2}{6} \]
\[ R_B = \frac{\omega l}{6} \]

Taking moments about B,

\[ \Sigma M_B = 0; \]
\[ R_A \times l = \left( \frac{\omega l}{2} \right) \left( \frac{2l}{3} \right) = \frac{\omega l^2}{3} \]
\[ R_A = \frac{\omega l}{3} \]

**Check**

To check the computations, substitute in Eq. 01, we have

\[ R_A + R_B = \left( \frac{\omega l}{6} \right) + \left( \frac{\omega l}{3} \right) = \frac{\omega l}{2} \]

Hence O.K.

**Shear Force Values**

\[ V_A = R_A = \frac{\omega l}{3} \]
\[ V_B = \frac{\omega l}{3} - \frac{\omega l}{2} = -\frac{\omega l}{6} \text{ and } V_B = -\frac{\omega l}{6} + \frac{\omega l}{6} = 0 \]

**Location of Zero Shear Force**

Consider a section at a distance \( x \) from left support and load intensity at that
section $\omega_s$ is given by $\omega_s = \left(\frac{x}{l}\right)\phi$

and Shear Force at that section is given by

$$V_x = \frac{1}{2} \omega_s x - R_B = \left(\frac{\omega x^2}{2l}\right) - \left(\frac{\omega l}{6}\right) x = 0 \text{ or } x = \frac{l}{\sqrt{3}}$$

**Bending Moment Values**

$M_A = 0$

$M_B = 0$

Bending Moment will be maximum at Zero Shear Force and

$$M_c = R_B \left(\frac{\omega l^2}{6l} - \frac{\omega l}{6l} \frac{l}{\sqrt{3}}\right)$$

$$= \left[\frac{\omega l^2}{6l}\right] \left(1 - \frac{1}{3}\right) = \frac{\omega l^2}{9\sqrt{3}}$$

**Cantilever with Point Load**

The reactions can be obtained from the conditions of equilibrium.

$\sum V = 0; \ R_A = W$

Taking moments about A,

$M_A = -W(l-a)$

Shear Force Values

$V_B = 0$

$V_C = 0$

$V_C = 0 - W = -W$

$V_A = -W$

$V_A = -W + W = 0$

**Bending Moment Values**

$M_B = 0$

$M_C = 0$

$M_A = -W(l-a)$

**Cantilever with Uniformly Distributed Load (UDL)**

The reactions can be obtained from the conditions of equilibrium.
\[ \sum V_A = 0; \quad R_A = \omega l \]

Taking moments about A,
\[ M_A = -\omega l \times \left( \frac{1}{2} \right) = -\frac{\omega l^2}{2} \]

Shear Force Values
\[ V_B = 0 \]
\[ V_A = -\omega l \]
\[ V_A = -\omega l + \omega l = 0 \]

Bending Moment Values
\[ M_B = 0 \]
\[ M_A = -\omega l \times \left( \frac{1}{2} \right) = -\frac{\omega l^2}{2} \]

**Cantilever with Uniformly Varying Load (UVL)**

**Case (i)**

The reactions can be obtained from the conditions of equilibrium.
\[ \sum V_A = 0; \quad R_A = \frac{\omega l}{2} \]

Taking moments about A,
\[ M_A = -\left( \frac{\omega l}{2} \right) \times \left( \frac{1}{2} \right) = -\frac{\omega l^2}{3} \]

Shear Force Values
\[ V_B = 0 \]
\[ V_A = \frac{\omega l}{2} \]
\[ V_A = \frac{\omega l}{2} - \frac{\omega l}{2} = 0 \]

Bending Moment Values
\[ M_B = 0 \]
\[ M_A = -\left( \frac{\omega l}{2} \right) \times \left( \frac{1}{2} \right) = -\frac{\omega l^2}{6} \]

Consider a section at a distance \( x \) from free end and load intensity at that section \( \omega_x \) is given by
\[ \omega_x = \left( \frac{x}{l} \right) \omega \]
Shear Force at that section is given by

\[ V_x = \frac{1}{2} \omega_x x = \left( \frac{\omega x^2}{2l} \right) \]

Bending Moment at that section is given by

\[ M_x = -\left[ \frac{1}{2} \omega_x x \right] \left( \frac{x}{3} \right) = -\left( \frac{\omega x^3}{6l} \right) \]

**Case (ii)**

The reactions can be obtained from the conditions of equilibrium.

\[ \Sigma \omega_A = 0; \ R_A = \frac{\omega l}{2} \]

Taking moments about A,

\[ M_A = \left( \frac{\omega l}{2} \right) \times \left( \frac{2l}{3} \right) = \frac{\omega l^2}{3} \]

Shear Force Values

\[ V_B = 0 \]

\[ V_A = \frac{\omega l}{2} \]

\[ V_A = \frac{\omega l}{2} - \frac{\omega l}{2} = 0 \]

Bending Moment Values

\[ M_B = 0 \]

\[ M_A = \left( \frac{\omega l}{2} \right) \times \left( \frac{2l}{3} \right) = \frac{\omega l^2}{3} \]

Consider a section at a distance \( x \) from free end and load intensity at that section \( \omega_x \) is given by

\[ \omega_x = \left( \frac{x}{l} \right) \omega \]

Shear Force at that section is given by

\[ V_x = R_A - \frac{1}{2} \omega_x x = \left( \frac{\omega l}{2} \right) - \left( \frac{\omega x^2}{2l} \right) \]

Bending Moment at that section is given by

\[ M_x = R_A x - \left[ \frac{1}{2} \omega_x x \right] \left( \frac{x}{3} \right) - M_A = \left( \frac{\omega l}{2} \right) x - \left( \frac{\omega x^3}{6l} \right) - \left( \frac{\omega l^2}{3} \right) \]

**Cantilever with Partial Uniformly Distributed Load (UDL)**

The reactions can be obtained from the conditions of equilibrium.
\[ \Sigma V_A = 0; \quad R_A = \omega b \]

Taking moments about A,

\[ M_A = -\omega b \left( \frac{a + b}{2} \right) \]

Shear Force Values

\[ V_B = 0 \]
\[ V_D = 0 \]
\[ V_C = -\omega b \]
\[ V_A = -\omega b \]
\[ V_A = -\omega b + \omega b = 0 \]

Bending Moment Values

\[ M_B = 0 \]
\[ M_D = 0 \]
\[ M_C = -\omega b \left( \frac{b}{2} \right) = -\frac{\omega b^2}{2} \]
\[ M_A = -\omega b \left( \frac{a + b}{2} \right) \]

3.01. Draw the Shear Force and Bending Moment Diagram for a Cantilever beam subjected to concentrated loads as shown in Fig. 3.38.

From the conditions of equilibrium

\[ \Sigma V = 0; \quad R_A = 10 + 20 + 30 = 60 \text{ kN} \]
\[ \Sigma M = 10 \times 6 + 20 \times 3 + 30 \times 2 = 180 \text{ kN-m} \]

Shear Force Values at Salient Points

\[ V_D = 0 - 10 = -10 \text{ kN} \]
\[ V_C = -10 - 20 = -30 \text{ kN} \]
\[ V_B = -30 - 30 = -60 \text{ kN} \]
\[ V_A = -60 + 60 = 0 \text{ kN} \]

Bending Moment Values at Salient Points

\[ M_D = 0 \text{ kN-m} \]
\[ M_C = -10 \times 3 = -30 \text{ kN-m} \]
\[ M_B = -10 \times 4 - 20 \times 1 = - 60 \text{ kN-m} \]
\[ M_A = -10 \times 6 - 20 \times 3 - 30 \times 2 = - 180 \text{ kN-m} \]
3.02. A cantilever beam is subjected to loads as shown in Fig. 3.39. Draw SFD and BMD.

From the conditions of equilibrium
\[ \sum V_A = 0; \ R_A = 10 + 30 + 20 \times 5 = 140 \text{ kN (↑)} \]
\[ \sum M_A = 30 \times 2 + 10 \times 3 + (20 \times 5)\left(\frac{5}{2}\right) + 40 = 380 \text{ kN-m}. \]

**Shear Force Values at Salient Points**

\[ V_D = 0 \text{ kN} \]
\[ V_C = 0 - 20 \times 2 = -40 \text{ kN} \]
\[ V_C = -40 - 10 = -50 \text{ kN} \]
\[ V_B = -50 - 20 \times 1 = -70 \text{ kN} \]
\[ V_B = -70 - 30 = -100 \text{ kN} \]
\[ V_A = -100 - 20 \times 2 = -140 \text{ kN} \]
\[ V_A = -140 + 140 = 0 \text{ kN} \]

**Bending Moment Values at Salient Points**

As there is applied moment at section D, there will be two moments at that section and hence
\[ M_{DR} = 0 \]
\[ M_{DL} = 0 - 40 = -40 \text{kN-m} \]
\[ M_C = -20 \times 2 \times 1 - 40 = -80 \text{kN-m} \]
\[ M_B = -20 \times 3 \times 1.5 - 10 \times 1 - 40 = -140 \text{kN-m} \]
\[ M_A = -20 \times 5 \times 2.5 - 10 \times 3 - 20 \times 2 - 40 = -360 \text{kN-m} \]
3.03. Draw BMD and SFD for the cantilever beam shown in Fig. 3.40.

Locate the point of contra flexure if any,

![Diagram of BMD and SFD for a cantilever beam]

From the conditions of equilibrium

\[ \Sigma V_A = 0; R_A = 30 + \left( \frac{1}{2} \right) \times 20 \times 2 = 50 \text{ kN (↑)} \]

\[ \Sigma M_A = 30 \times 2 + \left( \frac{1}{2} \right) (20 \times 2) \left( \frac{3 + \frac{2}{3}}{3} \right) - 100 = 33.33 \text{ kN-m} \]

**Shear Force Values at Salient Points**

\[ V_D = 0 \text{ kN} \]

\[ V_C = 0 - \left( \frac{1}{2} \right) (20 \times 2) = -20 \text{ kN} \]

\[ V_B = -20 \text{ kN} \]


\[ V_B = -20 - 30 = -50 \text{ kN} \]
\[ V_A = -50 \text{ kN} \]
\[ V_A = -50 + 50 = 0 \text{ kN} \]

**Bending Moment Values at Salient Points**

As there is applied moment at section B, there will be two moments at that section and hence

- \[ M_D = 0 \text{ kN} \]
- \[ M_C = -\left(\frac{1}{2}\right) (20 \times 2) \left(\frac{2}{3}\right) = -13.33 \text{ kN-m} \]
- \[ M_{BR} = -\left(\frac{1}{2}\right) (20 \times 2) \left(1 + \frac{2}{3}\right) = -33.33 \text{ kN-m} \]
- \[ M_{BL} = -33.33 + 100 = + 66.67 \text{ kN-m} \]
- \[ M_A = -\left(\frac{1}{2}\right) (20 \times 2) \left(3 + \frac{2}{3}\right) - 30 \times 2 + 100 = -33.33 \text{ kN-m} \]

**Points of contraflexure:**

\[ \frac{x}{33.33} = \frac{2-x}{66.67} \text{ or } x = 0.67 \text{ m} \]

It lies at 0.67m and 2m right of the left support.

**Bracket Connections**

There can be following types of bracket connections which can be converted to load and moment.

![Fig.3.41 Bracket Connections](image)

The types of brackets are vertical and L bracket as shown in Fig. 3.41. Apply two equal, opposite and collinear forces at the joint where the load gets transferred to the beam. The two forces \((F)\) acting equal and opposite separated by a distance will form a couple equal to the product of Force and the distance between the forces along with the remaining Force.
3.04. An overhanging beam ABC is loaded as shown in Fig. 3.42. Draw the shear force and bending moment diagrams. Also locate point of contraflexure. Determine maximum +ve and —ve bending moments. (Jan-06)

The reactions can be obtained from the conditions of equilibrium.

\[ \sum V_A = 0; \quad R_A + R_B = 2 \times 6 + 2 = 14 \text{kN} \]

Taking moments about A,

\[ \sum M_A = 0; \quad 4R_B = (2 \times 6) \left( \frac{6}{2} \right) + 2 \times 6 \quad \text{or} \quad R_B = \frac{48}{4} = 12 \text{kN} \]

Similarly taking moments about B,

\[ \sum M_B = 0; \quad 4R_B + 2 \times 2 + \left( 2 \times 2 \right) \left( \frac{2}{2} \right) = (2 \times 4) \left( \frac{4}{2} \right) \quad \text{or} \quad R_A = \frac{8}{4} = 2 \text{kN} \]

**Check**

Substituting in Eq. 01, we have \( R_A + R_B = 2 + 12 = 14 \text{kN} \) (O.K.)

**Zero Shear Force**

Consider a section at a distance \( x \) where Shear Force is zero as shown in Fig. 3.42.

From similar triangles, we have

\[ \frac{2}{x} = \frac{6}{(4-x)} \]

\[ x = 1 \text{m} \]

**Bending Moment Values**

\[ M_A = 0 \]

\[ M_B = -2 \times 2 - 2 \times 2 \times \left( \frac{2}{2} \right) = -8 \text{kN} \quad \text{(Negative because Sagging)} \]

\[ M_C = 0 \]

Bending Moment at zero Shear Force will be either Maximum or Minimum.

\[ M_x = 2x - \frac{2 \times x^2}{2} = 2x - x^2 = 1 \text{kNm} \]

Maximum positive BM is 1kNm at 1 m to right of left support and negative BM is 8kNm at right support.

**Point of Contraflexure:** Bending Moment equation at section \( y \) is

\[ M_y = 2y - \frac{2 \times y^2}{2} = 2y - y^2 \Rightarrow 0 \quad \text{or} \quad y = 2 \text{m} \]
3.05. Draw the Shear Force and Bending Moment Diagram for the loaded beam shown in Fig. 3.4. Find the Maximum bending moment.

The reactions can be obtained from the conditions of equilibrium.

\[ \sum V_A = 0; \quad R_A + R_B = 40 \times 4 = 160 \text{kN} \quad (01) \]

Taking moment about A,

\[ \sum M_A = 0; 8R_B = (40 \times 4) \left(1 + \frac{4}{2}\right) \text{ or } R_B = \frac{480}{8} = 60 \text{kN} \]
The reactions can be obtained from the conditions of equilibrium.

\[ \Sigma V_A = 0; \quad R_A + R_B = 40 \times 4 = 160 \text{kN} \tag{01} \]

Taking moment about A,

\[ \Sigma M_A = 0; \quad 8R_B = (40 \times 4) \left(1 + \frac{4}{2}\right) \quad \text{or} \quad R_B = \frac{480}{8} = 60 \text{kN} \]

Similarly taking moment about B,

\[ \Sigma M_B = 0; \quad 8R_A = (40 \times 4) \left(3 + \frac{4}{2}\right) \quad \text{or} \quad R_A = \frac{800}{8} = 100 \text{kN} \]

**Check**

Substituting in Eq. 01, we have \( R_A + R_B = 100 + 60 = 160 \text{kN} \) (O.K.)

**Zero Shear Force**

Consider a section at a distance \( x \) where Shear Force is zero as shown in Fig. 3.43

From similar triangles, we have

\[ \frac{100}{x} = \frac{60}{(4-x)} \quad \text{or} \quad x = 2.5 \text{m} \]

\[ V_o = 1 + 2.5 = 3.5 \text{m} \] from right support.
Bending Moment Values

\[ M_B = 0 \]
\[ M_D = 60 \times 3 = 180 \text{kN} \]
\[ M_C = 60 \times 7 - (40 \times 4) \left( \frac{4}{2} \right) = 100 \text{kN} \]
\[ M_A = 0 \]

Bending Moment at zero Shear Force will be either Maximum or Minimum.

\[ M_x = 100 \times (1 + x) - \frac{40 \times x^2}{2} = 100 \times (1 + x) - 20 \times x^2 = 225 \text{kNm} \]

3.06. Draw the Shear Force and Bending Moment Diagram for the loaded beam shown in Fig. 3.44. Also locate the Point of Contraflexure. Find and locate the Maximum +ve and —ve Bending Moments.

The reactions can be obtained from the conditions of equilibrium.

\[ \sum V_A = 0; \quad R_C + R_D = 40 + 20 = 60 \text{kN} \]  \hspace{1cm} (01)

Taking moment about C,

\[ \sum M_C = 0; \quad 4R_D + 2 \times 40 = 20 \times 6 \]
\[ \text{or} \quad R_D = \frac{40}{4} = 10 \text{kN} \]

Similarly taking moments about D,

\[ \sum M_D = 0; \quad 4R_C + 20 \times 2 = 40 \times 6 \]
\[ \text{or} \quad R_C = \frac{200}{4} = 50 \text{kN} \]

Check

Substituting in Eq. 01, we have \( R_C + R_D = 50 + 10 = 60 \text{ kN} \) (O.K.)

Zero Shear Force is at right support

Bending Moment Values

\[ M_B = 0 \]
\[ M_D = -20 \times 2 = -40 \text{kN-m} \]
\[ M_C = -40 \times 2 = -80 \text{kNm} \]
\[ M_A = 0 \]

Maximum Moments: Maximum negative BM is 80 kNm at the left support.
3.07. Draw BMD and SFD for the loaded beam shown in Fig. 3.45. Also locate the Point of contraflexure and Maximum +ve and —ve Bending Moment.

The reactions can be obtained from the conditions of equilibrium.

Taking moment about A,
\[ \sum V_A = 0; \quad R_A + R_B = 3 + 5 + 2 \times 6 = 20 \text{kN} \]

\[ \sum M_A = 0; 6R_B + 3 \times 2 = (2 \times 6) \left( \frac{6}{2} \right) + 5 \times 8 \quad \text{or} \quad R_B = \frac{70}{6} = 11.67 \text{kN} \]

Similarly taking moment about B,
\[ \sum M_B = 0; 6R_A + 5 \times 2 = (2 \times 6) \left( \frac{6}{2} \right) + 3 \times 8 \quad \text{or} \quad R_A = \frac{50}{6} = 8.33 \text{kN} \]

**Check:** Substituting in Eq. 01, we have \( R_A + R_B = 11.67 + 8.33 = 20 \text{kN} \) (O.K.)
Check: Substituting in Eq. 01, we have \( R_A + R_B = 11.67 + 8.33 = 20 \) kN (O.K.)

**Zero Shear Force**

Consider a section at a distance \( x \) where Shear Force is zero as shown in Fig. 3.45.

From similar triangles, we have

\[
\frac{5.33}{x} = \frac{6.67}{6-x} \quad \text{or} \quad x = 2.67 \text{m}
\]

**Bending Moment Values**

\[
M_D = 0
\]

\[
M_B = -5 \times 2 = -10 \text{kN}
\]

\[
M_A = -3 \times 2 = -6 \text{kN}
\]

\[
M_C = 0
\]

Bending Moment at zero Shear Force will be either Maximum or Minimum.

\[
M_x = 8.33 \times x - 3(2 + x) - \frac{2 \times x^2}{2} = 8.33 \times x - 3(2 + x) - \frac{2 \times x^2}{2} = 1.11 \text{kNm}
\]
Points of Contraflexure:

Bending moment at section \( y \) from the left support is given by

\[
M_y = 8.33y - 3\times(2 + y) - \frac{2y^2}{2} \text{ or } y^2 - 5.33y + 6 = 0 \text{ and } y = 1.61 \text{m and 3.72m}
\]

Hence the points at 1.61m and 3.72m to right of left support.

3.08. Draw the BMD and SFD for the loaded beam shown in Fig. 3.46.

The reactions can be obtained from the conditions of equilibrium.

\[
\Sigma V_A = 0; \quad R_A + R_B = 20 \text{kN}
\]

Taking moment about A,

\[
\Sigma M_A = 0; 3R_B = 20 \times 4 + 10
\]

\[
R_B = \frac{90}{3} = 30 \text{kN}
\]

Similarly taking moments about B,

\[
M_B = 0; \quad 3R_A + 10 + (20 \times 1) = 0
\]

\[
R_A = -\frac{30}{3} = -10 \text{kN}
\]

Check

Substituting in Eq. 01, we have \( R_A + R_B = -10 + 30 = 20 \text{ kN} \) (O.K.)

Bending Moment Values

\( M_D = 0 \)

\[
M_B = -20 \times 1 = -20 \text{kNm} \quad \text{ (Negative because Sagging)}
\]

\[
M_C = -20 \times 2 + 30 \times 1 = -10 \text{kNm}
\]

\[
M_C = -10 - 10 = -20 \text{kNm or} \quad \text{ (By considering right side forces)}
\]

\[
M_C = -10 \times 2 = 20 \text{kNm} \quad \text{ (By considering left side forces)}
\]

\( M_A = 0 \)
An overhang beam ABC is loaded as shown in Fig. 3.47. Draw BMD and SFD.

The reactions can be obtained from the conditions of equilibrium.

\( \sum V_A = 0; \quad R_A + R_B = 4 \times 3 + 12 = 24 \text{kN} \)

Taking moment about A,

\[ \sum M_A = 0; \quad 6R_B = 12 \times 9 + (4 \times 3) \left( \frac{3}{2} + \frac{3}{3} \right) \text{ or } R_B = \frac{162}{6} = 27 \text{kN} \]

Similarly taking moments about B,

\[ M_B = 0; \quad 6R_A + 12 \times 3 = (4 \times 3) \left( \frac{3}{2} \right) \text{ or } R_A = -\frac{18}{6} = -3 \text{kN} \]

**Check**

Substituting in Eq. 01, we have \( R_A + R_B = -3 + 27 = 24 \text{ kN} \) (O.K.)

**Bending Moment Values**

\( M_D = 0 \)

\[ M_B = -12 \times 3 = -36 \text{kNm} \quad \text{(Negative because Sagging)} \]

\[ M_C = -3 \times 3 = -6 \text{kNm} \]

\( M_A = 0 \)
3.09. Draw SFD and BMD for the beam shown in Fig. 3.48. Determine the maximum BM and its location. Locate the points of contraflexure. (July 02)

The reactions can be obtained from the conditions of equilibrium.

\[ \sum V_A = 0; \quad R_A + R_B = 20 \times 3 + 40 = 100 \text{kN} \]

Taking moment about A,

\[ \sum M_A = 0; 6R_B = (20 \times 3) \left( \frac{3}{2} \right) + 40 \times 3 + 120 \quad \text{or} \quad R_B = \frac{330}{6} = 55 \text{kN} \]

Similarly taking moments about B,

\[ M_B = 0; \quad 6R_A = 40 \times 3 + (20 \times 3) \left( 3 + \frac{3}{2} \right) - 120 \quad \text{or} \quad R_A = \frac{270}{6} = 45 \text{kN} \]

**Check**

Substituting in Eq. 01, we have \( R_A + R_B = 45 + 55 = 100 \text{kN} \) (O.K.)
Bending Moment Values

\[ M_B = 0 \]

\[ M_{D_x} = 55 \times 1.5 = 82.5 \text{kNm} \]

\[ M_{D_x} = 82.5 - 120 = -37.5 \text{kNm} \quad \text{(By considering right side forces)} \]

\[ M_{D_x} = 45 \times 4.5 - (20 \times 3) \left( 1.5 + \frac{3}{2} \right) - 40 \times 1.5 = -37.5 \text{kNm} \quad \text{(By left side forces)} \]

\[ M_C = 55 \times 3 - 120 = 45 \text{kNm} \quad \text{(By considering right side forces)} \]

\[ M_C = 45 \times 3 - (20 \times 3) \left( \frac{3}{2} \right) = 45 \text{kNm} \quad \text{(By left side forces)} \]

\[ M_A = 0 \]

Points of Contraflexure

Consider a section at a distance \( x \) where BM is changing its sign as shown in Fig. 3.49. From similar triangles, we have

\[ \frac{45}{x} = \frac{37.5}{(1.5 - x)} \]

\[ x = 0.818 \text{m} \]

The Points of contraflexure are located at 3.818m and 4.5m from the left support.

3.10. A beam ABCDE is 12m long simply supported at points B and D. Spans AB=DE=2m is overhanging. BC=CD=4m. The beam supports a udl of 10kN/m over AB and 20kN/m over CD. In addition it also supports concentrated load of 10kN at E and a clockwise moment of 16kNm at point C. Sketch BMD and SFD. (Aug 05)

The reactions can be obtained from the conditions of equilibrium.

\[ \sum V_A = 0; \quad R_B + R_D = 10 \times 2 + 20 \times 4 + 10 = 110 \text{kN} \quad \text{(01)} \]

Taking moment about B,

\[ \sum M_B = 0; \quad 8R_D + (10 \times 2) \left( \frac{2}{2} \right) = 10 \times 10 + (20 \times 4) \left( 4 + \frac{4}{2} \right) + 16 \text{ or } R_D = \frac{576}{8} = 72 \text{kN} \]

Similarly taking moment about D,

\[ \sum M_D = 0; \quad 8R_B + 10 \times 2 + 16 = (10 \times 2) \left( 8 + \frac{2}{2} \right) + (20 \times 4) \left( \frac{4}{2} \right) \text{ or } R_B = \frac{304}{8} = 38 \text{kN} \]

Check

Substituting in Eq. 01, we have \( R_B + R_D = 38 + 72 = 110 \text{ kN} \) (O.K.)

Zero Shear Force
Consider a section at a distance $x$ where Shear Force is zero as shown in Fig. 3.50.

From similar triangles, we have

\[ \frac{12}{x} = \frac{68}{4-x} \quad \text{or} \quad x = 0.6 \text{m} \]

**Bending Moment Values**

\[ M_E = 0 \]

\[ M_D = -10 \times 2 = -20 \text{kN} \]

\[ M_{C_R} = 72 \times 4 - 10 \times 6 - (20 \times 4) \left( \frac{4}{2} \right) = 68 \text{kNm} \]

\[ M_{C_L} = 68 - 16 = 52 \text{kNm} \quad \text{(From right side forces)} \]

\[ M_{C_L} = 38 \times 4 - (10 \times 2) \left( \frac{4 + \frac{2}{2}}{2} \right) = 52 \text{kNm} \quad \text{(From left side forces)} \]

\[ M_B = -(10 \times 2) \left( \frac{2}{2} \right) = -20 \text{kNm} \]

\[ M_A = 0 \]

Bending Moment at zero Shear Force will be either Maximum or Minimum.
\[ M_x = 72 \times (4 - x) - 10(2 + 4 - x) \times \frac{20 \times (4 - x)^2}{2} = 72 \times (4 - 0.6) - 10(2 + 4 - 0.6) - 10(4 - 0.6)^2 = 75.2 \text{kNm} \]

**Point of Contraflexures**

Consider a section at a distance \( z \) where Bending Moment is zero as shown in Fig. 3.49. From similar triangles, we have

\[
\frac{20}{z} = \frac{52}{4 - z} \quad \text{and} \quad z = 1.1 \text{m}
\]

Bending Moment at Section \( y \) from point D is zero and can be written as

\[
M_y = 72 \times y - 10(2 + y) - \frac{20 \times y^2}{2} = 0
\]

\[
= 72 \times y - 10(2 + y) - 10 \times y^2 = 62y - 10y^2 - 20 = 0 \quad \text{and} \quad y = 0.341 \text{m}
\]

3.11. Draw the Shear Force and Bending Moment Diagrams for the beam shown in Fig. 3.50. Locate the point of contraflexure if any. (Feb 04)

The reactions can be obtained from the conditions of equilibrium.

\[ \Sigma V_A = 0; \quad R_A + R_B = (10 \times 5) + 80 + 80 + (16 \times 2.5) = 250 \text{kN} \]

Taking moment about A,

\[
\Sigma M_A = 0; 12.5R_B = (10 \times 5) \left( \frac{5}{2} \right) + 80 \times 5 + 80 \times 7.5 + (16 \times 2.5) \left( \frac{12.5 + 2.5}{2} \right) \]

\[
R_B = \frac{1675}{12.5} = 134 \text{kN}
\]

Similarly taking moments about B,

\[
\Sigma M_B = 0; 12.5R_A + (16 \times 2.5) \left( \frac{2.5}{2} \right) = (10 \times 5) \left( \frac{7.5 + 5}{2} \right) + 80 \times 7.5 + 80 \times 5 =
\]

\[
R_A = \frac{1450}{12.5} = 116 \text{kN}
\]

**Check**

Substituting in Eq. 01, we have \( R_A + R_B = 116 + 134 = 250 \text{ kN} \) (O.K.)

**Bending Moment Values**

\[ M_E = 0 \]

\[ M_D = - (16 \times 2.5) \left( \frac{2.5}{2} \right) = -50 \text{ kNm} \]

\[ M_C = 134 \times 5 - (16 \times 2.5) \left( \frac{5 + 2.5}{2} \right) = 425 \text{ kNm} \]

\[ M_B = 116 \times 5 - (10 \times 5) \left( \frac{5}{2} \right) = 455 \text{ kNm} \]

\[ M_A = 0 \]
Point of Contraflexure

Consider a section at a distance $y$ from the right support where Bending Moment is zero as shown in Fig. From similar triangles, we have

$$\frac{50}{y} = \frac{425}{(5-y)} \quad \text{and} \quad z = 0.526m$$
3.12. From the given shear force diagram shown in the Fig. 3.50, develop the load intensity diagram and draw the corresponding bending moment diagram indicating the salient features. (Jan 08)

The vertical lines in Shear force diagram represent vertical load, horizontal lines indicate generally no load portion, inclined line represents udl and parabola indicates uniformly varying load.

To generate load intensity diagram, the computations are shown in Fig. 3.50. The vertical line from the horizontal line below the line indicates negative value and vice versa. To check whether the applied moments are there in the loading diagram, we can take algebraic sum of moments of all the loads about any point and if there is a residue from the equation it indicates the applied moment in the opposite rotation to be applied anywhere on the beam.

**Check**

Taking Moments about B, we have

\[ \Sigma M_B = 0; \ 40 \times 3 + 90 \times 8 - 20 \times 10 - (20 \times 8) \left( \frac{8}{2} \right) = 0 \]

**Note:** Hence there is no applied moment or couple and if there is any residue from the equation like \( +M \) kNm then there is an applied moment of \( M \) kNm clockwise and vice versa.

**Bending Moment Values**

\[ M_D = 0 \]

\[ M_C = -20 \times 2 = -40 \text{ kNm} \quad \text{(Negative due to hogging moment)} \]

\[ M_B = -40 \times 3 = -120 \text{ kNm} \quad \text{(Negative due to hogging moment)} \]

\[ M_A = 0 \]

Maximum Bending Moment occurs at zero shear force which is located at a distance \( x \) from the left support as shown in Fig. From similar triangles, we have

\[ \frac{90}{x} = \frac{70}{(8-x)} \quad \text{or} \quad x = 4.5 \text{m} \]

Maximum Bending Moment at the section \( x \) is

\[ M_x = 130x - 40 \times (3 + x) - \frac{20x^2}{2} = 130x - 40 \times (3 + x) - x^2 \]

\[ = 130 \times 4.5 - 40 \times (3 + 4.5) - 4.5^2 = 264.75 \text{kNm} \]
3.13. A beam 6m long rests on two supports with equal overhangs on either side and carries a uniformly distributed load of 30kN/m over the entire length of the beam as shown in Fig. 3.51. Calculate the overhangs if the maximum positive and negative bending moments are to be same. Draw the SFD and BMD and locate the salient points. (Jan 07)

The reactions can be obtained from the conditions of equilibrium.

As the loading is symmetrical $R_A = R_B$ and hence

$$\sum V_A = 0; R_B + R_C = 2R_B = 2R_C = 30 \times (6 + 2a)$$

$$R_B = R_C = \frac{30 \times 6}{2} = 90\text{kN}$$

Bending Moment at any section $x$ from the left end is given by

$$M_x = 90(x-a) - \frac{30x^2}{2} \text{ or } 90(x-a) - 15x^2$$

From the given problem, maximum positive and negative bending moments are to be same, which occurs at zero shear force sections. From the above loading diagram, it can be seen that the zero shear force occurs at support and at centre (as the loading...
is symmetrical). Hence substituting \( x = \alpha \) and \( 3 \), we get maximum +ve and —ve Bending Moment.

\[
M_B = -15\alpha^2
\]

\[
M_E = 90(3-\alpha) - 15(3)^2 = 90(3-\alpha) - 135
\]

Equating the absolute values of above two equations, we have

\[
15\alpha^2 = 90(3-\alpha) - 135 \text{ or } \alpha^2 + 6\alpha - 9 = 0 \text{ and } \alpha = 1.243 \text{m}
\]

**Bending Moment Values**

\[
M_D = 0
\]

\[
M_C = -\frac{30 \times 1.243^2}{2} = -23.176 \text{kNm}
\]

\[
M_B = -\frac{30 \times 1.243^2}{2} = -23.176 \text{kNm}
\]

\[
M_A = 0
\]

\[
M_E = 90(3 - 1.243) - \frac{30 \times 1.243^2}{2} = 23.176 \text{kNm}
\]

**Points of Contraflexure:**

\[
M_x = 90(x - 1.243) - 15x^2 = 6(x - 1.243) - x^2 \Rightarrow 0 \text{ or } x = 1.76 \text{m and 4.24m}
\]

The points of contraflexure are at 1.76m and 4.24m from left end.

3.14. Draw the Shear Force and Bending Moment Diagram for a simply supported beam subjected to uniformly varying load shown in Fig. 3.52.

The trapezoidal load can be split into udl and uvl (triangular load) as shown in Fig. 3.43.

\[
\sum V_A = 0; \quad R_A + R_B = (15 \times 6) + \left(\frac{1}{2}\right)(10 \times 6) = 120 \text{kN} \quad 01
\]

Taking moment about A,

\[
\sum M_A = 0; \quad 6R_B = (15 \times 6)\left(\frac{5}{2}\right) + \left(\frac{1}{2}\right)(10 \times 6)\left(\frac{2}{3}\times 6\right) \text{ or } R_B = \frac{390}{6} = 65 \text{kN}
\]

Similarly taking moments about B,

\[
\sum M_B = 0; \quad 6R_A = (15 \times 6)\left(\frac{5}{2}\right) + \left(\frac{1}{2}\right)(10 \times 6)\left(\frac{3}{6}\right) + 80 \times 7.5 + 80 \times 5 \text{ or } R_A = \frac{330}{6} = 55 \text{kN}
\]

**Check**

Substituting in Eq. 01, we have \( R_A + R_B = 55 + 65 = 120 \text{ kN} \) (O.K.)
Consider a section $x$ at a distance $x$ from the left support as shown.

The intensity of $uvl$ at $x$ is given by

$$\omega_x = \left(10 \times \frac{x}{6}\right) = 1.67x \text{kN/m}$$

$$V_x = 55 - 15x - \frac{1.67x^2}{2} = 55 - 15x - \frac{5}{6}x^2 \text{kN}$$

At $x = 2\text{m}$, $V_2 = 55 - 15 \times 2 - \frac{5}{6} \times 2^2 = 21.67\text{kN}$

At $x = 3\text{m}$, $V_3 = 55 - 15 \times 3 - \frac{5}{6} \times 3^2 = 2.5\text{kN}$

At $x = 5\text{m}$, $V_5 = 55 - 15 \times 5 - \frac{5}{6} \times 5^2 = -40.83\text{kN}$

Zero Shear Force = $V_o = 55 - 15 \times x - \frac{5}{6} \times x^2 = 0$ solving we get, $x = 3.124\text{m}$
Bending Moment Values

Bending Moment Equation at any section \( x \) from left support

Consider a section \( x \) at a distance \( x \) from the left support as shown.

\[
M_x = 55x - \frac{15x^2}{2} - \left( \frac{1.67x^2}{2} \right) \left( \frac{x}{3} \right) = 55x - 7.5x^2 - \frac{5}{18} x^3 \text{kNm}
\]

\[
M_x = 55 \times -7.5x^2 - \frac{5}{18} x^3
\]

\[
M_B = 0
\]

\[
M_A = 0
\]

Maximum Bending Moment occurs at SF = 0, i.e. \( x = 3.124 \text{m} \)

\[
M_x = 55 \times 3.124 - 7.5 \times 3.124^2 - \left( \frac{5}{18} \right) \times 3.124^3 = 90.156 \text{kNm}
\]

3.15. A beam ABCD 20m long is loaded as shown in Fig. 3.53. The beam is supported at B and C with a overhang of 2m to the left of B and a overhang of \( a \)m to the right of support C. Determine the value of \( a \) if the midpoint of the beam is point of inflexion and for this alignment plot BM and SF diagrams indicating the important values.

The reactions can be obtained from the conditions of equilibrium.

\[
\sum V_A = 0; \quad R_B + R_C = 5\omega + \omega \times 20 = 25\omega \text{kN} \quad \text{(01)}
\]

Taking moment about B,

\[
\sum M_B = 0; (18 - a)R_C + (5\omega) \times 2 + \left( \frac{\omega \times 2^2}{2} \right) = \left( \omega \times \frac{(20 - 2)^2}{2} \right)
\]

\[
(18 - a)R_C = 150\omega \text{ or } R_C = \frac{150\omega}{(18 - a)}
\]

Similarly taking moment about C,

\[
\sum M_C = 0; (18 - a)R_B + \left( \frac{\omega a^2}{2} \right) = (5\omega)(20 - a) + \left( \frac{\omega \times (20 - a)^2}{2} \right)
\]

\[
(18 - a)R_B = 300\omega - 25\omega a \text{ or } R_B = \frac{\omega(300 - 25a)}{(18 - a)}
\]

Check

Substituting in Eq. 01, we have

\[
R_B + R_C = \frac{150\omega}{(18 - a)} + \frac{\omega(300 - 25a)}{(18 - a)} = 25\omega \text{ (O.K.)}
\]
**Point of contraflexure**

Consider a section at a distance \( x \) from left support as shown in Fig. 3.53. Bending moment at this section is given by

\[
M_x = R_B \times (x-2) - 5\omega x - \frac{\omega x^2}{2} = \bigg[ \frac{\omega (300-25a)}{18-a} \bigg] \times (x-2) - 5\omega x - \frac{\omega x^2}{2}
\]

From the given data, this is zero at \( x = 10 \)m. Hence

\[
\bigg[ \frac{\omega (300-25a)}{18-a} \bigg] \times (x-2) - 5\omega x - \frac{\omega x^2}{2} = 0
\]

\[
\bigg[ \frac{(300-25a)}{18-a} \bigg] \times 8 - 5 \times 10 - \frac{10^2}{2} = 0
\]

\[
\bigg[ \frac{(300-25a)}{18-a} \bigg] = 12.5
\]

\[
300-25a = 225 - 12.5a \quad \text{or} \quad a = 6 \text{m}
\]

\[
R_B = \frac{\omega (300-25a)}{18-a} = \frac{\omega (300-25 \times 6)}{18-6} = 12.5\omega
\]

\[
R_C = \frac{150\omega}{18-a} = \frac{150\omega}{18-6} = 12.5\omega
\]

**Zero Shear Force**

Consider a section at a distance \( y \) where Shear Force is zero as shown in Fig. 3.53. From similar triangles, we have

\[
\frac{5.5}{y} = \frac{6.5}{12-y} \quad \text{or} \quad y = 5.5 \text{m}
\]

**Bending Moment Values**

\( M_D = 0 \)

\[
M_C = -\omega \times \frac{6^2}{2} = -18\omega
\]

\[
M_B = -5\omega \times 2 - \omega \times \frac{2^2}{2} = -12\omega
\]

\( M_A = 0 \)

\[
M_E = 12.5\omega \times 5.5 - 5\omega \times (5.5+2) - \frac{\omega (5.5+2)^2}{2} = 3.125\omega
\]

Another point of contraflexure is

\[
M_x = \bigg[ \frac{\omega (300-25 \times 6)}{18-6} \bigg] \times (6-2) - 5\omega \times 6 - \frac{\omega 6^2}{2}
\]
3.16 For the beam AC shown in Fig. 3.54, determine the magnitude of the load $P$ acting at C such that the reaction at supports A and B are equal and hence draw the Shear force and Bending moment diagram. Locate points of contraflexure. (July 08)

The reactions can be obtained from the conditions of equilibrium.

$$\sum V_A = 0; \quad R_A + R_B = 45 \times 4 + P \quad \text{(01)}$$

From the given data, $R_A = R_B$ and substituting in Eq. 01, $2R_A = 2R_B = 180 + P$

Taking moment about A,

$$\sum M_A = 0; \quad 6R_B = 7P + (45 \times 4) \left(\frac{4}{2}\right) + 30 \text{ or } 6R_B = 7P + 390$$

Substituting from Eq. 01,

$$3(180 + P) = 7P + 390 \text{ or } P = 37.5\text{kN}$$

**Check**

Similarly taking moments about B,

$$\sum M_B = 0; \quad 6R_A + P \times 1 + 30 = (45 \times 4) \left(\frac{2 + \frac{4}{2}}{2}\right)$$

$$6R_A = 690 - P$$

Substituting from Eq. 01, $3(180 + P) = 690 - P$ or $P = 37.5\text{kN}$

Hence O.K.
\[ 2R_A = 2R_B = 180 + 37.5 = 217.5 \text{kN} \]
\[ R_A = R_B = 108.75 \text{kN} \]

**Zero Shear Force**

Consider a section at a distance \( x \) where Shear Force is zero as shown in Fig. 3.54.

From similar triangles, we have

\[ \frac{108.75}{x} = \frac{71.25}{4-x} \quad \text{or} \quad x = 2.417 \text{m} \]

**Bending Moment Values**

\[ M_C = 0 \]
\[ M_B = -37.5 \times 1 = -37.5 \text{kNm} \] (Negative because Sagging)
\[ M_{Dx} = 108.75 \times 2 - 37.5 \times 3 = 105 \text{kNm} \]
\[ M_{Dx} = 108.75 \times 4 - (45 \times 4) \left( \frac{4}{2} \right) = 75 \text{kNm} \] (From left side forces)
\[ M_{Dx} = 105 - 30 = 75 \text{kNm} \] (From Right side forces)
\[ M_A = 0 \]

Maximum Bending moment occurs at zero shear force, i.e. at \( x = 2.417 \)

\[ M_x = 108.75 \times -\frac{45 \times x^2}{2} = 108.75 \times 2.417 - \frac{45 \times 2.417^2}{2} = 131.41 \text{kNm} \]

---

Fig. 3.54
3.16. Draw the bending moment and shear force diagrams for a prismatic simply supported beam of length \( L \), subjected to a clockwise moment \( M \) at the centre of the beam and a uniformly distributed load of intensity \( q \) per unit length acting over the entire span. (Jan 09)

The reactions can be obtained from the conditions of equilibrium.

\[ \Sigma V_A = 0; \quad R_A + R_B = q \times L \text{kN} \quad \text{(01)} \]

Taking moment about A,

\[ \Sigma M_A = 0; \quad R_B \times L + M = \frac{q \times L^2}{2} \]

\[ R_B = \frac{q \times L}{2} - \frac{M}{L} \]

Similarly taking moment about B,

\[ \Sigma M_B = 0; \quad R_A \times L = \frac{q \times L^2}{2} + M \]

\[ R_A = \frac{q \times L}{2} + \frac{M}{L} \]

**Check**

Substituting in Eq. 01, we have

\[ R_A + R_B = \frac{q \times L}{2} + \frac{M}{L} + \frac{q \times L}{2} + \frac{M}{L} = qL \text{ (O.K.)} \]

**Zero Shear Force**

Consider a section at a distance \( x \) where Shear Force is zero as shown in Fig. 3.55. From similar triangles, we have

\[ \left[ \frac{qL}{2} + \frac{M}{L} \right] \frac{x}{L} = \left[ \frac{qL}{2} + \frac{M}{L} \right] \frac{(L - x)}{L} \quad \text{or} \quad x = \left[ \frac{L + \frac{M}{qL}}{2} \right] \]

\[ = \frac{qL^2}{8} + \frac{M}{2} + \frac{M^2}{2qL^2} \]

**Bending Moment Values**

\( M_B = 0 \)

\( M_A = 0 \)

Bending Moment at zero Shear Force will be either Maximum or Minimum.

\[ M_x = \left[ \frac{q \times L}{2} + \frac{M}{L} \right] x - \frac{q}{2} x^2 = \left[ \frac{q \times L}{2} + \frac{M}{L} \right] \left[ \frac{L}{2} + \frac{M}{qL} \right] - \frac{q}{2} \left[ \frac{L}{2} + \frac{M}{qL} \right]^2 \]

\[ M_{\text{max}} = \frac{qL^2}{8} + \frac{M}{2} + \frac{M^2}{2qL^2} \]
3.17. For the loaded beam shown in Fig. 3.56, Draw the Shear Force and Bending Moment Diagram. Find and locate the Maximum +ve and —ve Bending Moments. Also locate the Point of Contraflexures. Detail the procedure to draw the SFD and BMD. (July 09)

It can be seen the loading is symmetrical and the Reactions are equal. From the conditions of equilibrium

\[ \sum V_A = 0; \]

\[ R_A + R_B = 2R_A = 2R_B = 2 \times \left( 20 + \frac{1}{2} \times (10 \times 2) \right) + 20 \times 2 \text{ or } R_A = R_B = 50 \text{kN} \]

**Bending Moment Values**

\[ M_F = M_C = 0 \]

\[ M_A = M_B = -20 \times 2 = -40 \text{kNm} \]

\[ M_{D_L} = M_{F_a} = 50 \times 2 - 20 \times 4 - \left( \frac{1}{2} \times 10 \times 2 \right) \left( \frac{2 \times 2}{3} \right) = 6.67 \text{kNm} \]

\[ M_{D_e} = M_{F_e} = 6.67 - 10 = -3.33 \text{kNm} \]

**Maximum Bending Moment and Points of Contraflexure**

**Maximum Bending Moment**

Bending Moment at any section \( x \) in the region DE is given by

\[ M_x = 50x - 20(x + 2) - \left( \frac{1}{2} \times 10 \times 2 \right) \left( x - \frac{2}{3} \right) - 20 \left( \frac{x - 2}{2} \right) - 10 \]

The Maximum bending moment occurs at zero shear force.

i.e. \( x = (5 - 2) = 3 \text{ m} \)

\[ M_x = 50 \times 3 - 20(3 + 2) - \left( \frac{1}{2} \times 10 \times 2 \right) \left( 3 - \frac{2}{3} \right) - 20 \left( \frac{3 - 2}{2} \right) - 10 = 6.67 \text{kNm} \]

**Shear Force Diagram**

1. Draw a horizontal line \( C_1F_2 \) equal to the length of the beam 10m to some scale, under the beam CF as shown.
2. Start the Shear force line from left extreme edge \( C_1 \). Draw \( C_1C_2 \) under the vertical load 20kN acting at C downward equal to some scale. To start with, the shear force at \( C_1=0 \) and at \( C_2 \), the Shear force \( = 0 - 20 \) (-ve as it is acting downward) = -20 kN.
3. There is no load in the region CA and hence under this region, the shear force line \( C_2A_1 \) will be a horizontal line parallel to beam axis.
4. At A, there is a reaction $R_A$ which is treated as vertical load = 50kN and hence the shear force line $A_1A_2 = 50kN$ to some scale and the shear force at $A_2 = -20 + 50 (+$ as it is upward) $= +30$ kN.

5. There is a uvl in the region AD and the shear force line will be a parabola in this region. The parabola will be tangential to vertical at $A_2$ as there is relatively higher load intensity at A and will be parallel to horizontal at $D_1$ as the load intensity is lesser at D. Hence the curve is sagging. The vertical distance from $A_2$ to $D_1$ is equal to the total load equivalent to uvl, i.e. $\frac{1}{2} \times 10 \times 2 = 10kN$ and the shear force at $D_1 = 30 - 10$ (- as it is downward) $= +20$ kN.

6. There is an udl in the region DE and hence the shear force line is inclined from $D_1$ to $E_1$. The vertical distance from $D_1$ to $E_1$ is equal to the total load equivalent to udl, i.e. $20 \times 2 = 40kN$ and the shear force at $E_1 = 20 - 40$ (- as it is downward) $= -20$ kN.

7. There is a uvl in the region EB and the shear force line will be a parabola in this region. The parabola will be tangential to horizontal at $E_1$ as there is relatively lower load intensity at E and will be parallel to vertical at $B_1$ as the load intensity is higher at B. Hence the curve is hogging. The vertical distance from $E_1$ to $B_1$ is
equal to the total load equivalent to uvl, i.e. \( \frac{1}{2} \times 10 \times 2 = 10 \text{kN} \) and the shear force at \( B_1 = -20 - 10 \) (as it is downward) = -30 kN.

8. At B, there is a reaction \( R_B \) which is treated as vertical load = 50kN and hence the shear force line \( B_1B_2 = 50 \text{kN} \) to same scale and the shear force at \( B_2 = -30 + 50 \) (+ as it is upward) = +20 kN.

9. There is no load in the region BF and hence under this region, the shear force line \( B_2F_1 \) will be a horizontal line parallel to beam axis.

10. Draw \( F_1F_2 \) under the vertical load 20kN acting at F downward equal to same scale. The shear force at \( F_2 = 20 - 20 = 0 \) (-ve as it is acting downward). Note that for the Shear Force Diagram to be precise, the shear force line must finally join the horizontal axis. If there is any shortage or surplus, the shear force diagram must be redrawn.

11. The portion of the shear force diagram above the horizontal axis is +ve and the one below the horizontal axis is –ve.

**Bending Moment Diagram**

1. The Bending Moment is zero at the extreme edges of the beam unless there is an applied moment or couple acting at the edges, Hence the Moment at \( C = M_C = 0 \) i.e. at \( C_3 \).

2. The Bending moment at A is -40 kNm and hence the bending moment line is inclined under the no load portion CA (it can be either horizontal or inclined depending on the moments at the corresponding ends of the portion in the region).

3. The region AD has a uvl and hence the bending moment line will be a cubic parabola (the index of BM is always one more than SF at any section and hence bending moment line is inclined under horizontal shear force line, parabola under inclined shear force line and cubic parabola under parabolic shear force line). The parabola joins the bending moment values at \( A_3 \) is -40kNm and at \( D_3 \) is +6.67kNm (Bending moment to the left of D). The cubic parabola will be parallel to vertical at \( A_3 \) and parallel to horizontal at \( D_3 \) as the absolute value of shear force at \( A_2 = 30 \text{kN} \) (more) compared to that at \( D_1 = 20 \text{kN} \).

4. The bending moment line is always a vertical line under the applied moment or couple. There is an clockwise applied moment of 10kNm acting at D and hence it is hogging. The vertical line \( D_3D_4 \) is downward and equal to the applied
moment to the same scale = 10kNm. The Bending moment value at $D_4 = -3.37$ kNm

5. The region DG is acted upon by udl, the shear force line is inclined and the bending moment line will be a parabola from $D_4$ to $G_3$. The parabola is joining Bending moment at $D_4 = -3.37$ to that at $G_3 = 6.67$kNm. The bending moment line will be tangential to vertical at $D_4$ and tangential to horizontal at $G_3$ as the shear force at $D_1 = 20$kN which is relatively higher than at $G$ which is 0.

6. The region GE is acted upon by udl, the shear force line is inclined and the bending moment line will be a parabola from $G_3$ to $E_3$. The parabola is joining Bending moment at $G_3 = 6.67$ to that at $E_3 = -3.37$kNm. The bending moment line will be tangential to horizontal at $G_3$ and tangential to vertical at $E_3$ as the absolute shear force at $G = 0$kN which is relatively lesser than at $E_3 = 3.37$kNm.

7. There is an anti-clockwise applied moment of 10kNm acting at E and hence it is sagging. The vertical line $E_3E_4$ is upward and equal to the applied moment to the same scale = 10kNm. The Bending moment value at $E_4 = 6.67$ kNm

8. The region EB has a uvl and hence the bending moment line will be a cubic parabola. The parabola joins the bending moment values at $E_4$ is 6.67kNm (Bending moment to the right of E) and at $B_3$ is -40kNm. The cubic parabola will be tangential to horizontal at $E_4$ and parallel to vertical at $B_3$ as the absolute value of shear force at $E_1 = 20$kN (less) compared to that at $B_1 = 30$kN.

9. The Bending moment at B is -40 kNm and hence the bending moment line is inclined under the no load portion BF to join the horizontal axis at $F_3$ where the bending moment is zero.
Question paper problems of Mechanical Engineering 06ME34

3.19 Draw the shear force and bending moment diagrams for a overhanging beam shown in Fig. 3.57. Find and locate the points of contraflexure. (July 09)

The reactions can be obtained from the conditions of equilibrium.

\[ \sum V_A = 0; \quad R_B + R_D = 10 \times 2 + 40 + \frac{1}{2} \times 20 \times 2 + 20 = 100 \text{kN} \quad (01) \]

Taking moment about B,

\[ \sum M_B = 0; 4R_D + 10 \times 2 \left( \frac{2}{2} \right) = 40 \times 2 + \left( \frac{1}{2} \times 20 \times 2 \right) \left( \frac{2 \times 2}{3} \right) + 20 \times 6 \]

\[ R_D = \frac{246.67}{4} = 61.67 \text{kN} \]

Similarly taking moment about D,

\[ \sum M_D = 0; 4R_B + (20 \times 2) = (10 \times 2) \left( \frac{4 + 2}{2} \right) + 40 \times 2 + \left( \frac{1}{2} \times 20 \times 2 \right) \left( \frac{2 \times 1}{3} \right) \]

\[ R_B = \frac{153.33}{4} = 38.33 \text{kN} \]

Check

Substituting in Eq. 01, we have \( R_B + R_D = 38.33 + 61.67 = 100 \text{ kN} \) (O.K.)

Bending Moment Values

\[ M_E = 0 \]

\[ M_D = -20 \times 2 = -40 \text{kN} \]

\[ M_C = 61.67 \times 2 - 20 \times 4 - \left( \frac{1}{2} \times 20 \times 2 \right) \left( \frac{2 \times 2}{3} \right) = 16.67 \text{kNm} \]

\[ M_B = -(10 \times 2) \left( \frac{2}{2} \right) = -20 \text{kNm} \]

\[ M_A = 0 \]

Points of Contraflexures

Bending moment at any section \( x \) from the left support

For region CD

\[ M_x = 38.33x - (10 \times 2)(x + 1) - 40(x - 2) - \left( \frac{1}{2} \times 20 \times \left( \frac{x - 2}{2} \right) \right) \left( \frac{2}{3} \right)(x - 2) \]

For Point of contraflexure, \( M_x = 0 \), solving, we get \( x = 2.713 \text{m} \)

For region BC \( M_x = 38.33x - (10 \times 2)(x + 1) \)

For Point of contraflexure, \( M_x = 0 \), solving, we get \( x = 1.09 \text{m} \)
From second method, consider the similar triangles between BC,

\[ \frac{x}{20} = \frac{2-x}{16.67} \text{ or } x = 1.09 \text{m} \]

For the beam shown in Fig. 3.58, draw the shear force and bending moment diagram and locate the Point of contraflexure if any. (Jan 09)

The reactions can be obtained from the conditions of equilibrium.

\[ \sum V_A = 0; \ R_B + R_D = 10 \times 2 + 30 + 40 + 20 \times 4 = 170 \text{kN} \quad (01) \]

Taking moment about B,

\[ \sum M_B = 0; 6R_D = (10 \times 2) \left(\frac{2}{4} + 30 \times 2 + 40 \times 4 + (20 \times 4) \left(\frac{4 + \frac{4}{2}}{2}\right) \right) \text{ or } R_D = \frac{720}{6} = 120 \text{kN} \]

Similarly taking moment about D,

\[ \sum M_D = 0; 6R_B = (10 \times 2) \left(\frac{4 + \frac{2}{2}}{2} + 30 \times 4 + 40 \times 2 \right) \text{ or } R_B = \frac{300}{6} = 50 \text{kN} \]

Check

Substituting in Eq. 01, we have \( R_B + R_D = 50 + 120 = 170 \text{ kN} \) (O.K.)

Bending Moment Values

\[ M_E = 0 \]

\[ M_D = -(20 \times 2) \left(\frac{2}{2}\right) = -40 \text{kN} \]
\[ M_C = 120 \times 2 - (20 \times 4) \left( \frac{4}{2} \right) = 80 \text{kNm} \]
\[ M_B = 50 \times 2 - (10 \times 2) \left( \frac{2}{2} \right) = 80 \text{kNm} \]
\[ M_A = 0 \]

**Points of Contraflexures**

*Bending moment* at any section \( x \) from the left support

For region CD

\[
M_x = 38.33x - (10 \times 2)(x+1) - 40(x-2) - \left( \frac{1}{2} \times 20 \times \frac{(x-2)^2}{2} \right) - \left( \frac{2}{3} \right)(x-2)
\]

For Point of contraflexure, \( M_x = 0 \), solving, we get \( x = 2.713 \text{m} \)

For region BC \( M_x = 38.33x - (10 \times 2)(x+1) \)

For Point of contraflexure, \( M_x = 0 \), solving, we get \( x = 1.09 \text{m} \)

From second method, consider the similar triangles between BC,

\[
\frac{x}{20} = \frac{2-x}{16.67} \quad \text{or} \quad x = 1.09 \text{m}
\]

3.21 For the beam shown in Fig. 3.59, obtain SFD and BMD. Locate Points of contraflexure, if any. (July 09)

The reactions can be obtained from the conditions of equilibrium.
\[ \sum V_A = 0; \quad R_B + R_D = 5 \times 8 + 50 = 90 \text{kN} \]  

Taking moment about B,
\[ \Sigma M_B = 0; 16R_B + 120 = (5 \times 8) \left( \frac{8}{2} \right) + 50 \times 12 + 160 \text{ or } R_B = \frac{800}{16} = 50 \text{kN} \]

Similarly taking moment about D,
\[ \Sigma M_D = 0; 16R_D + 160 = (5 \times 8) \left( \frac{8}{2} \right) + 50 \times 4 + 120 \text{ or } R_D = \frac{640}{16} = 40 \text{kN} \]

Check

Substituting in Eq. 01, we have \( R_B + R_D = 40 + 50 = 90 \text{ kN} \) (O.K.)

**Bending Moment Values**

- \( M_{DR} = 0 \)
- \( M_{AR} = -120 \text{kNm} \)
- \( M_C = 50 \times 4 - 160 = 40 \text{kNm} \)
- \( M_B = 50 \times 8 - 50 \times 4 - 160 = 40 \text{kNm} \)
- \( M_{AL} = -160 \text{kNm} \)

\[ M_{AL} = 0 \]

Points of Contraflexures

*Bending moment* at any section \( x \) from the left support

For region AB

\[ M_x = 40x - \left( \frac{5x^2}{2} \right) - 120 = 0 \text{ or } x = 4 \text{m} \]
Point of contraflexure is \( x = 4 \) m from the left support.

For region CD \( M_y = 50y - 160 = 0 \) or \( y = 3.2 \) m

For Point of contraflexure is \( y = 3.2 \) m from the right support.

From second method, consider the similar triangles between CD

\[ \frac{y}{160} = \frac{4 - y}{40} \] or \( y = 3.2 \) m

A beam ABCD, 8m long has supports at A and at C which is 6m from point A. The beam carries a UDL of 10kN/m between A and C. At point B a 30kN concentrated load acts 2m from the support A and a point load of 15kN acts at the free end D. Draw the SFD and BMD giving salient values. Also locate the point of contra-flexure if any. (14)(July 2015)

From the conditions of equilibrium, we have algebraic sum of vertical forces to be zero.

\[ \uparrow + \Sigma V = 0; \quad R_A + R_C = 30 + 15 + (10)(6) = 105 \text{ kN} \ (\uparrow) \]

Algebraic sum of moments about any point is zero. Taking moments about A, we get

\[ \Sigma M_A = 0; \quad 6R_C = (30)(2) + (15)(8) + \left[ (10)(6) \right] \left( \frac{6}{2} \right) = 360 \text{ kN} \]

\[ R_C = 60 \text{ kN} \ (\uparrow) \]

Taking moments about C, we get

\[ \Sigma M_C = 0; \quad 6R_A + (15)(2) = (30)(4) + \left[ (10)(6) \right] \left( \frac{6}{2} \right) = 270 \text{ kN} \]

\[ R_A = 45 \text{ kN} \ (\uparrow) \]

Check: \( R_A + R_C = 45 + 60 = 105 \text{ kN} \ (\uparrow) \)

Shear Force Diagram can be directly drawn. Bending Moment values:

Unless there are end moments of the beam, the Moments are zero at ends of the beam.

\[ M_A = 0 \text{ and } M_D = 0 \]

\[ M_B = (45)(2) - \left[ (10)(2) \right] \left( \frac{2}{2} \right) = 70 \text{ kNm} \]

\[ M_C = -(15)(2) = -30 \text{ kNm} \]
To locate the point of contra-flexure where the bending moment changes its sign, consider the section to be at a distance $x$ towards left of the right support as shown. The bending moment at the section is given by

$$M_x = 60x - (15)(2 + x) - (10)(x)\left(\frac{x}{2}\right) \Rightarrow 0$$

$$45x - 30 - 5x^2 = 0$$

Solving, $x = 0.725m$ and $8.275m$

Hence the point of contra-flexure is at $0.725m$ to left of right support.

To draw the Shear force and bending moment diagrams for the Fig. shown

From the conditions of equilibrium, we have algebraic sum of vertical forces to be zero.

$$\sum V = 0; \quad R_A + R_B = (15)(2) + 40 + (10)(2) = 90 \text{kN} \uparrow$$

From the conditions of equilibrium, we have algebraic sum of moments about any point is zero. Taking moments about A, we get

$$\sum M_A = 0; \quad 8R_B = [(15)(2)]\left[1 + \frac{2}{2}\right] + (40)(1 + 2 + 1) + [(10)(2)]\left[8 + \frac{2}{2}\right] = 400 \text{kN}$$

$$R_B = 50 \text{kN} \uparrow$$

Taking moments about B, we get
\[ \Sigma M_B = 0; \quad 8R_A + \left[ (10)(2) \right] \left( \frac{2}{2} \right) = (40)(4) + \left[ (15)(2) \right] \left( 4 + 1 + \frac{2}{2} \right) = 340 \text{ kN} \]

\[ R_A = 40 \text{ kN} (\uparrow) \]

Check: \( R_A + R_B = 40 + 50 = 90 \text{ kN} \ (\uparrow) \)

Shear Force Diagram can be directly drawn.

**Bending Moment values:**

Unless there are end moments of the beam, the Moments are zero at ends of the beam.

\[ M_A = 0 \text{ and } M_C = 0 \]

\[ M_D = (40)(1) = 40 \text{ kNm} \]

\[ M_E = (40)(3) - \left[ (15)(2) \right] \left( \frac{2}{2} \right) = 90 \text{ kNm} \]

\[ M_F = (40)(4) - \left[ (15)(2) \right] \left( 1 + \frac{2}{2} \right) = 100 \text{ kNm} \]

\[ M_B = -\left[ (10)(2) \right] \left( \frac{2}{2} \right) = -20 \text{ kNm} \]

To locate the point of contra-flexure where the bending moment changes its sign, consider the section to be at a distance \( x \) towards left of the right support as shown.
moment inclined line is crossing zero line as a straight line forming two alternate triangles which are similar. Hence using similar triangle properties

\[
\frac{4 - x}{x} = \frac{100}{20}
\]

Solving, \(x = 0.67\) m

Hence the point of contra-flexure is at 0.67 m to left of right support.

Draw Shear force and Bending moment Diagram for the beam shown in Fig.

From the conditions of equilibrium, we have algebraic sum of vertical forces to be zero.

\[\uparrow + \Sigma V = 0; \quad R_A + R_B = (20)(4) + 80 = 160 \text{ kN (\uparrow)}\]

\[\Sigma M_A = 0; \quad 8R_B = [(20)(4)]\left(\frac{4}{2}\right) + (80)(4 + 2) = 640 \text{ kN}

\[R_B = 80 \text{ kN (\uparrow)}\]
Algebraic sum of moments about any point is zero. Taking moments about A, we get

\[ \Sigma M_A = 0; \quad 8R_B = \left[ (20)(4) \right] \left( \frac{4}{2} \right) + (80)(4 + 2) = 640 \text{kN} \]

\[ R_B = 80 \text{kN} (\uparrow) \]

Taking moments about B, we get

\[ \Sigma M_B = 0; \quad 8R_A = \left[ (20)(4) \right] \left( 4 + \frac{4}{2} \right) + (80)(2) = 640 \text{kN} \]

\[ R_A = 80 \text{kN} (\uparrow) \]

Check: \( R_A + R_B = 80 + 80 = 160 \text{ kN} \) (\( \uparrow \))

Shear Force Diagram can be directly drawn.

**Bending Moment values:**

Unless there are end moments of the beam, the Moments are zero at ends of the beam.

\[ M_A = 0 \text{ and } M_B = 0 \]

\[ M_C = (80)(4) - \left[ (20)(4) \right] \left( \frac{4}{2} \right) = 160 \text{kNm} \]

\[ M_D = (80)(2) = 160 \text{kNm} \]